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Working Paper 2409

June 2024

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Was Keynes right? A reconsideration of the effect of a protective tariff under stagnation*

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June 26, 2024

Abstract

This paper first presents a dynamic model that features both real and monetary aspects of international trade and is capable of dealing with both full employment and secular unemployment. The model is then utilized to examine the effect of a tariff on the terms of trade, the trade pattern, real consumption and employment of labor. It is shown that with full employment in both countries, a tariff by the home country improves its terms of trade and increases its national welfare at the expense of the foreign country. These results however are reversed in the presence of unemployment in both countries. We also examine the asymmetric cases and calibrate our model to evaluate numerically the effect of large tariff changes. The main finding is that the tariff only worsens the economy when it is already stagnant.

Key Words: demand shortage, unemployment, tariffs, secular stagnation
***JEL* Classification**: E24, E31, F13, F41, J20

*We acknowledge financial support from the Joint Usage/Research Center at ISER (Osaka University) and KIER (Kyoto University), and JSPS KAKENHI Grants (20K01631, 20H05631, 21K13261, 22K01511).
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1 Introduction

The notion that a tariff can increase the levying country's welfare dates back to the writings of Robert Torrens and J. S. Mill during the 1820s (Irwin 1996). Although their demonstration centered on the classical price-specie-flow mechanism, the rigorous proof of the validity of their idea was not completed until the end of that century, when the reciprocal demand techniques and indifference curve analysis were invented by neoclassical economists such as Marshall and Edgeworth and applied to the barter model of trade under the assumption of perfect wage flexibility and full employment.[1]

If its economy has been stagnant with chronic unemployment, is it possible for the country to create jobs and increase income by imposing a tariff on imports? This question was first raised by none other than J.M. Keynes during the Great Depression. With his policy options severely curtailed by Great Britain's adherence to the gold standard, Keynes opted for the advocacy of a uniform tariff on all imports in order to combat the deflation ravaging the U.K. economy (Eichengreen 1984). Even today, stagnation is not a thing of the past. Japan, for example, has been trapped in a severe recession since the early 1990s, notwithstanding the Bank of Japan's efforts to resuscitate the ailing economy through an expansionary monetary policy (Ono 2010, 2019).

The main objective of this paper is thus to reexamine the question raised by Keynes: can a country pull itself out of the stagnation by levying a tariff on all its imports? To address this question, we first present a model that integrates real and monetary aspects of international trade. The real or barter model of trade builds on the Ricardian model extended by Dornbusch, Fischer and Samuelson (DFS for short, 1977). Thus, the model has two countries producing a continuum of goods with labor under linear technologies and trading according to the Ricardian comparative advantage. The model with a continuum of goods has the obvious advantage over the standard two-good model in that it enables us to study not only the effect of a tariff on industry output (internal margins) but also the effect on destruction and creation of industries in each country (external margins).[2]

To incorporate monetary variables into our analysis, the secular stagnation model developed by Ono (1994, 2001) for a closed economy is adapted to an international setting. Thus, the two models share three important features. First, consumers are assumed to maximize their lifetime utility by choosing real consumption and real money balances at each instant. Second, the marginal utility of real money balance holdings is assumed to be bounded below by a positive value, which implies the possibility of a liquidity trap and chronic unemployment. Third, nominal wages are assumed to adjust sluggishly when there is unemployment although perfectly flexible under full employment. All other markets including one for international assets clear instantaneously.[3]

[1]See Kaldor (1940) for the first demonstration of a welfare-improving tariff using the offer curve analysis.

[2]The Ricardian model with a continuum of goods is used extensively to investigate the effect of trade policy. See for example, Opp (2010) and Naito (2012).

[3]Recently there has been a renewed interest in research into secular stagnation. See, for example, Ono and Ishida (2014), Illing, Ono and Schlegl (2018), Michau (2018), Michaillat and Saez (2022), and Hashimoto, Ono and Schlegl (2023).

The integrated model is then utilized to examine the effect of a tariff on the terms of trade, the trade pattern and the levels of real consumption and unemployment, if there is unemployment in either country. With the possibility of unemployment in both countries, there are three possible equilibrium outcomes to be investigated: (i) full employment in both the home country and the foreign country, (ii) unemployment in both countries, (iii) unemployment in the home country with full employment in the foreign country. In all these cases the flexible exchange rate system is assumed.

Our first finding is that in all the three cases above the classical result holds, i.e., by levying a tariff the home country increases its relative real wage (the terms of trade). In addition, as a tariff makes some goods non-traded, the range of goods each country exports shrinks. This effect of a tariff on external margins is familiar from the DFS model with a continuum of goods. However, other effects of a tariff depend crucially on the labor market conditions. Consider first the case in which labor is fully employed in the home country. Then a tariff increases real consumption (and keeps full employment) at home. The effects of a home tariff on the foreign economy however depends on the employment condition there. If foreign labor is fully employed, then real consumption decreases in the foreign country as its terms of trade deteriorates. If instead foreign labor is unemployed, then foreign consumption and employment increase because falling prices expand demands for foreign goods.

What if the home country suffers secular stagnation? Then a tariff results in a fall in real consumption and a rise in unemployment in sharp contrast with the Keynes's prescription for a tariff. The effect of a home tariff on the foreign economy again depends on the employment condition there. If foreign labor is fully employed, a home tariff reduces foreign consumption by turning the terms of trade against the foreign country. If foreign labor is unemployed, however, a deterioration in the terms of trade makes foreign goods cheaper and expands their demands, causing foreign employment and consumption to increase.

We now relate our work to the relevant literature. Our model is most closely related to the DFS model, which, although static, also incorporates monetary aspects of trade into the Ricardian model with a continuum of goods by invoking the quantity theory of money. Although the two monetary models differ much in structure, under flexible exchange rates, a tariff has the identical effect on the home country economy under full employment; namely, a tariff improves the home country's terms of trade and increases its national welfare, while narrowing the range of goods it exports. However the two models diverge when there is unemployment. In the DFS model, unemployment is caused by strict nominal wage rigidity, so with the assumed constancy of the velocity of money, the employment level is completely determined by the supply of national money. Hence, in their analysis a tariff has no effect on employment; it only causes an equiproportional appreciation of the home currency to countervail the rise in its relative wage.

Our paper is also closely related to the work by Ono (2014), which tackles the tariff question in a monetary model like ours. However, his model is based on the two-good Ricardian model and his analysis is limited to the symmetric cases (full employment or unemployment in both countries). Although his model cannot shed light on the effect of a tariff on trade patterns, Ono

(2014) nonetheless obtains the results similar to ours, namely, with full employment in both countries, a tariff by the home country improves its terms of trade and increases its national welfare at the expense of the foreign country while the results are reversed with unemployment in both countries. Our analysis thus complements his analysis as we extend investigation to the asymmetric cases and also examine the effect of large tariff changes numerically.[4]

There is also much work investigating the effect of tariffs without a specific barter model of trade. For example, Mundell (1966) utilized the model, which would later turn into the Mundell-Fleming model, to examine the effect of a tariff, finding that under flexible exchange rates with nominal wage rigidity a tariff causes a depreciation of the levying country's money and a reduction in quantities of its output and employment. Eichengreen (1983) demonstrated the validity of Mundell's finding even with downward real wage rigidity, conditional on the tariff revenues being rebated to domestic consumers in a lump-sum fashion. These analyses are static and may also be subject to the usual criticism for want of microfoundations. Recently, there has been a well-developed open economy macroeconomics literature studying macroeconomic fluctuations within RBC and DSGE frameworks (Obstfeld and Rogoff 1996, Galí and Monacelli 2005, and Uribe and Schmitt-Grohe 2017). These researches focused on business fluctuations due to various shocks around a full-employment steady state. In the recent dynamic trade literature, Naito (2021) used an endogenous growth model under the lab-equipment specification and found the cases where optimal tariffs can be zero even for a large country. Bergin and Corsetti (2023) considered a model with international production chains and investigated the optimal monetary response to exogenous tariff policy shocks. These studies, however, do not treat aggregate demand deficiency.

The remainder of this paper proceeds as follows. Section 2 outlines the features of the model. Section 3 examines the effects of tariffs on the terms of trade, the trade pattern (the external margins), real consumption and employment under flexible exchange rates. Section 4 presents the calibrated model. The final section summarizes our findings and concludes the paper.

2 The model

We consider the continuous-time model with two countries, which we call the home and the foreign country. For the remainder of the paper, we omit the time notation t unless ambiguities arise. The real side of the model is based on the Ricardian model from DFS (1977). There is a continuum of goods, which are indexed by i and arranged on the unit interval $[0, 1]$.

2.1 Firms

A typical home firm can produce $y(i)$ units of good i by employing $l(i)$ units of home labor under the Ricardian technology:

$$y(i) = \frac{1}{a(i)} l(i),$$

[4] Ono (2006, 2018) utilizes a similar model with two goods to investigate the international repercussions of changes in other parameters such as the liquidity preferences and government purchases.

where $a(i)$ denotes the constant unit labor requirement for commodity $i \in [0,1]$. The home firm's objective is to maximize its profit, $P(i)y(i) - Wl(i)$, for given nominal price $P(i)$ and nominal wage W. The first-order condition gives

$$P(i) = a(i)W \text{ if } y(i) > 0, \qquad P(i) < a(i)W \text{ if } y(i) = 0.$$

Thus, if good i is produced in the home country, we have

$$\frac{P(i)}{P} \equiv p(i) = a(i)w, \tag{1}$$

where P is the nominal price index, $p(i)$ is the real price of commodity i and $w(\equiv W/P)$ is the real wage in the home country. Similar relations hold in the foreign country. If we let the asterisk (*) denote the corresponding foreign variables, the ratio of the home wage to the foreign wage is given by

$$\omega \equiv \frac{W}{\varepsilon W^*} = \frac{w}{w^*}\frac{P}{\varepsilon P^*}, \tag{2}$$

where ε is the nominal exchange rate (the home currency price of the foreign exchange).

Let $A(i)$ denote the relative productivity of home labor for good i. Rearrange the goods in order of decreasing home country productivity so that the derivative $A'(i)$ is negative, that is,

$$A(i) \equiv a^*(i)/a(i); \quad A'(i) < 0, \quad i \in [0,1]. \tag{3}$$

This implies that the home country specializes in producing goods with lower indices $i \in [0, I]$ whereas the foreign country specializes in higher-indexed goods $[I^*, 1]$. The two borderline goods, I and I^* are identified by the fact that the domestic price equals the trade cost-adjusted import price, namely,

$$\begin{aligned} \text{Home products } i &\in [0, I]\text{: } P(I) = (1+\tau)\xi\varepsilon P^*(I) \;\Rightarrow\; A(I) = z \equiv \frac{\omega}{(1+\tau)\xi}, \\ \text{Foreign products } i &\in [I^*, 1]\text{: } P^*(I^*) = (1+\tau^*)\xi\frac{P(I^*)}{\varepsilon} \;\Rightarrow\; A(I^*) = z^* \equiv (1+\tau^*)\xi\omega, \end{aligned} \tag{4}$$

where $\xi(\geq 1)$ is an iceberg trade cost (Samuelson, 1954), τ and $\tau^*$are the uniform ad valorem tariff rates on the CIF (Cost, Insurance and Freight) basis. From (4), z, z^*, I, and I^* satisfy

$$z^* \geq z, \quad I(= A^{-1}(z)) \geq I^*(= A^{-1}(z^*)). \tag{5}$$

Thus, goods $i \in [I^*, I]$ are produced in both countries and hence not traded.

From (1) and (4), the real prices in each country are

$$\begin{aligned} \text{Home:} \qquad p(i) &= \begin{cases} a(i)w \text{ for } i \in \left[0, A^{-1}(z)\right], \\ \frac{a^*(i)w}{z} \text{ for } i \in \left[A^{-1}(z), 1\right], \end{cases} \\ \text{Foreign:} \qquad p^*(i) &= \begin{cases} z^* a(i) w^* \text{ for } i \in \left[0, A^{-1}(z^*)\right], \\ a^*(i)w^* \text{ for } i \in \left[A^{-1}(z^*), 1\right]. \end{cases} \end{aligned} \tag{6}$$

2.2 Consumers

We let L and L^* denote the numbers of consumers in the home and foreign country, respectively. Each consumer is endowed with one unit of labor.

Every consumer is infintely-lived and obtains momentary utilities $u^{(*)}(c^{(*)})$ from consumption index $c^{(*)}$ defined below and $v^{(*)}(m^{(*)})$ from real money balance holdings $m^{(*)}$. We use notation $(*)$ to avoid repetitions for each country. Thus, they have the lifetime utility:

$$U^{(*)} = \int_0^\infty \left(u^{(*)}(c^{(*)}) + v^{(*)}(m^{(*)}) \right) \exp(-\rho t) dt, \tag{7}$$

$$u^{(*)\prime} > 0,\ u^{(*)\prime\prime} < 0;\ v^{(*)\prime} > 0,\ v^{(*)\prime\prime} \leq 0;$$

where ρ is the common subjective discount rate and primes denote derivatives. The consumption index is common across home and foreign consumers and symmetric across all varieties:

$$c^{(*)} = \left(\int_0^1 c^{(*)}(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1, \tag{8}$$

where $c^{(*)}(i)$ is individual demand for commodity i and σ stands for the elasticity of substitution.

Home consumers maximize this consumption index subject to the real consumption expenditure e:

$$e = \int_0^1 p(i)c(i)di,$$

where the real commodity prices are given by (6). Solving the optimization problem, we get the individual demand $c(i)$ for commodity i

$$c(i) = \left(\frac{p(i)^{-\sigma}}{\int_0^1 p(j)^{1-\sigma} dj} \right) e.$$

Substituting this to (8) gives

$$c = \left(\int_0^1 p(j)^{1-\sigma} dj \right)^{\frac{1}{\sigma-1}} e.$$

Foreign consumer behavior is similarly obtained.

Indexing the real prices so as to make the consumption utility independent of the prices yields $c = e$ and $c^* = e^*$; hence,

$$\int_0^1 p(j)^{1-\sigma} dj = 1, \quad \int_0^1 p^*(j)^{1-\sigma} dj = 1, \tag{9}$$

and

$$c(i) = p(i)^{-\sigma} c, \quad c^*(i) = p^*(i)^{-\sigma} c^*. \tag{10}$$

From (6) and (9), we obtain the real wages:

$$w = w(z) \equiv \left(\int_0^{A^{-1}(z)} a(i)^{-(\sigma-1)} di + z^{\sigma-1} \int_{A^{-1}(z)}^1 a^*(i)^{-(\sigma-1)} di \right)^{\frac{1}{\sigma-1}},$$

$$w^* = w^*(z^*) \equiv \left(z^{*-(\sigma-1)} \int_0^{A^{-1}(z^*)} a(i)^{-(\sigma-1)} di + \int_{A^{-1}(z^*)}^1 a^*(i)^{-(\sigma-1)} di \right)^{\frac{1}{\sigma-1}}. \quad (11)$$

Let $\delta^{(*)}$ denote the fraction of income home (or foreign) consumers spend on home commodities. Then we can apply (6), (10) and (11) to write

$$\delta = \frac{\int_0^{A^{-1}(z)} p(i)c(i)di}{c} = \frac{\int_0^{A^{-1}(z)} a(i)^{-(\sigma-1)} di}{\int_0^{A^{-1}(z)} a(i)^{-(\sigma-1)} di + z^{\sigma-1} \int_{A^{-1}(z)}^1 a^*(i)^{-(\sigma-1)} di} \equiv \hat{\delta}(z),$$

$$\delta^* = \frac{\int_0^{A^{-1}(z^*)} p(i)c(i)di}{c^*} = \frac{\int_0^{A^{-1}(z^*)} a(i)^{-(\sigma-1)} di}{\int_0^{A^{-1}(z^*)} a(i)^{-(\sigma-1)} di + z^{*(\sigma-1)} \int_{A^{-1}(z^*)}^1 a^*(i)^{-(\sigma-1)} di} \equiv \hat{\delta}(z^*), \quad (12)$$

from which with (5) we obtain

$$\hat{\delta}' < 0, \quad \delta \geq \delta^*,$$

which implies that home consumers spend a larger fraction of their incomes on the home-produced goods than foreigners do. Note also that $w^{(*)}$ in (11) and $\delta^{(*)}$ in (12) satisfy

$$\frac{w'z}{w} = 1 - \delta, \quad \frac{w^{*\prime}z^*}{w^*} = -\delta^*. \quad (13)$$

Each consumer's objective is to maximize (7) subject to the flow budget equation and the asset constraint:

$$\begin{aligned} \dot{b} &= rb + wx - c - Rm + s, \qquad b = m + f, \\ \dot{b}^* &= r^*b^* + w^*x^* - c^* - R^*m^* + s^*, \qquad b^* = m^* + f^*, \end{aligned} \quad (14)$$

where $x^{(*)} \in [0, 1]$ is the employment rate and $s^{(*)}$ is a lump-sum subsidy (or tax if negative). $x^{(*)}$ also represents the actual employment of labor per capita, since each consumer's labor endowment is normalized to 1 and the actual employment is determined on the short side of the labor market. Note also that the total assets $b^{(*)}$ consist of real money balances $m^{(*)}$ and international assets (or debts) $f^{(*)}$. To keep things simple, initial international asset holdings (i.e. initial debts) are assumed to be zero.[5] Thus,

$$f_0 = 0, \quad f_0^* = 0. \quad (15)$$

[5] The effect of a change in the terms of trade on foreign asset-debt holdings depends on the form of the asset-debt holdings: home or foreign assets, and real or nominal terms. To avoid this unnecessary complexity, we assume that foreign asset-debt holdings are negligible.

The real and nominal interest rates on international assets, $r^{(*)}$ and $R^{(*)}$, satisfy

$$R = r + \pi, \quad R^* = r^* + \pi^*, \tag{16}$$

where $\pi^{(*)} \equiv \dot{P}^{(*)}/P^{(*)}$. The non-arbitrage condition on international assets gives

$$R = \frac{\dot{\varepsilon}}{\varepsilon} + R^*. \tag{17}$$

We now state the current-value Hamiltonian of a home consumer:

$$H = u(c) + v(m) + \varsigma\left(rb + wx - c - Rm + s\right),$$

where ς is the costate variable for b. Maximization of the Hamiltonian yields the first-order optimality conditions:

$$u'(c) = \varsigma, \quad v'(m) = \varsigma R, \quad \dot{\varsigma} = (\rho - r)\varsigma.$$

Having in mind that a foreign consumer's behavior is similarly described, from the above conditions we obtain

$$\eta\frac{\dot{c}}{c} + \rho + \pi = R = \frac{v'(m)}{u'(c)}, \quad \eta^*\frac{\dot{c}^*}{c^*} + \rho + \pi^* = R^* = \frac{v^{*\prime}(m^*)}{u^{*\prime}(c^*)}, \tag{18}$$

where $\eta^{(*)} \equiv -u^{(*)\prime\prime}(c^{(*)})c^{(*)}/u^{(*)\prime}(c^{(*)})$ is the elasticity of marginal utility with respect to consumption $c^{(*)}$. The transversality condition is satisfied when

$$\lim_{t\to\infty} \varsigma b \exp(-\rho t) = 0, \quad \lim_{t\to\infty} \varsigma^* b^* \exp(-\rho t) = 0.$$

2.3 Governments

Suppose that the home (foreign) government imposes a tariff on imported goods and rebates the tax revenue equally to all residents in a lump-sum fashion. Then each government's lump-sum tax-cum-subsidy $s^{(*)}$ is

$$s = \frac{\tau}{1+\tau}(1-\delta)c, \quad s^* = \frac{\tau^*}{1+\tau^*}\delta^* c^*. \tag{19}$$

The nominal money supply $M^{S(*)}$ is assumed to be held constant throughout the analysis.

2.4 Market adjustments

Given the consumer, firm and government behaviors discussed above, the money and international asset markets adjust perfectly and instantaneously to ensure that the following conditions hold continuously:[6]

$$\text{Money markets:} \quad mL = \frac{M^S}{P}, \quad m^* L^* = \frac{M^{S*}}{P^*}, \tag{20}$$

$$\text{International asset market:} \quad fL + \frac{\varepsilon P^*}{P} f^* L^* = 0. \tag{21}$$

[6]From Walras's law for stock variables, Equation (21) is valid if Equation (20) holds.

The commodity market adjustment is also assumed to be perfect; hence,

$$\frac{l(i)}{a(i)}\,(=y(i)) = \begin{cases} c(i)L + \xi c^*(i)L^* & \text{for } i \in [0, I^*], \\ c(i)L & \text{for } i \in [I^*, I], \end{cases}$$

$$\frac{l^*(i)}{a^*(i)}\,(=y^*(i)) = \begin{cases} c^*(i)L^* & \text{for } i \in [I^*, I], \\ \xi c(i)L + c^*(i)L^* & \text{for } i \in [I, 1]. \end{cases}$$

In contrast, the labor market is segmented internationally. The nominal wage adjustment is perfect if full employment prevails but sluggish if there is unemployment. Thus, the wage adjustment is the following:

$$\begin{aligned} &\text{When} \quad x^{(*)} = 1, \quad \text{wages are perfectly adjusted;} \\ &\text{When} \quad x^{(*)} < 1, \quad \frac{\dot{W}^{(*)}}{W^{(*)}} = \alpha^{(*)}(x^{(*)} - 1), \end{aligned} \tag{22}$$

where $\alpha^{(*)}$ represents the adjustment speed of the nominal wage $W^{(*)}$ when there is unemployment.[7] Because total revenues equal total labor costs under linear technology and perfect competition, and $\delta^{(*)}$ is each country's expenditure ratio on the goods produced by the home country, we have

$$W\int_0^I l(i)di = \int_0^I P(i)y(i)di = P\delta cL + \varepsilon P^* \frac{\delta^* c^*}{(1+\tau^*)} L^*,$$

$$W^*\int_{I^*}^1 l^*(i)di = \int_{I^*}^1 P^*(i)y^*(i)di = \left(\frac{P}{\varepsilon}\right)\frac{(1-\delta)c}{(1+\tau)}L + P^*(1-\delta^*)c^*L^*.$$

Therefore, using (2) and (4), we find the employment rate $x^{(*)}$ to equal

$$\begin{aligned} x &= \frac{\int_0^I l(i)di}{L} = \frac{\delta c}{w} + \left(\frac{L^*}{L}\right)\frac{\xi \delta^* c^*}{z^* w^*}, \\ x^* &= \frac{\int_{I^*}^1 l^*(i)di}{L^*} = \left(\frac{L}{L^*}\right)\frac{z\xi(1-\delta)c}{w} + \frac{(1-\delta^*)c^*}{w^*}. \end{aligned} \tag{23}$$

Given that the money supply is constant in each country, Equation (20) allows us to represent changes in $m^{(*)}$ by the differential equation

$$\dot{m}^{(*)}/m^{(*)} = -\pi^{(*)}. \tag{24}$$

Each country's current account per capita $CA^{(*)}$ generates international asset dynamics per capita $\dot{f}^{(*)}$, which can be expressed, after substituting (19) and (24) into the budget Equation

[7] This asymmetry in the inflation process is a fundamental element of the recent stagnation models including the contributions of Schmitt-Grohe and Uribe (2016, 2017) and Michau (2018), among others. Ono and Ishida (2014) provide a micro-foundation for our wage adjustment equation under the condition that stagnation occurs in steady state.

(14), as

$$\begin{aligned}\dot{f} &= CA = && rf + wx - \frac{1+\tau\delta}{1+\tau}c, \\ \dot{f}^* &= CA^* = && r^*f^* + w^*x^* - \frac{1+\tau^*(1-\delta^*)}{1+\tau^*}c^*. \end{aligned} \tag{25}$$

3 Policy effects

We assume that the economy is initially in steady state and examine the effects of tariff $\tau^{(*)}$ on consumption and specialization patterns of the two countries. From the properties of the dynamics proven in Appendix A, we obtain the following lemma:

Lemma 1. *Whether there is full employment or unemployment, the economy always stays in a steady state and jumps to a new steady state immediately in response to any exogenous disturbances.*

Lemma 1 implies that international asset remains at the initial value for any t:

$$f = f_0, \quad f^* = f_0^*.$$

Lemma 1 also allows us to focus on the steady state in the following analysis. For simplicity, we examine only the effects of small import tariffs in the neighborhood of the free trade equilibrium, where

$$\tau = 0, \quad \tau^* = 0.$$

Because $\dot{f} = \dot{f}^* = 0$ in the steady state, (15) and (25) yield

$$\frac{1+\tau\hat{\delta}(z)}{1+\tau}c = w(z)x, \quad \frac{1+\tau^*(1-\hat{\delta}(z^*))}{1+\tau^*}c^* = w^*(z^*)x^*. \tag{26}$$

3.1 Full employment in both countries ($x = x^* = 1$)

We start by considering the benchmark case, in which full employment prevails in both countries ($x = x^* = 1$). Substituting (26) to (23) with $x = 1$ yields the equilibrium relative wage ω:

$$\frac{1-\hat{\delta}(z)}{1+\tau\hat{\delta}(z)}\omega L = \frac{\hat{\delta}(z^*)}{1+\tau^*(1-\hat{\delta}(z^*))}L^*. \tag{27}$$

Recall from (12) that $\delta^{(*)}$ depends on ω through $z^{(*)}$ in (4). Thus, (27) determines the equilibrium ω as a function of tariff rates. By differentiating (27) with respect to τ and using the fact that $z^{(*)}$ in (4) satisfies

$$\begin{aligned} &z(\omega,\tau) = \frac{\omega}{(1+\tau)\xi}, \quad z_\omega = \frac{1}{\xi}, \quad z_\tau = -\frac{\omega}{\xi}, \\ &z^*(\omega,\tau^*) = (1+\tau^*)\xi\omega, \quad z^*_\omega = \xi, \quad z^*_{\tau^*} = \omega\xi, \end{aligned} \tag{28}$$

in the neighborhood where $\tau^{(*)} = 0$, we obtain the effect of τ on the relative real wage:

$$\frac{d\omega}{d\tau} = \frac{\delta(1-\delta) - \frac{\omega\hat{\delta}'(z)}{\xi}}{\frac{L^*}{\omega L}(\frac{\delta^*}{\omega} - \hat{\delta}'(z^*)\xi) - \frac{\hat{\delta}'(z)}{\xi}} > 0. \tag{29}$$

We then get the effect of τ on real consumption from (11), (13) and (26) in which $x = x^* = 1$:

$$\frac{dc}{d\tau} = w(1-\delta)\frac{1}{\omega}\frac{d\omega}{d\tau} > 0, \quad \frac{dc^*}{d\tau} = -w^*\delta^*\frac{1}{\omega}\frac{d\omega}{d\tau} < 0.$$

It is concluded that by levying a tariff, the home country increases its relative real wage and real consumption while decreasing consumption in the foreign country. In short, a tariff has the standard beggar-thy-neighbor effect under full employment in both countries.

We next examine the effect of a tariff on the ranges of goods each county produces (external margins). From (3), (4) and (29), we find

$$\frac{dI^*}{d\tau} = \frac{\xi}{A'(I^*)}\frac{d\omega}{d\tau} < 0,$$

$$\frac{dI}{d\tau} = \frac{\omega}{\xi A'(I)}\left(\frac{1}{\omega}\frac{d\omega}{d\tau} - 1\right) \gtreqless 0.$$

Since the foreign country produces goods in the range $[I^*, 1]$, the first result above says that the imposition of a tariff by the home country expands the range of goods foreigners produce because a rise in the home relative wage makes foreign exports more competitive. In contrast, the effect of the home country's external margin is more complicated. On the one hand, cheaper foreign goods make home firms unprofitable at the margin. On the other, however, the tariff shields home firms from this terms-of-trade effect, expanding the range of the goods they produce. We summarize the above results as follows:

Proposition 1. *Assume full employment in both countries. Then, an increase in the home import tariff improves the home current account CA, which raises the relative home wage (ω) and the relative prices of home commodities, implying an improvement in the home terms of trade. Therefore, home consumption increases while foreign consumption decreases. The higher relative prices of home commodities expand the range of commodities the foreign country produces but has an ambiguous effect on the range of home commodities. Namely,*

$$\tau \uparrow \rightarrow \ \omega \uparrow, c \uparrow, c^* \downarrow, I^* \downarrow, I(?).$$

3.2 Stagnation in both countries ($x < 1$ and $x^* < 1$)

We next consider the case of aggregate demand shortage ($x < 1$ and $x^* < 1$) in both countries. Even with unemployment, the economy always stays in a steady state and jumps immediately to a new steady state when disturbed, as mentioned in Lemma 1. Thus, we can focus on the steady state in the following analysis.

With unemployment, the nominal wage, nominal prices and the price level continue to decline according to

$$\pi^{(*)} \equiv \frac{\dot{P}^{(*)}}{P^{(*)}} = \frac{\dot{W}^{(*)}}{W^{(*)}} = \alpha^{(*)}(x^{(*)} - 1). \tag{30}$$

However, since the nominal wage and prices fall at the same rate, the real wage $w^{(*)}$ given in (11) and ω remain constant.

A falling price level continuously expands real balances. However, as demonstrated in Ono (2001), a continuous increase in real balances may fail to inspire spending and guide the economy to full employment if the desire for real money balances is insatiable. This occurs if the marginal utility of real balances is bounded away from zero, i.e.,

$$\lim_{m^{(*)} \to \infty} v^{(*)\prime}(m^{(*)}) = \beta^{(*)} > 0,$$

where $\beta^{(*)}$ is a positive constant.[8] In such a case, (18) may be invalid for any $m^{(*)}$. Specifically, if we let $c^{(*)f}$ denote the consumption level consistent with full employment ($x^{(*)} = 1$), there cannot be a steady state equilibrium with full employment if

$$\rho < \frac{\beta^{(*)}}{u^{(*)\prime}(c^{(*)f})} \left(< \frac{v^{(*)\prime}(m^{(*)})}{u^{(*)\prime}(c^{(*)f})} \text{ for any } m^{(*)} \right). \tag{31}$$

Then, the only steady state equilibrium we have is with persistent unemployment,

As proven by Ono (1994, 2001), when $0 < x^{(*)} < 1$, nominal price $P^{(*)}$ continues to decline, and real money balances continue to increase, causing $v^{(*)\prime}(m)$ to converge to $\beta^{(*)}$.[9] Thus, in the steady state (18), (23) and (30) give us these equations:

$$\frac{\beta^{(*)}}{u^{(*)\prime}(c^{(*)})} = \rho + \alpha^{(*)}(x^{(*)} - 1) \quad \to \quad x^{(*)} = x^{(*)}(c^{(*)}), \tag{32}$$

$$x = \frac{\hat{\delta}(z)c}{w(z)} + \left(\frac{L^*}{L}\right) \frac{\xi \hat{\delta}(z^*)c^*}{z^* w^*(z^*)} \quad \to \quad c = h(c^*, z, z^*), \tag{33}$$

$$x^* = \left(\frac{L}{L^*}\right) \frac{z\xi(1 - \hat{\delta}(z))c}{w(z)} + \frac{(1 - \hat{\delta}(z^*))c^*}{w^*(z^*)} \quad \to \quad c^* = h^*(c, z, z^*), \tag{34}$$

which can be solved for c and c^* for given (z, z^*).

$$c = c(z, z^*), \qquad c^* = c^*(z, z^*). \tag{35}$$

Applying $c^{(*)}$ and $x^{(*)}(c^{(*)})$ given by (32), (33), (34) and (35) to (25) in which $r = \rho$ and

[8] See Ono (1994: 4–8) for a discussion on the insatiable utility of money in the history of economic thought (e.g., Veblen, Marx, Simmel, Keynes) and its economic implications. The validity of this property is empirically supported by Ono (1994: 34–8) using the GMM (generalized method of moments), by Ono, Ogawa and Yoshida (2004) using parametric and nonparametric methods, and more extensively by Akesaka, Mikami and Ono (2024) using a nationally extensive survey of Japanese households.

[9] The deflation path (money expansion path) is compatible with the transversality condition, since the nominal interest rate $R = \beta/u'(c)$ is positive, so that $\dot{m}/m = -\pi = -R + \rho < \rho$ holds.

$\dot{f}^{(*)} = 0$ yields

$$\dot{f} = \rho f_0 + w(z)x(c(z,z^*)) - \frac{1+\hat{\delta}(z)\tau}{1+\tau}c(z,z^*) = 0,$$

$$\dot{f}^* = \rho f_0^* + w^*(z^*)x^*(c^*(z,z^*)) - \frac{1+\left(1-\hat{\delta}(z^*)\right)\tau^*}{1+\tau^*}c^*(z,z^*) = 0, \tag{36}$$

where $f_0^{(*)} = 0$ from (15). Noting that $z^{(*)}$ satisfies (28), the current account functions are assumed to satisfy

$$\text{Marshall-Lerner condition:} \quad \frac{\partial \dot{f}}{\partial \omega} < 0 \quad \left(\frac{\partial \dot{f}^*}{\partial \omega} > 0\right), \tag{37}$$

which implies that an increase in the relative home wage depreciates the home country's current account. Then, the relative wage (or relative price) must satisfy the equation $\dot{f} = 0$; otherwise, the transversality condition would not be satisfied.[10] This adjustment of ω is attained by an initial jump of the exchange rate ε through $\omega \equiv W/\varepsilon W^*$, because neither W nor W^* in (22) can jump under stagnation. Thus, the equilibrium relative wage ω is found by solving $\dot{f} = 0$. This equation in turn implies that f remains constant at the initial level $f_0(=0)$. Once the equilibrium ω is determined, the equilibrium values of c, c^*, x, x^*, I and I^* are obtained as functions of tariffs τ and τ^*.

Now we can state the effect of a tariff by the home country (see Appendix B.1 for the proof).

Proposition 2. *Suppose that there is chronic unemployment in both countries. Then, an increase in the home import tariff improves the home current account ($CA \uparrow$) and induces an appreciation of the home currency ($\varepsilon \downarrow$), which raises the relative home wage ($\omega \uparrow$) and the international relative prices of home commodities. The tariff also reduces demand for home commodities, causing more unemployment and decreasing consumption in the home country. By contrast, there is an increase in demand for foreign commodities due to a fall in relative prices, which increases employment and consumption in the foreign country, and expands the range of products $1 - I^*$ produced there. The home country may or may not produce a greater variety of products because the protective effect of a tariff is militated against by a fall in prices of imports. Namely,*

$$\tau \uparrow \to CA \uparrow \to \omega \uparrow, \quad (x,c)\downarrow, (x^*,c^*)\uparrow, I^*\downarrow, I(?).$$

Thus, when both countries are stagnant, a tariff is a shoot-yourself-in-the-foot policy.

3.3 The asymmetric case ($x < 1$ and $x^* = 1$)

Finally, we consider the asymmetric case, where the home country faces unemployment while the foreign economy achieves full employment. In this case, (31) holds in the home country but

[10] Clearly, this property holds as the two countries are assumed to have the same subjective discount rate, and, thus, there is no international borrowing and lending in the international asset market. Ikeda and Ono (1992) consider the dynamics of the international asset when the subjective discount rate is heterogeneous across countries.

not in the foreign country. Hence,

$$\rho < \frac{\beta}{u'(c^f)} \quad \text{and} \quad \rho > \frac{\beta^*}{u^{*\prime}(c^{*f})}. \tag{38}$$

Because the foreign country achieves full employment, by setting $x^* = 1$ in the second equation of (23) we get c^* as a function of (c, z, z^*):

$$c^* = \frac{w^*}{1-\delta^*}\left(1 - \frac{L}{L^*}\frac{z\xi(1-\delta)c}{w}\right).$$

In contrast, (33) holds in the home country. By substituting the c^* obtained above into (33), we find that c satisfies

$$\frac{\beta}{u'(c)} = \rho + \alpha\left(\frac{\hat{\delta}(z)c}{w(z)} + \frac{\xi\hat{\delta}(z^*)}{z^*(1-\hat{\delta}(z^*))}\left(\frac{L^*}{L} - \frac{z\xi(1-\hat{\delta}(z))c}{w(z)}\right) - 1\right) \to c = \hat{c}(z, z^*). \tag{39}$$

Differentiating $\hat{c}(z, z^*)$ given above and using (11), (12), (13), and (33) yields

$$\hat{c}_z = \frac{\left(\hat{\delta}'(z) - \frac{\delta(1-\delta)}{z}\right)c}{(1-\delta^*)wx' - (\delta - \delta^*)} < 0, \quad \hat{c}_{z^*} = \left(\frac{L^*}{L}\right)\frac{\left(\hat{\delta}'(z^*) - \frac{\delta^*(1-\delta^*)}{z^*}\right)\frac{wc^*}{\omega w^*}}{(1-\delta^*)wx' - (\delta - \delta^*)} < 0, \tag{40}$$

where x' is the derivative obtained from $x(c)$ in (32). Substituting $\hat{c}(z, z^*)$ for c in the first equation of (25), where $r = \rho$ and $\dot{f} = 0$, and then substituting $x^* = 1$ to the second equation of (25) where $r^* = \rho$ and $\dot{f}^* = 0$, we obtain

$$\begin{aligned} \dot{f} &= \rho f_0 + w(z)x(\hat{c}(z, z^*)) - \frac{1+\tau\hat{\delta}(z)}{1+\tau}\hat{c}(z, z^*) = 0, \\ \dot{f}^* &= \rho f_0^* + w^*(z^*) - \frac{1+\tau^*(1-\hat{\delta}(z^*))}{1+\tau^*}c^* = 0, \end{aligned} \tag{41}$$

where $f_0^{(*)} = 0$ from (15) and $x(c)$ is given by (32).

Because $z(\omega, \tau) = \omega/[(1+\tau)\xi]$ and $z^*(\omega, \tau^*) = (1+\tau^*)\xi\omega$, as defined in (4), the first equation of (41) gives the equilibrium ω as a function of (τ, τ^*). Consequently, c, $x(c)$, c^*, I, and I^* are also determined as functions of (τ, τ^*). The Marshall-Lerner condition in the present case is assumed as follows:

$$\frac{\partial \dot{f}}{\partial \omega} < 0.$$

Under this condition we obtain the following effects of changes in τ (proof in Appendix B.2):

Proposition 3. *Suppose that there is unemployment in the home country and full employment in the foreign country. An increase in the home import tariff improves the home current account ($CA \uparrow$) and induces an appreciation of the home currency ($\varepsilon \downarrow$), which raises the relative home wage ($\omega \uparrow$) and the international relative prices of home commodities. A tariff also reduces demand for home commodities and home employment, thereby decreasing home consumption and aggravating the stagnation. In the foreign country, where there is full employment, a deteriora-*

tion in its terms of trade decreases consumption, although cheaper prices help foreigners produce a greater variety of goods. The home country may or may not produce a greater variety of products because the protective effect of a tariff is militated against by a fall in prices of imports. Namely,

$$\tau \uparrow \rightarrow CA \uparrow \rightarrow \omega \uparrow, c \downarrow, x \downarrow, c^* \downarrow (x^* = 1), I^* \downarrow, I(?).$$

The effects of imposition of a tariff τ^* by the fully-employed foreign country against the stagnant home country are summarized as follows (proof in Appendix B.2):

Proposition 4. *An increase in the foreign import tariff τ^* worsens the home current account ($CA \downarrow$), depreciates the home currency ($\varepsilon \uparrow$), and lowers the home relative wage ($\omega \downarrow$). Therefore, the domestic demand for home commodities expands, stimulating home employment and consumption. With the home wage lower, the home country produces a greater variety of products. An improvement in the foreign terms of trade, which is caused by a rise in τ^*, increases consumption in the foreign country but it may or may not produce a greater variety of goods because the protective effect of the tariff is mitigated by a fall in import prices. Namely,*

$$\tau^* \uparrow \rightarrow CA \downarrow \rightarrow \omega \downarrow, c \uparrow, x \uparrow, c^* \uparrow (x^* = 1), I^*(?), I \uparrow .$$

To summarize the findings up to this point: a tariff never extricates the country itself from stagnation. Worse, a tariff is always a shoot-yourself-in-the-foot policy for a stagnant country, whether the other country has full employment or not.

4 The effects of tariffs and trade costs

In the previous section, we analytically derived the effects of small tariffs. The present section extends the analysis to the case of large tariff changes. Since the analysis yields ambiguous results for large-tariff cases, we resort to numerical analysis. We thus specify the functional forms of the model and then calibrate its parameters. We set a time period to be one year.

4.1 Model specification and calibration

We start by assuming that the labor input coefficients for each industry are distributed exponentially as in

$$a(i) = \theta e^{\gamma i}, \qquad a^*(i) = \theta^* e^{-\gamma^* i}, \tag{42}$$

where θ and θ^* are the scale (cost) parameters and γ and γ^* the rate parameters respectively. Thus,

$$A(i) \equiv \frac{a^*(i)}{a(i)} = \frac{\theta^*}{\theta} e^{-(\gamma+\gamma^*)i}.$$

We next calibrate the four distributional parameters θ, θ^*, γ and γ^* as follows. We first choose the scale parameters so that each country's most productive industry has the unit labor requirement of one, i.e. $a(0) = 1$ and $a^*(1) = 1$. We next assume symmetry in distribution so that $a(i) = a^*(1 - i)$.This implies that two countries are equally productive in the good at the

Table 1: Parameter calibration

Parameter	Calibration	Reference or moment
Time preference rate	$\rho = \rho^* = 0.04$	Standard value
Elasticity of substitution	$\sigma = \sigma^* = 5$	Bernard et al. (2003), Broda and Weinstein (2006)
Marginal utility of money	β, β^*	target = 5% output gap under stagnation
Speed of wage adjustment	$\alpha = \alpha^* = 0.01$	for feasibility
Relative country size	$L/L^* = 1$	symmetry, equally sized countries
Initial foreign debt position	$f_0 = 0$	zero net foreign debt position
Iceberg trade cost	$\xi = 1.7$	Anderson and van Wincoop (2004), Novy (2003)
Tariffs	$\tau, \tau*$	policy variable
Scale parameter (home)	$\theta = 1$	normalization such that $a(0) = 1$
Scale parameter (foreign)	$\theta^* = 5$	normalization such that $a^*(1) = 1$
Rate parameter (home)	$\gamma = 1.61$	target relative productivity of $A(0) = 5$
Rate parameter (foreign)	$\gamma^* = 1.61$	symmetric distributions, i.e. $a(i) = a^*(1-i)$

midpoint of the unit line, i.e. $A(0.5) = 1$. Lastly, we choose the relative productivity $A(0) = 5$. Applying these criteria simultaneously, we obtain these distributional parameters: $\theta = 1$, $\theta^* = 5$, $\gamma = 1.61$ and $\gamma^* = 1.61$.

As for consumer preferences, we assume logarithmic utility functions for consumption, i.e. $u(c) = \ln c$ and $u^*(c^*) = \ln c^*$, and set the time preference rates at $\rho = \rho^* = 0.04$. This choice is standard in the literature and equivalent to an annual real interest rate of $r = 4\%$ in steady state. The elasticity of substitution is set to $\sigma = \sigma^* = 5$ following Bernard, Eaton, Jensen and Kortum (2003) and Broda and Weinstein (2006). Finally, the bounds on the marginal utility of money β and β^* have no effect on real allocation under full employment, but determine its feasibility. Therefore, we set them so as to obtain a specific value of the output gap $x, x^* \in (0, 1)$.

The parameters α and α^* determine the degree of downward nominal wage rigidity under stagnation. We assume symmetry and set $\alpha = \alpha^* = 0.01$, which satisfies the feasibility restriction $\rho > \alpha$. Consistent with the empirical evidence on wage and price rigidities for Japan, this calibration implies deflationary dynamics under stagnation.

With respect to the external environment, we assume an equal country size, i.e., $L/L^* = 1$, and an initial net foreign asset position of $f_0 = 0$. Finally, the iceberg trade cost is initially set to $\xi = 1.7$ based on the findings of Anderson and van Wincoop (2004) and Novy (2013). Later, we also simulate the effects of a reduction in the trade cost parameter. The model calibration is summarized in Table 1.

4.2 Numerical solutions of the model

The model specification above gives us the solutions for each country's borderline products I and I^*:

$$I = \frac{-1}{\gamma + \gamma^*} \ln\left(z \frac{\theta}{\theta^*}\right), \; I^* = \frac{-1}{\gamma + \gamma^*} \ln\left(z^* \frac{\theta}{\theta^*}\right), \tag{43}$$

where

$$z \equiv \frac{\omega}{(1+\tau)\xi}, \quad z^* \equiv \omega(1+\tau^*)\xi.$$

We require the equilibrium to satisfy $I < 1$ and $I^* > 0$. Using the expressions in (42) for labor coefficients and (43) for I and I^*, we use (11) and (12) to determine the real wages and expenditures shares, respectively, as functions of ω. These expressions are affected by demand shortage only to the extent that equilibrium ω varies across the three possible cases considered below.

The case of ($x = x^* = 1$): We numerically obtain the steady state ω by solving (27), where expenditure shares are functions of ω as discussed above. We then find consumption from (26):

$$c = \frac{1+\tau}{1+\tau\hat{\delta}(z)} w(z), \quad c^* = \frac{1+\tau^*}{1+\tau^*(1-\hat{\delta}(z^*))} w^*(z^*). \tag{44}$$

The case of ($x < 1, x^* < 1$): Equations (32), (33), (34) and (35) give employment $x(c)$ and $x^*(c^*)$ and consumption $c(z, z^*)$ and $c^*(z, z^*)$. We then numerically obtain the steady state ω by solving the first equation of (36) in which $f_0 = 0$ as follows:

$$c(z, z^*) = \frac{1+\tau}{1+\tau\hat{\delta}(z)} w(z) x(c(z, z^*)),$$

where z and z^* are functions of ω from (4).

The case of ($x < 1, x^* = 1$): In the asymmetric stagnation case, foreign consumption c^* is determined by (44) as a function of z^*, while home employment and consumption are given by (32), (33) and (39) as $x(c)$ and $\hat{c}(z, z^*)$. As in the previous case, steady state ω is obtained from

$$\hat{c}(z, z^*) = \frac{1+\tau}{1+\tau\hat{\delta}(z)} w(z) x(\hat{c}(z, z^*)).$$

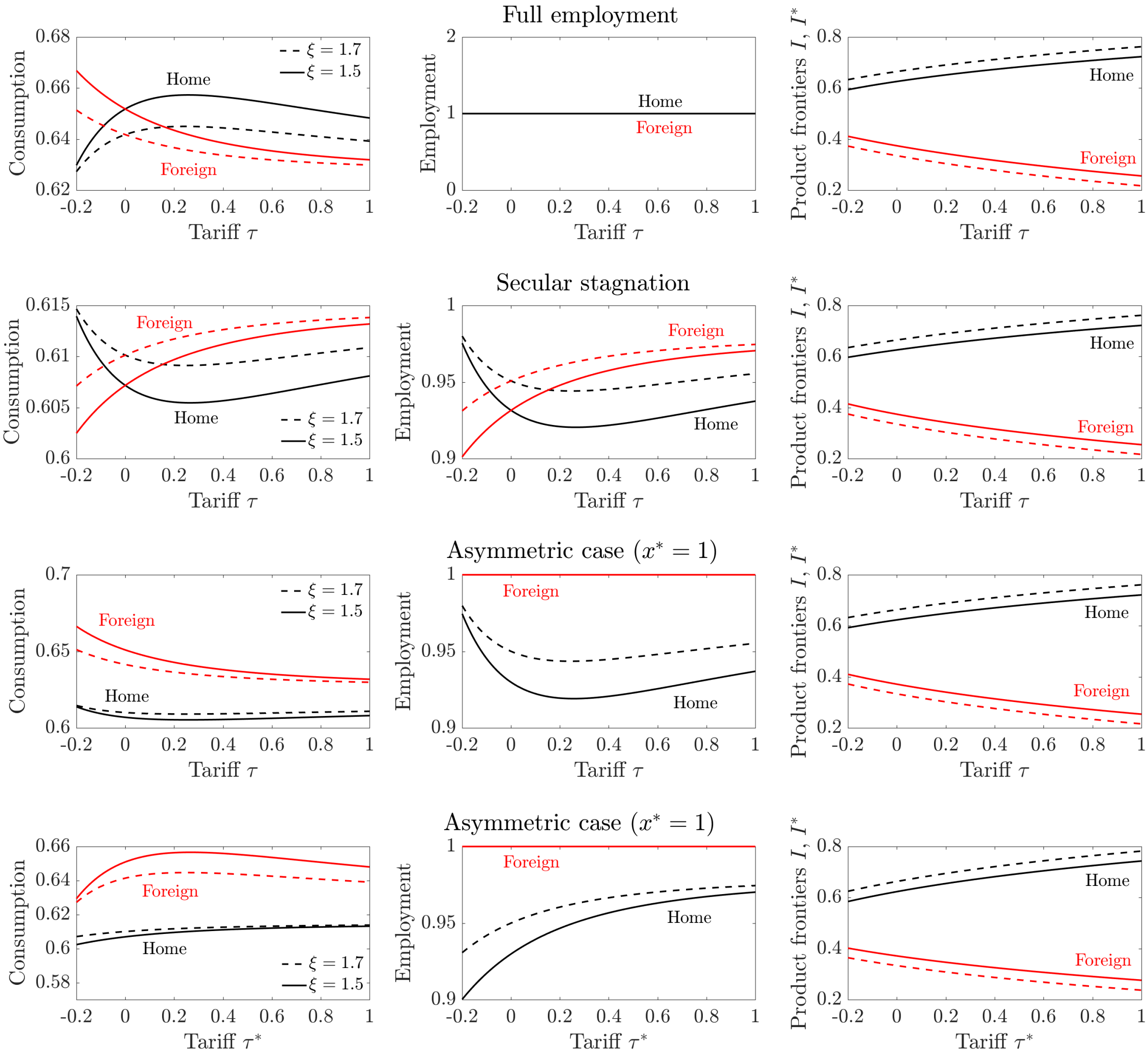
4.3 Effects of trade tariffs and iceberg trade costs

We report our calibration results in Figure 1. Each panel in the figure displays two sets of curves: home and foreign variables. The solid curves are based on the iceberg trade cost at $\xi = 1.5$ while the broken curves are obtained at $\xi = 1.7$.

The first row of Figure 1 shows the effects of the home tariff τ (at $\tau^* = 0$) when labor is fully employed in both countries. As shown in the first panel, the home tariff increases home consumption until it reaches $\tau = 24.4\%$, showing the robustness of our analytical results obtained under a small tariff. However, a further increase in the tariff rate decreases home consumption. This is an expected result from the classical optimum tariff literature. On the other hand, foreign consumption is found to be monotonically decreasing for the relevant parameter values. This shows that our analytical result is robust at any tariff rate.

In the second row, we see the effects of the home tariff when both countries are stagnant. As the home tariff increases, home consumption falls until $\tau = 24.8\%$ and then begins to increase at higher tariff rates. This verifies that our analytical prediction of a tariff as a shoot-yourself-in-the-foot policy extends up to $\tau = 24.8\%$. In contrast, foreign consumption monotonically increase

Figure 1: Effects of trade tariffs τ and iceberg trade costs ξ



with the tariff, demonstrating the robustness of our analytical results beyond the incipient rate. Further, changes in each country's employment mimic changes in consumption (see the second panel).

The third row illustrates the effects of the home tariff τ when foreign labor is fully employed but home labor is not. As consistent with our analytical results, home consumption declines until the home tariff reaches $\tau = 24.3\%$. However, with further increases in the tariff, home consumption also begins to increase. This result implies the existence of the worst tariff (pessimum) as opposed to the optimum tariff. Although fully employed, the foreign country also sees its real consumption fall as the home country raises its tariff rate. The second panel shows that home employment behaves like home consumption, first decreasing but eventually turning up.

The last row shows the effects of the tariff τ^* by the fully-employed foreign country against the stagnant home country. Foreign consumption increases when the tariff rate is low and then begins to fall as the rate rises high enough. This is consistent with the classical optimal tariff literature. Home consumption and employment continue to increase, like foreign consumption and employment in the second row (where both countries are stagnant).

Thus, it is concluded that by levying (not too high) tariffs the country can increase its real consumption only if it achieves full employment. Put another way, when its economy is stagnant, the country cannot expect the tariff to increase its employment or real consumption. The trading partner's consumption decreases if it is fully employed, and increases if it is stagnant, whether the tariff-imposing country's labor is fully employed or not.

Figure 1 also displays the effect of a tariff on the external margins. As illustrated by the rightmost panels in each row, higher tariffs are shown to widen the variety of local products in both countries, thereby turning more goods to being non-traded and diminishing the extent of international specialization.

Figure 1 can also shed light on the effect of globalization, usually associated with falling in transportation costs in the literature (e.g., Baldwin, 2001). It is shown that regardless of the trading partner's employment condition, lower transportation costs have negative impacts on consumption and employment for a stagnant country and positive effects on consumption in a fully-employed country. Our results thus imply that globalization (or lower transportation costs) may not benefit every country in the world. Finally, we observe that lower transportation costs promote international specialization in all the cases.

5 Conclusion

We examine whether the imposition of import tariffs can increase real consumption and decrease unemployment when the economy is stagnant. To that end, we develop a dynamic model of international trade based on the money-in-utility model of secular stagnation by Ono (1994, 2001) and the continuum-of-good Ricardian trade model by Dornbusch, Fischer and Samuelson (1977), and then examine the effect of a tariff on the terms of trade, the trade pattern, real consumption and employment of labor under flexible exchange rates. Our main finding is that the tariff is no cure for stagnation. In fact, it is shown that the imposition of a tariff causes more unemployment and diminishes real consumption, calling into question Keynes's advocacy of the tariff to increase employment in the U.K. during the Great Depression. The effect of the tariff on the foreign country is also investigated. If there is full employment in the foreign country, then the home tariff harms the foreign economy by reducing its real consumption. In contrast, if the foreign country has chronic unemployment, the home tariff boosts the foreign economy by increasing employment and real consumption there.

Although our results contrast sharply with Keynes's view of tariffs, it is recalled that his advocacy of tariffs was constrained by the U.K.'s adherence to the gold standard whereas our model is based on flexible exchange rates. Thus, to be fair to Keynes, our analysis should be extended to the case of the gold standard. We intend to undertake this extension in the near future.

Appendix

Appendix A: Dynamic System

In the text the effects of trade policies are examined in the neighborhood where $\tau = \tau^* = 0$. From $W^{(*)}/P^{(*)} \equiv w^{(*)}$ with (13), we have

$$\frac{\dot{W}}{W} - \pi = \frac{w'z}{w}\frac{\dot{\omega}}{\omega} = (1-\delta)\frac{\dot{\omega}}{\omega}, \;\; \frac{\dot{W}^*}{W^*} - \pi^* = \frac{w^{*\prime}z^*}{w^*}\frac{\dot{\omega}}{\omega} = -\delta^*\frac{\dot{\omega}}{\omega}. \tag{45}$$

In addition, from the definition of $\omega \equiv W/\varepsilon W^*$ with (16), (17), and (45), the following equation holds:

$$\frac{\dot{\omega}}{\omega} = -\frac{\dot{\varepsilon}}{\varepsilon} + \frac{\dot{W}}{W} - \frac{\dot{W}^*}{W^*} = -(R-R^*) + \pi + (1-\delta)\frac{\dot{\omega}}{\omega} - \pi^* + \delta^*\frac{\dot{\omega}}{\omega}$$

$$\Rightarrow \; r - r^* = -(\delta-\delta^*)\frac{\dot{\omega}}{\omega}. \tag{46}$$

A.1 Full employment case ($x = x^* = 1$)

From (23) in which $x = x^* = 1$, we obtain each country's consumption as a function of ω and easily confirm that

$$\frac{\partial c}{\partial \omega} = \frac{w(\Gamma_1 cL + \Gamma_2 c^* L^*)}{\omega(\delta-\delta^*)L} > 0, \;\; \frac{\partial c^*}{\partial \omega} = -\frac{w^*(\Gamma_1 cL + \Gamma_2 c^* L^*)}{(\delta-\delta^*)L^*} < 0,$$

$$\Gamma_1 \equiv \frac{-\hat{\delta}'(z)z + \delta(1-\delta)}{w} > 0, \;\; \Gamma_2 \equiv \frac{-\hat{\delta}'(z^*)z^* + \delta^*(1-\delta^*)}{\omega w^*} > 0, \tag{47}$$

because $\delta > \delta^*$ in the presence of tariffs and transportation costs. Then, from (18) with (16), we have

$$\eta\frac{\dot{c}}{c} - \eta^*\frac{\dot{c}^*}{c^*} = r - r^*. \tag{48}$$

Using (46) with $\dot{c}^{(*)} = \frac{\partial c^{(*)}}{\partial \omega}\dot{\omega}$, where $\frac{\partial c^{(*)}}{\partial \omega}$ is given in (47), (48) can be rewritten as

$$\left(\eta\frac{\omega}{c}\frac{\partial c}{\partial \omega} - \eta^*\frac{\omega}{c^*}\frac{\partial c^*}{\partial \omega} + \delta - \delta^*\right)\frac{\dot{\omega}}{\omega} = 0.$$

Therefore, in this case, the relative wage remains at constant for any time; $\omega = \bar{\omega}$. Note that $\bar{\omega}$ is determined such that $\dot{f} = 0$, where $\dot{f}$ is given by the first equation of (25) with $x = 1$. As such, the economy always stays in the steady state to satisfy the transversality condition.

With no tariffs and zero transportation costs, $\delta = \delta^*$ and then ω is straightforwardly determined by (23). Thus, c and c^* immediately jumps to the respective levels that attains $\dot{f} = 0$ and $\dot{f}^* = 0$.

A.2 Stagnation case ($x < 1$ and $x^* < 1$)

From $\omega \equiv W/\varepsilon W^*$ with $R^{(*)} = \frac{\beta^{(*)}}{u^{(*)\prime}(c^{(*)})}$, (17), and (22), we have

$$\frac{\dot{\omega}}{\omega} = -\frac{\dot{\varepsilon}}{\varepsilon} + \frac{\dot{W}}{W} - \frac{\dot{W}^*}{W^*} = -r^H + r^F,$$

$$r^H \equiv \frac{\beta}{u'(c)} - \alpha(x-1), \quad r^F \equiv \frac{\beta^*}{u^{*\prime}(c^*)} - \alpha^*(x^*-1), \tag{49}$$

where x and x^* satisfy (23). Using $R^{(*)} = \frac{\beta^{(*)}}{u^{(*)\prime}(c^{(*)})}$, (16), (22), (45), and (49), the real interest rate in each country is given by

$$r^{(*)} = R^{(*)} - \pi^{(*)} = \delta^{(*)} r^H + (1-\delta^{(*)}) r^F. \tag{50}$$

From (18) with (16) and (50) in the neighborhood of small trade barriers ($\delta \approx \delta^*$), we obtain the following relationships:

$$r = r^* \;\; \rightarrow \;\; \eta\frac{\dot{c}}{c} = \eta^*\frac{\dot{c}^*}{c^*} = -\frac{\dot{\varsigma}}{\varsigma} = -\frac{\dot{\varsigma}^*}{\varsigma^*} \;(= r-\rho)\,, \quad \text{where } \varsigma^{(*)} = u^{(*)\prime}(c^{(*)}),$$

$$\varsigma^* = \kappa\varsigma, \quad \kappa = \text{constant over time},$$

$$\kappa = \frac{u^{*\prime}(c^*)}{u'(c)} \quad \rightarrow \quad c^* = \check{c}^*(c,\kappa); \quad \frac{\partial \check{c}^*}{\partial c} = \kappa\frac{u''(c)}{u^{*\prime\prime}(c^*)} > 0. \tag{51}$$

The dynamics of c is obtained from (18) and (50) as follows:

$$\frac{\dot{c}}{c} = \frac{1}{\eta}\left(\delta r^H + (1-\delta) r^F - \rho\right). \tag{52}$$

Once κ is given, the first equation in (49) and (52), in which x and x^* satisfy (23) and $c^* = \check{c}^*(c,\kappa)$ is given by (51), yield an autonomous dynamic system of (ω, c) in the stagnation case. In the steady state ($\dot{\omega} = \dot{c} = 0$), the following condition holds

$$r^H = r^F = r = \rho.$$

Taking a linear expansion and evaluating the dynamic system of (ω, c) around the steady-state values yields the following Jacobian matrix:

$$\begin{bmatrix} \omega(-r^H_\omega + r^F_\omega) & \omega(-r^H_c + r^F_c) \\ \frac{c}{\eta}\left(\delta r^H_\omega + (1-\delta) r^F_\omega\right) & \frac{c}{\eta}\left(\delta r^H_c + (1-\delta) r^F_c\right) \end{bmatrix}.$$

From (23) and (49), we obtain

$$r^H_\omega = \frac{\alpha}{w\omega}\left(\delta(1-\delta) - \hat{\delta}'(\omega)\omega\right)(cL + c^*L^*)\frac{1}{L},$$

$$r^F_\omega = -\frac{\alpha^*}{w^*\omega}\left(\delta(1-\delta) - \hat{\delta}'(\omega)\omega\right)(cL + c^*L^*)\frac{1}{L^*},$$

$$r_c^H = \frac{\alpha}{w}\left((wx' - 1) + 1 - \delta - \delta \frac{L^*}{L}\frac{\partial \check{c}^*}{\partial c}\right),$$

$$r_c^F = \frac{\alpha^*}{w^*}\left((w^* x^{*\prime} - (1-\delta))\frac{\partial \check{c}^*}{\partial c} + (1-\delta)\frac{L}{L^*}\right),$$

where $x^{(*)\prime}$ represents

$$x^{(*)\prime} = \frac{-\beta^{(*)} u^{(*)\prime\prime}}{\alpha^{(*)}\left(u^{(*)\prime}\right)^2},$$

which is the same as $x^{(*)\prime}(c^{(*)})$ derived from (32). The eigenvalues $\mu = (\mu_1, \mu_2)$ of this system solve the following characteristic equation:

$$\begin{vmatrix} \omega(-r_\omega^H + r_\omega^F) - \mu & \omega(-r_c^H + r_c^F) \\ \frac{c}{\eta}\left(\delta r_\omega^H + (1-\delta) r_\omega^F\right) & \frac{c}{\eta}\left(\delta r_c^H + (1-\delta) r_c^F\right) - \mu \end{vmatrix} = 0,$$

from which we find that the eigenvalues (μ_1, μ_2) satisfy

$$\mu_1 \mu_2 = -\frac{c}{\eta}\frac{\alpha\alpha^*}{ww^*}\frac{1}{LL^*}\left(\delta(1-\delta) - \hat{\delta}'(\omega)\omega\right)(cL + c^*L^*)\left((wx' - 1)L + (w^*x^{*\prime} - 1)L^*\frac{\partial \check{c}^*}{\partial c}\right), \quad (53)$$

where $\partial \check{c}^*/\partial c$ is given in (51).

We next show that $\mu_1\mu_2 < 0$ under the Marshall-Lerner condition (37). From (31), the left-hand side of (32) is larger than its right-hand side when $c^{(*)}$ is large enough to achieve full employment ($x^{(*)} = 1$). Thus, in order for the solution of c(or c^*) to exist even if c^*(or c) is very small, the left-hand side must be smaller than the right-hand side when c(or c^*) is zero. This also implies that the partial derivative of the right-hand side of (32) with respect to $c^{(*)}$ is smaller than that of the left-hand side. Using (33) and (34), we find that these properties are formally exhibited as follows:

$$\rho - \alpha > 0, \quad \left(\frac{-\beta u''}{\alpha\left(u'\right)^2}\right) w - \delta = wx' - \delta > 0;$$

$$\rho - \alpha^* > 0, \quad \left(\frac{-\beta^* u^{*\prime\prime}}{\alpha^*\left(u^{*\prime}\right)^2}\right) w^* - (1-\delta^*) = w^*x^{*\prime} - (1-\delta^*) > 0. \quad (54)$$

Under (54), in order for the two functions $c = h(c^*, z, z^*)$ and $c^* = h^*(c, z, z^*)$ given in (33) and (34) to intersect so that c and c^* exist for given z and z^*, the two functions h and h^* must satisfy

$$\left(\frac{\partial h}{\partial c^*}\right)\left(\frac{\partial h^*}{\partial c}\right) < 1,$$

which reduces to

$$\Upsilon \equiv \left(wx' - \delta\right)\left(w^*x^{*\prime} - 1\right) + \delta^*\left(wx' - 1\right) > 0. \quad (55)$$

From (12), (13), (33), (34) and (55), we find that c and c^* given in (35) satisfy

$$c_z < 0, \;\; c_{z^*} < 0; \;\; c_z^* > 0, \;\; c_{z^*}^* > 0, \quad (56)$$

i.e., rises in the home relative wage ω, causing z, z^* and the prices of home commodities to increase, decrease home labor demand, worsen home deflation and reduce home consumption while they increase foreign labor demand, alleviate foreign deflation and stimulate foreign consumption. From the first equation in (36), the Marshall-Lerner condition (37) gives

$$\frac{\partial \dot{f}}{\partial \omega} = w' z_\omega x + (wx' - 1)\left(c_z z_\omega + c_{z^*} z^*_\omega\right) < 0.$$

Using (4), (13), (28), (56), and the above condition, we find $wx' - 1 > 0$. Because the same condition holds with respect to the foreign current account, we find

$$wx' - 1 > 0, \qquad w^* x^{*\prime} - 1 > 0, \tag{57}$$

implying that $\mu_1\mu_2$ represented by (53) is negative; hence, one of the two values is positive and the other is negative.

Therefore, once κ and ω_0 are given, the dynamic equilibrium paths of ω and c are determined; hence, the paths of $c^*(c,\kappa)$, $w(\omega)$ and x are also determined. Substituting them into $\dot{f}$ given in (25), time integrating it and applying the no-Ponzi game condition to the result gives a relationship between $\kappa(= u^{*\prime}(c^*)/u'(c))$ and ω_0. It implies that a choice of ω_0 determines the relative magnitude of c and c^*. Thus, we choose ω_0 so that the current account given by Equation (25) remains zero. This prevents both countries from manipulation of foreign exchanges ε_0 to affect ω_0 and thereby disincentivizes expropriation of foreign assets.[11]

A.3 Asymmetric case ($x < 1$ and $x^* = 1$)

Applying the stagnation condition $R = \frac{\beta}{u'(c)}$, (22), and (45) to (18) yields

$$\eta\frac{\dot{c}}{c} = \frac{\beta}{u'(c)} - \alpha(x-1) - \rho + (1-\delta)\frac{\dot{\omega}}{\omega}. \tag{58}$$

From (23) with $x^* = 1$, we have

$$c^* = \tilde{c}^*(c,\omega), \;\; \dot{c}^* = \frac{\partial \tilde{c}^*}{\partial c}\dot{c} + \frac{\partial \tilde{c}^*}{\partial \omega}\dot{\omega}, \tag{59}$$

$$\frac{\partial \tilde{c}^*}{\partial c} = -\frac{\omega w^*}{w}\frac{1-\delta}{1-\delta^*}\frac{L}{L^*} < 0, \;\; \frac{\partial \tilde{c}^*}{\partial \omega} = -\frac{w^*(\Gamma_1 cL + \Gamma_2 c^* L^*)}{(1-\delta^*)L^*} < 0,$$

where Γ_1 and Γ_2 are given in (47). From (18) with (16) and (46) in the neighborhood of small trade barriers ($\delta \approx \delta^*$), we obtain the following relationships:

$$r = r^* \;\;\rightarrow\;\; \eta\frac{\dot{c}}{c} = \eta^*\frac{\dot{c}^*}{c^*} = r - \rho \;\;\rightarrow\;\; \kappa = \frac{u^{*\prime}(c^*)}{u'(c)} = \text{constant over time.} \tag{60}$$

[11] This stability analysis is similar to that in Ono (2014), which treats a two-good Ricardian model with secular demand stagnation.

Equations (59) and (60) give

$$\frac{\partial \tilde{c}^*}{\partial \omega}\dot{\omega} = \left(\frac{\eta}{\eta^*}\frac{c^*}{c} - \frac{\partial \tilde{c}^*}{\partial c}\right)\dot{c}. \tag{61}$$

Substituting $\dot{\omega}$ given by (61) into (58) and utilizing the sign conditions of $\frac{\partial \tilde{c}^*}{\partial c}$ and $\frac{\partial \tilde{c}^*}{\partial \omega}$ in (59) gives

$$\Omega\dot{c} = \Delta(c,\kappa) \equiv \left(\frac{\beta}{u'(c)} - \alpha(x-1) - \rho\right), \tag{62}$$

$$\Omega \equiv \frac{\eta}{c} + \frac{(1-\delta)\left(\frac{\eta}{\eta^*}\frac{c^*}{c} - \frac{\partial \tilde{c}^*}{\partial c}\right)}{-\frac{\partial \tilde{c}^*}{\partial \omega}} > 0.$$

Furthermore, from the second equation in (23) in which $x^* = 1$ and (60), ω is determined by c and κ; hence from the first equation of (23), x is also determined by c and κ: $x = \tilde{x}(c,\kappa)$. Thus, Equation (62) provides an autonomous dynamic equation of c for given κ.

Because c^f satisfies (38), $\Delta(c,\kappa)$ defined by (62) satisfie

$$\Delta(c^f,\kappa) \equiv \frac{\beta}{u'(c^f)} - \rho > 0.$$

Therefore, for the steady-state level of c, which makes $\Delta(c,\kappa) = 0$, to exist in the range of $(0, c^f)$, one must have

$$\Delta(0,\kappa) < 0.$$

When $c = 0$, we have $c^* = u^{*\prime -1}(\kappa u'(0)) = 0$ from (60), leading to $x = 0$ from (23). Therefore, from (62) to which the property that $u'(0) = \infty$ is applied, one finds $\Delta(0,\kappa) = -(\rho - \alpha)$. For this value to be negative, as mentioned above, it must be valid that

$$\rho - \alpha > 0.$$

In this case $\Delta(c,\kappa)$ must be positively inclined with respect to c around the steady state, and, thus,

$$\frac{\partial \Delta(c,\kappa)}{\partial c} > 0.$$

That is, the dynamics given by (62) is unstable. Therefore, c jumps to the level that satisfies $\Delta(c,\kappa) = 0$, and the steady state of (62) is immediately reached. Finally, κ is determined such that $\dot{f} = 0$, where $\dot{f}$ is given by (41).[12]

Appendix B: Proofs of Propositions

The following calculations hold in the neighborhood where

$$\tau = 0, \;\; \tau^* = 0.$$

[12]This stability analysis is similar to that in Ono (2018), which treats a two-good Ricardian model with secular demand stagnation.

B.1 Proof of Proposition 2

Partially differentiating $\dot{f}$ in (36) with respect to τ and applying (28), (56) and (57) to the result gives

$$\frac{\partial \dot{f}}{\partial \tau} = (wx' - 1)c_z z_\tau > 0.$$

Therefore, we obtain

$$\frac{d\omega}{d\tau} = -\frac{\frac{\partial \dot{f}}{\partial \tau}}{\frac{\partial \dot{f}}{\partial \omega}} > 0, \tag{63}$$

implying that an increase in τ raises ω and lowers the international relative prices of import commodities. Assuming that τ^* remains to be zero, from (36), we obtain

$$w'xdz + (wx' - 1)dc + (1 - \delta)cd\tau = 0,$$

$$w^{*\prime}x^*dz^* + (w^*x^{*\prime} - 1)dc^* = 0.$$

Noting that $wx = c$ from (36) in the steady state, applying (13) and (28) to the above equations, and utilizing (57) yields

$$\frac{dc}{d\tau} = -\left(\frac{(1-\delta)wx}{(wx'-1)\omega}\right)\frac{d\omega}{d\tau} < 0, \quad \frac{dx}{d\tau} = x'\frac{dc}{d\tau} < 0;$$

$$\frac{dc^*}{d\tau} = \left(\frac{\delta^* w^* x^*}{(w^*x^{*\prime}-1)\omega}\right)\frac{d\omega}{d\tau} > 0, \quad \frac{dx^*}{d\tau} = x^{*\prime}\frac{dc^*}{d\tau} > 0.$$

From (3), (4), (28) and (63), the effects on the trade structures are

$$\frac{dI^*}{d\tau} = \left(\frac{z^*_\omega}{A'(I^*)}\right)\frac{d\omega}{d\tau} < 0,$$

$$\frac{dI}{d\tau} = \frac{1}{A'(I)}\left(z_\omega \frac{d\omega}{d\tau} + z_\tau\right),$$

$$z_\omega \frac{d\omega}{d\tau} + z_\tau = \frac{w}{-\frac{\partial \dot{f}}{\partial \omega}}\frac{1}{z^*\Upsilon}\left(\omega(1-\delta)x\Upsilon - \left(\frac{L^*}{L}\right)x^*(wx'-1)(w^*x^{*\prime}-1)\left(\delta^*(1-\delta^*) - \hat{\delta}'(z^*)z^*\right)\right),$$

where $A' < 0$ from (3) and $\Upsilon > 0$ from (55). Thus, the effect of τ on I is ambiguous while that on I^* is negative .

B.2 Proof of Propositions 3 and 4

In the case where the home country is under stagnation and the foreign country achieves full employment, from (28) and (40), $\dot{f}$ given by (41) satisfies

$$\frac{\partial \dot{f}}{\partial \omega} = w'xz_\omega + \left(wx' - 1\right)\left(\hat{c}_z z_\omega + \hat{c}_{z^*} z^*_\omega\right) < 0,$$

$$wx' - 1 > 0,$$

$$\frac{\partial \dot{f}}{\partial \tau} = (wx' - 1)\hat{c}_z z_\tau > 0; \quad \frac{\partial \dot{f}}{\partial \tau^*} = (wx' - 1)\hat{c}_{z^*} z^*_{\tau^*} < 0.$$

These equations yield

$$\frac{d\omega}{d\tau} = -\frac{\frac{\partial \dot{f}}{\partial \tau}}{\frac{\partial \dot{f}}{\partial \omega}} = \frac{-(wx' - 1)\hat{c}_z z_\tau}{w' x z_\omega + (wx' - 1)\left(\hat{c}_z z_\omega + \hat{c}_{z^*} z^*_\omega\right)} > 0,$$

$$\frac{d\omega}{d\tau^*} = -\frac{\frac{\partial \dot{f}}{\partial \tau^*}}{\frac{\partial \dot{f}}{\partial \omega}} = \frac{-(wx' - 1)\hat{c}_{z^*} z^*_{\tau^*}}{w' x z_\omega + (wx' - 1)\left(\hat{c}_z z_\omega + \hat{c}_{z^*} z^*_\omega\right)} < 0, \tag{64}$$

implying that an increase in the home tariff τ raises the home relative wage ω while an increase in the foreign tariff τ^* reduces ω. Because $c = \hat{c}(z, z^*)$ from (39), using (64) we obtain the effects of τ and τ^* on home consumption c as follows:

$$\frac{dc}{d\tau} = \left(\hat{c}_z z_\omega + \hat{c}_{z^*} z^*_\omega\right)\frac{d\omega}{d\tau} + \hat{c}_z z_\tau = \frac{w' x z_\omega \hat{c}_z z_\tau}{w' x z_\omega + (wx' - 1)\left(\hat{c}_z z_\omega + \hat{c}_{z^*} z^*_\omega\right)} < 0,$$

$$\frac{dc}{d\tau^*} = \left(\hat{c}_z z_\omega + \hat{c}_{z^*} z^*_\omega\right)\frac{d\omega}{d\tau^*} + \hat{c}_{z^*} z^*_{\tau^*} = \frac{w' x z_\omega \hat{c}_{z^*} z^*_{\tau^*}}{w' x z_\omega + (wx' - 1)\left(\hat{c}_z z_\omega + \hat{c}_{z^*} z^*_\omega\right)} > 0,$$

which straightforwardly yield

$$\frac{dx}{d\tau} = x'\frac{dc}{d\tau} < 0, \qquad \frac{dx}{d\tau^*} = x'\frac{dc}{d\tau^*} > 0.$$

Because $x^* = 1$ and $f_0^* = 0$, from the second equation of (41) we obtain

$$w^*(z^*) = \frac{1 + \tau^*(1 - \hat{\delta}(z^*))}{1 + \tau^*} c^*.$$

Therefore, using (13), (28) and (64), we obtain

$$\frac{dc^*}{d\tau} = w^{*\prime} z^*_\omega \frac{d\omega}{d\tau} < 0,$$

$$\frac{dc^*}{d\tau^*} = w^{*\prime} z^*_\omega \frac{d\omega}{d\tau^*} > 0.$$

Let us next examine the effects of τ and τ^* on I and I^*. From (3) and (4), we find

$$dI = \frac{dz}{A'(I)} = \frac{A(I)}{A'(I)}\left(\frac{d\omega}{\omega} - d\tau\right), \quad dI^* = \frac{dz^*}{A'(I^*)} = \frac{A(I^*)}{A'(I^*)}\left(\frac{d\omega}{\omega} + d\tau^*\right).$$

Using (3), (13), (28), (40) and (64), we obtain

$$\frac{dI}{d\tau^*} = \frac{A(I)}{A'(I)}\frac{d\omega}{\omega d\tau^*} > 0, \quad \frac{dI^*}{d\tau} = \frac{A(I^*)}{A'(I^*)}\frac{d\omega}{\omega d\tau} < 0,$$

$$\frac{dI}{d\tau} = -\left(\frac{A(I)}{A'(I)\frac{\partial \dot{f}}{\partial \omega}}\right)\left(w' x z_\omega + (wx' - 1)\hat{c}_{z^*} z^*_\omega\right),$$

$$\frac{dI^*}{d\tau^*} = \left(\frac{A(I^*) z_\omega}{A'(I^*) \frac{\partial \dot{f}}{\partial \omega}} \right) \left(w' x + (w x' - 1) \hat{c}_z \right).$$

Therefore, the effect of τ on I and the effect of τ^* on I^* are ambiguous because $\hat{c}_{z^*} < 0$ and $\hat{c}_z < 0$.

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