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Working Paper 2403

May 2024

11200 SW 8th Street, Miami, Florida 33199

https://economics.fiu.edu/

Immigration and Secular Stagnation[*]

by

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March 25, 2024

Abstract

We examine the effect of international immigration on the host-country economy in the dynamic model capable of generating full employment as well as secular unemployment in equilibrium. It is shown that the effect of immigration depends on the host country's employment conditions and immigration magnitudes. If full employment prevails initially, a small inflow of immigrants boosts aggregate demand and improves welfare for host-country residents; a massive influx of immigrants leads to secular stagnation. If unemployment prevails initially, immigration always decreases aggregate demand and worsens unemployment. Furthermore, remittances by immigrants are always harmful to the host country under full employment but can beneficial under stagnation.

JEL: F16, F22, J61

Keywords: Immigration, secular stagnation, unemployment, aggregate demand, remittances

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[*]The authors thank seminar participants at ISER (Osaka University), SWET (Hitostubashi University), and Wayne State, Kobe, and Osaka Prefecture Universities, for their valuable comments. Ono acknowledges financial support from the Joint Usage/Research Center at ISER, Osaka University, and the Grants-in-Aid for Scientific Research (#20H05631), the Japan Society for the Promotion of Science (JSPS).

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1. Introduction

According to the 2022 UN International Organization for Migration (IOM) report, in 2020 there were 281 million global migrants or 4 percent of the world population of 7.8 billion people.[1] Not very surprisingly, over the last five decades Europe and Northern America have been the most popular destinations for international migration. In the United States, there were 46.2 million immigrants in 2022, an increase from 13.1 million in 2000.[2] Similarly, 37.5 million people living in the EU in 2022 were born outside the EU.[3] The rapid increase in the number of international immigrants begs the question of how influxes of immigrants affect income, jobs, and welfare for host-country residents. The present paper addresses these concerns.

Although there already exists much work investigating such concerns in the literature, this strand of research has focused only on the "real side" of the economy. In contrast, the present paper examines the effect of immigration in the dynamic model which highlights both real and monetary aspects of the economy. The next two sections describe the model, in which agents maximize intertemporal utility with respect to consumption (of the aggregate good) and real money holdings,[4] taking as given their initial endowments of internationally traded assets. This model thus can be considered an extension of the closed-country models developed by Ono (1994, 2001), Ono and Ishida (2014) and Michau (2018) to an international setting, where the host country is open to the world's commodity, capital, and equity markets.[5] As such, our

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[1] WMR-2022.pdf (iom.int) If immigrants formed their own country, it would have been the world's fourth most populous country after China, India, and the United States.

[2] Migration Policy Institute : https://www.migrationpolicy.org/programs/data-hub/charts/immigrant-population-over-time

[3] https://ec.europa.eu/eurostat/statistics.../Migration_and_migrant_population_statistics. This is despite a 30 per cent drop from the 2019 figure due to the pandemic.

[4] Furness (1910) shows how (stone) money holdings affected the utility of Uap (Micronesia) islanders (reported in Friedman 1994).

[5] Hashimoto and Ono (2019) develop a model dealing with this type of small-open economies.

model shares two key features with those works, in particular, Ono (1994, 2001). Firstly, the marginal utility of real balances is bounded away from zero. Secondly, wage adjustment is not instantaneous downward although the labor market adjusts according to the Walrasian mechanism. It is worth noting however that it is not the sluggish wage adjustment but the bounded marginal utility of real balances that gives rise to secular unemployment in our model. To see this, suppose that there is unemployment so that the nominal wage and the nominal price continue to decrease, thereby expanding real balances. Without the boundedness assumption, the marginal utility of real balances keeps falling towards zero so that consumers eventually want to spend money rather than keep holding it. This increase in spending boosts aggregate demand and creates new jobs, thereby setting off a virtuous cycle that continues until full employment is reached. Now suppose that the marginal utility of real balances is bounded away from zero. Then, although the nominal price continues to fall, the marginal utility of real balances may not decrease low enough to stimulate enough spending to achieve full employment. In such cases, the virtuous cycle noted above is forestalled and the economy remains stagnant.

With the possibility of stagnation, the first question we ask is when unemployment prevails and when full employment is achieved. The benchmark model yields the following results. Full employment is achieved if the host country holds internationally traded equities below some threshold level; otherwise there prevails chronic unemployment.

In the subsequent sections we introduce immigration into our benchmark model and investigate our main question: how an influx of immigrants affects the host country economy. Immigrants are assumed to differ from host-country natives in two respects. First, immigrants arrive with a given number of internationally traded assets but without host-country currency. This implies that on arrival in the host country, immigrants must convert part of their international assets to host-country currency to satisfy their demands for real balances (in local currency). Second, immigrants remit part of their earnings to families and relatives back home whereas host country natives have no such desires. We take immigrants' remittances as a parameter of the model (i.e., obligations determined outside of the model) and investigate their effect on the host-country economy.

Our key findings can now be summarized as follows. (1) If full employment prevails in the host country initially, an influx of immigrants stimulates consumption and improve lifetime welfare for native residents as long as the number of immigrants is not too large, or immigrants are not too rich (possessing large sums of international equities). If immigrants are too numerous or too rich, then the host country can slide into secular stagnation. (2) If chronic unemployment prevails in the host country initially, an influx of immigrants unconditionally exacerbates unemployment and reduces consumption and income for natives. We note however that income losses do not necessarily mean that native residents are hurt by immigration because their real balances also grow faster. (3) The effect of immigrants' remittances also depends on the initial economic state of the host country. If full employment initially prevails, remittances decrease natives' consumption and lifetime welfare, whereas if unemployment prevails initially, remittances boost natives' consumption and employment.

We now review some relevant literature. As already mentioned, there exists an extensive literature, both formal and descriptive, that investigates various aspects of immigration and immigration policies. To save space, our review covers only formal studies. Early work in this line of research has regarded international immigration as a case of international factor mobility within standard factor-endowment trade models.[6] Subsequent research however has focused on features specific to international movements of labor per se. Pioneering in this endeavor, Ethier (1985) has examined the nature of temporary immigration as under guest-worker programs

[6] See e.g., Berry and Soligo (1969), Dixit and Norman (1980), and Markusen (1983).

administered in West Germany and elsewhere at that time. However, many temporary immigrants in Europe have opted to stay permanently in their host countries. Furthermore, the majority of todays' immigrants seem to be permanent settlers rather than temporary job seekers. Therefore, in this paper we study the effect of permanent immigration.

More recent work on immigration has turned attention to the presence of unemployment in host countries, investigating how immigrants and host-country immigration policies can affect unemployment of native workers.[7] To model unemployment, this strand of research has typically adopted the search-theoretic approach, where unemployment arises as an equilibrium phenomenon.[8] For example, Ortega (2000) and Miyagiwa and Sato (2019) have studied how changes in host country immigration policy affects endogenous immigration flows, whereas Liu (2010), Chassamboulli and Palivos (2014) and Battisti et al. (2018), have investigated the effect of an exogenous influx of immigrants on native workers' wages and welfare. In the present paper we also examine the impact of an exogenous inflow of immigrants on the destination economy but focus on involuntary unemployment instead of frictional unemployment as in the precursory work.

The remainder of this paper is organized in 7 sections. The next section describes the general environment of the model. Section 3 presents the benchmark model of a small open economy and studies its properties in the absence of immigration. Section 4 modifies the benchmark to deal with immigration. Section 5 studies the effect of immigration when the host country enjoys full employment prior to an influx of immigration. Section 6 extends the analysis to the case of secular stagnation. Section 7 studies the welfare impact of remittances.

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[7] Ethier (1986) has initiated research on illegal immigration. His model features unemployment due to fixed wages. Subsequent work on illegal immigration, with and without unemployment, includes Bonds and Chen (1987), Djajić (1997), Carter (1999), Woodland and Yoshida (2006), Liu (2010), Mangin and Zenou (2016), and Miyagiwa and Sato (2019), among others.

[8] Pissarides (2000) is the standard reference for equilibrium unemployment.

Section 8 concludes.

2. Environment

We consider a small open economy in a continuous infinite-time horizon. The country produces the aggregate good with labor and capital according to the neoclassical production function $F(L(t), K(t))$, where $K(t)$ and $L(t)$ are quantities of capital and labor used at time t, respectively. (The time index t is suppressed below unless ambiguities arise.) Assuming constant returns to scale, we can rewrite the function production F as

$$F(L, K) = f(n)K,$$

where

$$n \equiv L/K.$$

The country is open to the world capital market and capital moves across borders instantaneously. We assume, to keep things simple, that firms can rent capital freely from the world market at the real equity rate r. This implies that firms carry no state variables, so they maximize momentary profits $f(n)K - wn - rK$ at each instant, taking r and w (the real wage) as given. The first-order conditions are:

$$f'(n) = w, \tag{1}$$

$$f(n) - nf'(n) = r. \tag{2}$$

(Primes denote differentiation.) Under diminishing returns to factors, these equations uniquely determine n and w, given r. Note that the equilibrium n and w are independent of time.

All individuals are endowed with one unit of labor. Leisure yields no utility, so individuals prefer to supply their entire labor endowments to the labor market. However, they may be prevented from doing so by demand shortages. To allow for such possibilities, let σ denote the realized rate of employment ($0 < \sigma \leq 1$). Then the typical individual's realized real labor income equals σw. When $\sigma = 1$, there is full employment; otherwise, there is unemployment.

There are two types of individuals: native-born (indexed by h, or *host country*) and immigrants (indexed by i). All individuals $j(= h, i)$ have identical preferences, deriving momentary utility $u(c_j(t)) + v(m_j(t))$ from consuming $c_j(t)$ units of the aggregate good and holding real money balances $m_j(t)$ at time t. These subutility functions are assumed to satisfy:

Assumption 1:

(a) For all $c_j \geq 0$, $u(c_j)$ is strictly increasing, strictly concave and continuously differentiable, and satisfies the Inada conditions; i.e., $\lim_{c_j \to +0} u'(c_j) = \infty$ and $\lim_{c_j \to \infty} u'(c_j) = 0$.

(b) For all $m_j \geq 0$, $v(m_j)$ is continuously differentiable with positive first derivatives and weakly concave. Specifically, there is $\overline{m} > 0$ such that $v'(m_j)$ is strictly decreasing for all $m_j < \overline{m}$ and $v'(m_j) = \beta > 0$ for all $m_j \geq \overline{m}$.

We show in the next section that the presence of the lower bound $\beta > 0$ on the marginal utility of money $v'(m_j)$ is crucial for the existence of unemployment.[9]

The representative individual maximizes the utility functional:

$$\int_0^\infty \left(u(c_j) + v(m_j)\right) \exp(-\rho t)\, dt, \tag{3}$$

where ρ denotes the subjective discount rate, subject to two constraints. One is the stock budget constraint

$$a_j = m_j + b_j, \tag{4}$$

which indicates that an agent j can hold his real assets a_j in two forms: real money balances and real equities. Real money balances, m_j, yield no interest and are not traded internationally.

[9] Also see Ono (1994, 2001), Illing et al. (2018), and Hashimoto et al. (2023).

By contrast, real equities or "bonds," denoted by b_j, are traded in the world markets and yield the real return r per unit as mentioned above.

The second constraint faced by the agent is the flow-budget constraint:

$$\dot{a}_j = w\sigma + (ra_j - Rm_j) - c_j - \tau_j, \tag{5}$$

which describes how an agent's real asset holdings change over time (the "dot" over variables denotes time derivatives; e.g., $\dot{a}_j \equiv da_j/dt$). The first term on the right-hand side of (5) is the agent's realized labor income at the employment rate σ. The second is the real interest income from holding assets a_j, where R is the nominal interest rate. Letting $\pi = R - r$ be the rate of inflation (deflation if negative), we can rewrite this interest income $ra_j - Rm_j = rb_j - \pi m_j$. The remaining terms are consumption c_j and remittances $\tau_j (\geq 0)$.

The associated Hamiltonian is

$$H = u(c_j) + v(m_j) + \lambda(w\sigma + ra_j - Rm_j - c_j - \tau_j),$$

where λ is the co-state variable. The first-order conditions are

$$\lambda = u'(c_j),$$

$$\lambda R = v'(m_j),$$

$$\dot{\lambda} = (\rho - r)\lambda,$$

which combine to yield the optimality condition that c_j and m_j must fulfill at each instant:

$$\rho + \pi + \eta_j \frac{\dot{c}_j}{c_j} = R = \frac{v'(m_j)}{u'(c_j)}, \tag{6}$$

where $\eta_j \equiv -u'' c_j / u' > 0$ is the elasticity of marginal utility of real consumption. The left-hand side of (6) represents the intertemporal marginal rate of substitution, i.e., an individual's desire to consume (now instead of later). If this desire is less than the nominal interest rate R, the individual decreases his consumption. The right-hand side of (6) measures the intratemporal marginal rate of substitution between real balances and consumption, i.e., the desire to hold real balances (the "liquidity premium"). If this desire is greater than R, the individual sells bonds to hold more real balances. Equation (6) thus guarantees that nobody has the incentive to change his consumption level and real money balances at the optimum. In addition to (6), the optimal c_j and a_j also fulfill the transversality condition:

$$\lim_{t\to\infty} u'(c_j(t))a_j(t)\exp(-\rho t) = 0. \tag{7}$$

Turning to the money market, we assume that the home-country monetary authority keeps its money supply, assumed to be fixed at M^s (This assumption is slightly modified when we discuss the effect of immigration.) Equilibrium in the money market requires

$$M^s/P = m, \tag{8}$$

where m denotes the economy-wide real balances and P the nominal price of the aggregate good. Time differentiation of (8) yields

$$\dot{m} = -\pi m. \tag{9}$$

The small home country takes the international nominal price P^I of the aggregate good as given. Since the good is traded freely in the international market, the exchange rate e adjusts instantaneously to satisfy

$$P = eP^I. \tag{10}$$

The next assumption is standard in the model for a small open economy for the interior solution of the model to exist:

Assumption 2: $\rho = r$.

Setting $\rho = r(= R - \pi)$ in (6) yields

$$\frac{\dot{c}_j}{c_j} = 0,$$

implying that under Assumption 2 c_j is constant over time. This simplifies the optimality condition (6) to be rewritten as

$$\rho + \pi = R = \frac{v'(m_j)}{u'(c_j)} \quad \text{for } j = i, h. \tag{11}$$

Turning to the labor market, we adopt the conventional Walrasian wage adjustment mechanism. When labor is fully employed, wage adjustments are assumed instantaneous. By contrast, when there is unemployment, the nominal wage W is assumed to fall over time according to

$$\dot{W}/W = \alpha(\sigma - 1) < 0,$$

where the parameter $\alpha(> 0)$ represents the speed of adjustment.[10] Note that full employment or not, equations (1) and (2), and assumption 2 imply that P and W move in tandem to keep the real wage constant ($w = W/P$). Given that the money supply M^s remains constant over time, the above price adjustment process can be summarized as follows:[11]

$$\pi = 0 \quad \text{for} \quad \sigma = 1$$

$$\pi = \alpha(\sigma - 1) < 0 \quad \text{for} \quad \sigma < 1. \tag{12}$$

3. The benchmark model (without immigration)

This section presents the model without immigration. To keep the analysis simple, we assume that native-born individuals send or receive no remittances overseas (i.e., $\tau_h = 0$). We also normalize the native population to one without loss of generality. With this normalization the individual budget constraints (4) and (5) apply to the whole country in the absence of immigrants so write $m = m_h$. Substituting from (4) and applying (9), the flow budget constraint (5) can be writtten

$$\dot{b}_h = \rho b_h + w\sigma - c_h. \tag{13}$$

[10] Ono and Ishida (2014) present the microfoundations of wage adjustment mechanism that converges to such adjustment.

[11] Schmitt-Grohé and Uribe (2016, 2017) also assume a similar wage adjustment mechanism.

Furthermore, we prove, in Appendix A, that the economy is always in a steady state so that its current account is always balanced. Thus, $b_h = b_h^0$, where b_h^0 denotes the initial stock of real equities (bonds), i.e., the host country's initial capital endowment under the normalization of the native population. Substituting b_h^0 in (13) and letting $\dot{b}_h = 0$ yields

$$c_h = w\sigma + \rho b_h^0. \tag{14}$$

The right-hand side of (14) is the host country's total income, comprising the wage income and the interest income from the bond holdings. (14) implies that the native population consumes all its income. Call (14) the balanced-trade condition.

3.1. The benchmark model with full employment

Suppose the country enjoys full employment ($\sigma = 1$). Then the benchmark model has a recursive solution. First, setting $\sigma = 1$ in the balanced-trade condition (14) pins down the full-employment consumption level:

$$c_h = c_h^F \equiv w + \rho b_h^0.$$ [12]

Next, since the nominal price is constant under full employment, setting $\pi = 0$ in the optimality condition (11), we get

$$\rho = \frac{v'(m_h^F)}{u'(c_h^F)}, \tag{15}$$

which determines the equilibrium real money balances m_h^F. Then the money market-clearing condition (8) determines the nominal price P, given the money supply M^s. Finally, the exchange rate adjusts to satisfy the condition in (10).

Having solved the benchmark model for full employment, we turn investigate when full employment prevails. To that end, let us define the consumption level $\overline{c}$ by

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[12] We assume $b_h^0 > -w/\rho$ to ensure $c_h^F > 0$.

$$\rho = \frac{\beta}{u'(\overline{c})} \tag{16}$$

Then, (15) implies

$$\rho = \frac{v'(m_h)}{u'(c_h^F)} = \frac{\beta}{u'(\overline{c})}.$$

Since $v'(m_h) \geq \beta$ by Assumption 1(b), (16) implies that the full employment consumption is bounded above by $\overline{c}$:

$$c_h^F \equiv w + \rho b_h^0 \leq \overline{c}. \tag{17}$$

The converse of this result also holds as demonstrated in the next subsection. Thus, condition (17) is both necessary and sufficient for existence of an equilibrium with full employment. Moreover, this equilibrium is unique.

Proposition 1: Under assumptions 1 and 2 the model admits a unique equilibrium with full employment if and only if $c_h^F (\equiv w + \rho b_h^0) \leq \overline{c}$.

Recall that native-born individuals spend all their income on consumption. Therefore, if natives own too many bonds, their total income $(w + \rho b_h^0)$ may exceed the limit $\overline{c}$. In such a case, condition (17) is violated and hence by proposition 1 there cannot be an equilibrium with full employment.

3.2. The benchmark model with unemployment

We now turn to the case with unemployment ($\sigma < 1$). With unemployment, we have $\pi = \alpha(\sigma - 1) < 0$ by (12), so the nominal price P falls continuously, thereby increasing the real balances $m = M^s/P$ above the threshold level $\overline{m}$ such that $v'(m_h) = \beta$ holds in equilibrium. Thus, the optimality condition (11) is given by

$$\rho + \alpha(\sigma - 1) = \frac{\beta}{u'(c_h)}. \tag{18}$$

To keep the left-hand side of (18) positive for any $\sigma \in [0,1]$, we need $\rho > \alpha$; that is, the speed of wage adjustment cannot be too fast.

Since the current account (13) is always balanced, the balanced-trade condition (14) holds despite the on-going deflation, implying that the equilibrium consumption level and the employment rate are determined jointly by (14) and (18). We illustrate this in Figure 1 where (14) is presented by the straight line and (18) by the monotone-increasing curve.[13] The intersection point A gives us the equilibrium consumption and employment level, denoted by c_h^* and σ^*.

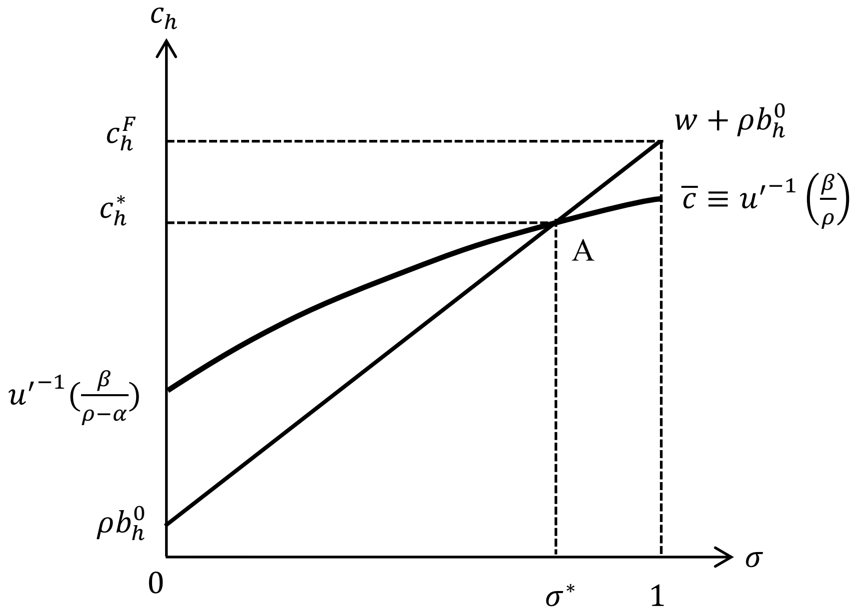

Figure 1:
Equilibrium with unemployment (benchmark)

It is straightforward now to ascertain the conditions for the existence of equilibrium with unemployment. There is unemployment ($0 < \sigma^* < 1$) if the straight line crosses the curve at

[13] The graph of (18) is strictly upward-sloping but not necessarily concave as drawn in Figure 1.

$\sigma < 1$. From Figure 1 this requirement is satisfied if

$$c_h^F \equiv w + \rho b_h^0 \ > \overline{c} = u'^{-1}(\beta/\rho), \tag{19}$$

Inverting condition (19), we get $\rho < \beta/u'(c_h^F)$. Thus, when (19) holds, we have the marginal desire to hold real balances $\beta/u'(c_h^F)$ under full employment exceeding the marginal desire to consume, ρ, and so there cannot be full employment in equilibrium.[14]

In the analysis we also want to rule out the possibility of zero employment. To that end we assume that

$$\rho b_h^0 < u'^{-1}(\beta/(\rho - \alpha)). \tag{20}$$

Condition (20) implies that the straight line lies strictly below the curve at $\sigma = 0$ as in Figure 1.[15]

The above discussion suggests that both conditions (19) and (20) are necessary for the existence of an equilibrium with unemployment. They are also sufficient because, if they both hold, an appeal to the intermediate-value theorem proves the existence of an equilibrium with $0 < \sigma < 1$.

Moreover, the equilibrium is unique if the straight line is steeper than the curve at the intersection point. A little algebra expresses this condition as

$$\Omega \equiv w + \left(\frac{\alpha}{\beta}\right)\frac{(u')^2}{u''} > 0. \tag{21}$$

where w is determined by (1) and (2). For the remainder of our analysis, we assume that condition (21) holds in the neighborhood of the equilibrium. (21) also leads to the following lemma, the proof of which is given in Appendix A.

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[14] This proves that (17) is also sufficient for the existence of an equilibrium with full employment, as alluded to in the paragraph leading to proposition 1.

[15] Figure 1 depicts the case in which $b_h^0 > 0$. That is just for the sake of presentation. If $b_h^0 < 0$, the straight line cuts the σ-axis at $\sigma > 0$. This however does not affect our analysis, given that condition (20) holds.

Lemma 1: When (21) holds, the economy is always in steady state; that is, $\dot{c}_i$, $\dot{a}_i$ and $\dot{b}$ are zero.

Lemma 1 implies that any parameter change immediately moves the economy to a new steady-state equilibrium without any transition phase.

Finally, we show that the transversality condition (7) is satisfied with unemployment even though real balances m_h keep expanding. Since the price falls at the rate $\pi = \alpha(\sigma - 1)$, we can write $m_h(t) = m_h(0)\exp(-\pi t)$. From (18) we also have

$$\pi = \alpha(\sigma - 1) = \frac{\beta}{u'(c_h)} - \rho (< 0).$$

Therefore,

$$\lim_{t\to\infty} u'(c_h)\, m_h(t) \exp(-\rho t) = \lim_{t\to\infty} u'(c_h)\, m_h(0) \exp\left(-\frac{\beta}{u'(c_h)} t\right) = 0.$$

Because b_h stays constant at b_h^0, the above result implies that

$$\lim_{t\to\infty} u'(c_h)\, a_h(t) \exp(-\rho t) = \lim_{t\to\infty} u'(c_h)\, (m_h(t) + b_h^0) \exp(-\rho t) = 0,$$

proving that the transversality condition (7) holds.

The next proposition summarizes our findings of this subsection so far.

Proposition 2: Under assumptions 1 and 2, there is an equilibrium with unemployment if and only if b_h^0 satisfies conditions (19) and (20). The equilibrium is unique under condition (21).

Contrary to Assumption 1(b), suppose that the marginal utility of real balances is bounded below by zero instead of $\beta > 0$. Then, as $m_h \to \infty$, $v'(m_h)$ approaches zero, so the optimality condition (18) is replaced by

$$\rho + \alpha(\sigma - 1) = \frac{v'(m_h)}{u'(c_h)}. \tag{22}$$

As expanding real balances drive $v'(m_h)$ down toward zero, c_h must keep increasing under condition (22). This continuous rise in consumption creates new jobs, raising the employment rate σ until full employment is achieved, at which time the nominal price halts its decline. Thus, unemployment cannot occur unless the marginal utility of real balances is bounded away from zero.

We now offer the intuitive explanation of Proposition 2. Suppose that the country's income $w + \rho b_h^0$ happens to equal $\overline{c}$. Then the straight line and the curve in Figure 1 meet at $\sigma = 1$, with the country consuming $\overline{c} = w + \rho b_h^0 \equiv c_h^F$ and holding $\overline{m}$ of real balances. This equilibrium with full employment satisfies the optimality condition:

$$\rho = \frac{v'(\overline{m})}{u'(c_h^F)} = \frac{\beta}{u'(\overline{c})}.$$

If the country has a greater income $(w + \rho b_h^0 > \overline{c})$, full employment can no longer be maintained. To show this, recall that consumption cannot exceed the limit $\overline{c}$; cf. (17). Consumption cannot be equal to $\overline{c}$, either, because then the income would exceed the consumption by $w + \rho b_h^0 - \overline{c} > 0$. If the country spends this excess income to purchase foreign assets, there is a perpetual current account surplus, causing the country's currency to appreciate, which makes the country's goods less competitive compared with foreign-produced goods, reducing labor demand and hence its national income. Actually, since the foreign and home goods are homogeneous, this currency appreciation is unobserved in equilibrium. Instead, adjustment occurs directly through a drop in employment rate σ. To keep the current account balanced, σ falls enough to bring the country's income down to equal its actual consumption level; i.e., $w\sigma + \rho b_h^0 = c_h$. This new consumption c_h is less than $\overline{c}$ by (16) and (18) because $\sigma < 1$.

The above discussion implies that $\rho b_h^0 = \overline{c} - w$ is the maximum interest income consistent with full employment. If native-born individuals more bonds such that $\rho b_h^0 > \overline{c} - w$, then unemployment occurs by Proposition 2. We report these results in

Corollary 1: (a) If $\rho b_h^0 \leq \overline{c} - w$, there is full employment.

(b) If $\rho b_h^0 > \overline{c} - w$, there is unemployment.

4. Immigration: an overview

We now extend the model to allow immigration from foreign countries. To that end we assume the following. At time $t = t_0$, a given number, say, x_i of immigrants enter the country (the subscript i denotes immigrants). Immigrants are each endowed with one unit of labor and maximize the same utility functional given in (3) as natives do. As mentioned in the introduction, however, immigrants differ from natives in two respects. First, a typical immigrant arrives endowed with b_i^0 units of internationally traded bonds but without any host-country currency. Thus, upon entry immigrants immediately exchange their bonds (or borrow against their future incomes) for local currency to satisfy their demands for real balances.

Suppose that the country's monetary authority increases its money supply to satisfy immigrants' demand for money. Then if each immigrant acquires m_i units of local-country currency, the monetary authority ends up with $x_i m_i$ units of additional international bonds, earning the interest income $r x_i m_i$. We assume that these interest earnings are rebated evenly to natives.[16] At the end of the day, it is as if each native's bond holding has increased from b_h^0 to $b_h^0 + x_i m_i$ (while each immigrant's bond holdings has fallen to $b_i^0 - m_i$).

The second way immigrants differ from natives is with respect to remittances. It is assumed that each immigrant remit $\tau_i (\geq 0)$ units of the aggregate good back home whereas natives have

[16] Because natives' income increases, they too want to increase money holdings. To meet this money demand, the monetary authority purchases bonds with newly issued money, further increasing the country's money stock. Simultaneously, it also rebates the interest earnings on the newly acquired bonds to natives. Such adjustment is instantaneously completed the moment the host country takes immigrants in. Thereafter, the money stock remains constant.

no such obligations ($\tau_h = 0$). We take τ_i as given and investigate its effect below.

With the abovementioned changes, we rewrite a native's flow asset constraint at $t > 0$ as

$$\dot{a}_h = \rho(b_h^0 + x_i m_i) - \pi m_h + w\sigma - c_h, \tag{23}$$

and an immigrant's counterpart as

$$\dot{a}_i = \rho(b_i^0 - m_i) - \pi m_i + w\sigma - c_i - \tau_i. \tag{24}$$

Adding up these equations over all residents and recalling that the native-born population size is one, we get the following aggregate flow budget constraint:

$$\dot{a}\left(= \dot{b} + \dot{m}\right) = \rho(b_h^0 + x_i b_i^0) - \pi m + w\sigma(1 + x_i) - (c_h + x_i c_i) - x_i \tau_i,$$

where $m \equiv m_h + x_i m_i$ (the variables without subscripts denote the aggregated values). Substituting from (9) and rearranging terms, we can rewrite the above constraint as

$$\dot{b} = \rho(b_h^0 + x_i b_i^0) + w\sigma(1 + x_i) - (c_h + x_i c_i) - x_i \tau_i = 0. \tag{25}$$

By lemma 1, the economy is always in steady state so the expression in (25) always equals zero as indicated. This is the balanced-trade equation adapted to an economy which accepts immigrants.

5. Full employment

Suppose that we have full employment in a post-immigration equilibrium. Since $\pi = 0$ under full employment, the corresponding optimality conditions are given by

$$\rho = \frac{v'(m_h)}{u'(c_h)} = \frac{v'(m_i)}{u'(c_i)} \left(= \frac{\beta}{u'(\overline{c})}\right), \tag{26}$$

where the last equality follows from (16). Since $v'\left(m_j\right) \geq \beta$, (26) implies that $c_j \leq \overline{c}$; as in the benchmark model the equilibrium consumption levels natives and immigrants are bounded above by $\overline{c}$. With the nominal price constant, the individual asset holdings do not change over time. Setting $\dot{a}_j = 0$ in the flow budget constraints (23) and (24) yields

$$c_h = \rho(b_h^0 + x_i m_i) + w, \tag{27}$$

$$c_i = \rho(b_i^0 - m_i) + w - \tau_i. \tag{28}$$

Equations (26), (27) and (28) can be solved for the equilibrium consumption and real balances, denoted by $\tilde{c}_j$ and $\widetilde{m}_j$ ($j = h, i$).

However, (26) is consistent with two types of equilibria, depending on whether $\widetilde{m}_i$ exceeds $\overline{m}$. If $\widetilde{m}_i \leq \overline{m}$, then (26) can be arranged to yield

$$\widetilde{m}_i = v'^{-1}\big(\rho u'(\tilde{c}_i)\big) \equiv \varphi(\tilde{c}_i) \quad \text{for } \widetilde{m}_i \leq \overline{m}; \quad \varphi'(\cdot) > 0, \quad \varphi(\overline{c}) = \overline{m}. \tag{29}$$

Substituting for $\widetilde{m}_i$ from (29) into (28), we get

$$\tilde{c}_i + \rho\varphi(\tilde{c}_i) = (\rho b_i^0 - \tau_i) + w, \tag{30}$$

which determines a unique $\tilde{c}_i$, given the monotonicity of $\varphi(c_i)$. Then, substituting $\tilde{c}_i$ into (29) we get the immigrant's real balances $\widetilde{m}_i = \varphi(\tilde{c}_i)$. Since $\varphi' > 0$ by (29), (30) implies

$$\rho b_i^0 - \tau_i \uparrow \;\Rightarrow\; \tilde{c}_i \uparrow, \;\; \widetilde{m}_i \uparrow \quad \text{for } \widetilde{m}_i \leq \overline{m}. \tag{31}$$

Let us call the term $(\rho b_i^0 - \tau_i)$ an immigrant's "net worth." Then (31) states that the richer an immigrant (the greater his net worth), the more he consumes and the more real balances he holds. Furthermore, it is straightforward to show that there is a unique net worth NW^0 equal to

$$NW^0 = (\overline{c} - w) + \rho\overline{m},$$

such that the immigrant consumes $\tilde{c}_i = \overline{c}$ and holds $\widetilde{m}_i = \overline{m}$. Then (31) is valid only for $\rho b_i^0 - \tau_i \leq NW^0$.

If $\rho b_i^0 - \tau_i > NW^0$, we have a second type of equilibrium, where the immigrant consumes exactly $\overline{c}$ while holding $\widetilde{m}_i > \overline{m}$. In this case, an immigrant's marginal utility from real balances must equal β. Therefore, if an immigrant consumes in excess of $\overline{c}$, we have $\rho < \beta/u'(c_i)$, that is, his desire to hold real balances exceeds his desire to consume, prompting him to sell more bonds, thereby reducing his interest income. In equilibrium, an immigrant's income must fall until it equals $\overline{c}$, i.e.,

$$\overline{c} = \rho(b_i^0 - m_i) + w - \tau_i.$$

This equation determines an immigrant's money balances:

$$\widetilde{m}_i = [\rho b_i^0 - \tau_i + w - \overline{c}]/\rho \;(> \overline{m}). \tag{32}$$

It is evident that for $\widetilde{m}_i > \overline{m}$ we have that

$$\rho b_i^0 - \tau_i \uparrow \;\Rightarrow\; \tilde{c}_i = \overline{c}, \;\; \widetilde{m}_i \uparrow. \tag{33}$$

(31) and (33) demonstrate that an increase in the immigrant's net worth always raises his real balances $\widetilde{m}_i$, whether $\widetilde{m}_i$ exceeds $\overline{m}$ or not.

The preceding discussion has assumed full employment in a post-immigration equilibrium. However, this is valid only if $x_i \widetilde{m}_i$ is sufficiently small, i.e., immigrants are neither too rich nor too numerous. Otherwise, natives would receive so much interest income from the monetary authority to the extent that their total income exceeds the consumption limit $\overline{c}$, violating the condition for full employment. With $\tilde{c}_h \leq \overline{c}$, (27) implies the following result:

Proposition 3: There exists a post-immigration equilibrium with full employment only if

$$\rho b_h^0 + w \leq \overline{c} - \rho x_i \widetilde{m}_i, \tag{34}$$

where $\widetilde{m}_i$ is determined by (29) and (30) or by (32).

Proposition 3 states that if condition (34) holds, there is full employment even if immigrants hold real balances $\widetilde{m}_i > \overline{m}$. This statement contrasts with Proposition 2, which states that when natives hold real balances above $\overline{m}$, there cannot be full employment. This difference can be understood as follows. Recall that immigrants can sell as many bonds as they want in order to reduce their income to equal $\overline{c}$. By contrast, even if natives sell their bonds to the monetary authority, their incomes remain unchanged because the monetary authority rebates all interest earnings on purchased bonds to them. This implies that natives' incomes can change only through adjustment in the employment rate σ.

Plugging $\widetilde{m}_i$ given above into (26) and (27), we find the native's equilibrium consumption and real money holdings:

$$\tilde{m}_h = v'^{-1}\big(\rho u'(\tilde{c}_h)\big) \equiv \varphi(\tilde{c}_h),$$

$$\tilde{c}_h = \rho(b_h^0 + x_i \tilde{m}_i) + w \; (\leq \overline{c}).$$

The second equation says that $\tilde{c}_h$ increases with immigrants' real balances $\tilde{m}_i$. This is evident because natives receive a greater interest income when immigrants exchange more bonds for local money. Then the first equation above implies that natives' real balances also increase. Moreover, the immigrant with a greater net worth $(\rho b_i^0 - \tau_i)$ holds more money; cf. (31) and (33). These facts establish the next proposition.

Proposition 4: Suppose there is full employment before immigration. Then, provided that (34) holds, immigration causes the country's economy to jump to a new steady state with full employment. Furthermore,

(a) Immigration increases natives' consumption and real balance holdings.

(b) The richer the immigrants (i.e., endowed with more bonds), the higher the natives' consumption level and real balance holdings.

(c) The greater remittances by immigrants, the smaller the natives' consumption and real balance holdings.

Proposition 4 has the intuitive explanation. Immigrants exchange international bonds for local currency. While the country's currency is not traded internationally, acquisitions of internationally traded bonds allow natives to import and consume more of the aggregate good. Because richer immigrants convert more international bonds, natives' consumption rises with immigrant's net worth, provided that condition (34) holds.

Notice that condition (34) in Proposition 3 is more stringent than the condition in Proposition 1. That is, (34) implies (17) and hence full employment exists in a post-immigration equilibrium only if there is full employment before immigration. The converse of this statement is false, however. As we already noted, even if (17) holds, an influx of too many or too rich immigrants can increase the interest income $\rho x_i \widetilde{m}_i$ rebated to natives such that condition (34) gets violated. In the next section we turn to investigate such cases.

6. Unemployment

In this section we first characterize a post-immigration equilibrium with unemployment. As stated earlier, in an equilibrium with unemployment ($\sigma < 1$) the nominal wage and nominal price keep falling according to (12), i.e., $\pi < 0$, such that natives end up holding more real balances than the threshold $\overline{m}$. This implies that in an equilibrium with unemployment the following optimality condition holds:

$$\rho + \alpha(\sigma - 1) = \frac{\beta}{u'(c_h)} = \frac{v'(m_i)}{u'(c_i)} \left(< \frac{\beta}{u'(\overline{c})} = \rho\right). \tag{35}$$

The inequality in (35) follows from (16) and implies that $c_j \leq \overline{c}$ for $j = h, i$. Given that the nominal price is falling, (35) suggests two possible cases to be distinguished, depending on the size of m_i.

The first case arises when $m_i < \overline{m}$. In this case, m_i stays constant, i.e., the immigrant's asset holding is not expanding despite deflation. Therefore, we can set $\dot{a}_i = 0$ in (24) to get

$$\rho b_i^0 - \tau_i + \sigma w - c_i - [\rho + \alpha(\sigma - 1)]m_i = 0. \tag{36}$$

Equations (35) and (36) determine an immigrant's optimal real balances and consumption level in terms of σ and his net worth $\rho b_i^0 - \tau_i$. Straightforward algebra yields

$$m_i = m_i(\sigma; \rho b_i^0 - \tau_i), \ \frac{\partial m_i}{\partial(\rho b_i^0 - \tau_i)} > 0 \quad \text{for} \ m_i < \overline{m}, \tag{37}$$

$$c_i = c_i(\sigma; \rho b_i^0 - \tau_i), \ \frac{\partial c_i}{\partial(\rho b_i^0 - \tau_i)} > 0 \quad \text{for} \ c_i < u'^{-1}\left(\frac{\beta}{\rho + \alpha(\sigma - 1)}\right). \tag{38}$$

Thus, an immigrant's consumption and real balances increase with his net worth.

The country's current account also remains balanced despite deflation, validating (25). We can thus substitute from (36), (37) and (38) into (25) to obtain, after arranging,

$$c_h = w\sigma + \rho b_h^0 + x_i[\rho + \alpha(\sigma - 1)]m_i(\sigma; \rho b_i^0 - \tau_i). \tag{39}$$

The following optimality condition from (35) also holds:

$$\rho + \alpha(\sigma - 1) = \frac{\beta}{u'(c_h)}, \tag{40}$$

(39) and (40) jointly determine the native's consumption $\hat{c}_h$ and the unemployment rate $\hat{\sigma}$. Note that (40) is identical to the optimality condition (18) under unemployment in the benchmark model (section 3).

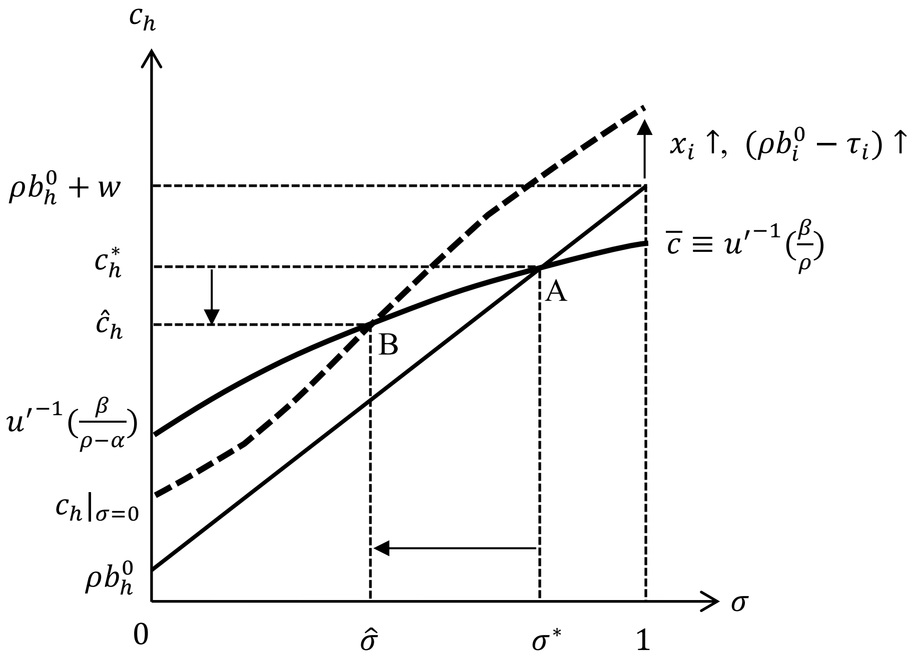

Figure 2:
Post- and pre-immigration equilibria with unemployment (first scenario)

In Figure 2, the broken curve through point B represents (39) while the solid curve depicts equation (40). The intersection point B determines the equilibrium values $\hat{c}_h$ and $\hat{\sigma}$. We can find an immigrant's consumption $\hat{c}_i$ and real balances $\hat{m}_i$ by substituting $\hat{\sigma}$ into (37) and (38). Figure 2 can also be used to derive the conditions guaranteeing the existence of an equilibrium with unemployment. To ensure that $\hat{\sigma} < 1$, the broken curve must take a greater value than the solid curve at $\sigma = 1$. This requirement is fulfilled if

$$c_h|_{\sigma=1} \equiv w + \rho\left(b_h^0 + x_i m_i(1; \rho b_i^0 - \tau_i)\right) > \overline{c},$$

where $c_h|_{\sigma=1}$ is the value of c_h in (39) for $\sigma = 1$. Since $x_i m_i > 0$, the above condition necessarily holds if

$$c_h^F \equiv \rho b_h^0 + w > \overline{c}.$$

i.e., the country has unemployment before immigration; cf. corollary 1. On the other hand, we have $\hat{\sigma} > 0$ only if

$$c_h|_{\sigma=0} \equiv \rho b_h^0 + (\rho - \alpha) x_i m_i(0; \rho b_i^0 - \tau_i) < u'^{-1}(\tfrac{\beta}{\rho-\alpha}),$$

i.e., the broken curve takes a smaller value than the solid curve at $\sigma = 0$.[17]

In Figure 2 point *A* indicates the equilibrium with unemployment before immigration as in Figure 1. A comparison shows that natives' equilibrium employment rate and consumption level are lower when there is immigration. Further, it is easy to check using (37) and (39) that an increase in the immigrant's net worth $\rho b_i^0 - \tau_i$ shifts the dotted curve upward in Figure 2. However, since (40) is unaffected, the solid curve remains intact. Hence, we conclude that the greater the immigrant's net worth, the lower the employment rate and a native's consumption level ($\hat{\sigma}$ and $\hat{c}_h$).

In addition, manipulation of (39) and (40) yields

$$dc_h = (w + \alpha x_i m_i) d\sigma + \tfrac{\beta x_i}{u'} dm_i,$$

[17] Given condition (20), this condition holds if x_i is not too large.

$$\alpha d\sigma = -\frac{\beta u''}{(u')^2} dc_h.$$

Combining these equations and using the definition of $\Omega(> 0)$ given in (21), we find that

$$\left(\frac{\beta x_i}{u'}\right) \frac{dm_i}{d(\rho b_i^0 - \tau_i)} = -(\Omega + \alpha x_i m_i) \frac{d\sigma}{d(\rho b_i^0 - \tau_i)} > 0,$$

where the inequality follows from $d\hat{\sigma}/d(\rho b_i^0 - \tau_i) < 0$.[18] Thus, an immigrant's real balances m_i increase with his net worth $(\rho b_i^0 - \tau_i)$, reaching $\overline{m}$ at some cutoff level. If an immigrant's net worth exceeds this cutoff level, he must hold $m_i > \overline{m}$. In such a case, (37) is no longer applicable, resulting in the second possibility.

The second case thus occurs when $m_i > \overline{m}$; i.e., if the immigrant is rich enough to hold real balances greater than $\overline{m}$. Since $v'(m_i) = \beta$, the optimality condition (35) is rewritten

$$\rho + \alpha(\sigma - 1) = \frac{\beta}{u'(c)} \quad \text{with} \quad c_h = c_i = c, \tag{41}$$

that is, in the second case immigrants and natives consume exactly the same quantity. Setting $c_h = c_i = c$ in (25), we can rewrite the host country's current account as

$$\dot{b} = \rho b_h^0 + x_i(\rho b_i^0 - \tau_i) + (1 + x_i)(w\sigma - c). \tag{42}$$

With the current account in balance, we can set the right-hand side of (42) equal to zero to get

$$c = w\sigma + \frac{\rho b_h^0 + x_i(\rho b_i^0 - \tau_i)}{1 + x_i}. \tag{43}$$

(41) and (43) jointly determine the equilibrium values of c and σ, which we denote by $\hat{c}(= \hat{c}_h = \hat{c}_i)$ and $\hat{\sigma}$. In Figure 3, the broken line through point C represents equation (43) while the solid curve traces out equation (41). Then the intersection point C indicates the equilibrium values, $\hat{c}_h(= \hat{c}_i)$ and $\hat{\sigma}$.

We have $0 < \hat{\sigma} < 1$ when the broken line lies above the curve near $\sigma = 1$ and below it near $\sigma = 0$ as illustrated in Figure 3. That is, existence of unemployment in the equilibrium

[18] This is readily verified in Figure 2.

requires that

$$c_{|\sigma=1} = w + \frac{\rho b_h^0 + x_i(\rho b_i^0 - \tau_i)}{1+x_i} > u'^{-1}\left(\frac{\beta}{\rho}\right) = \overline{c}, \tag{44a}$$

$$c_{|\sigma=0} = \frac{\rho b_h^0 + x_i(\rho b_i^0 - \tau_i)}{1+x_i} < u'^{-1}\left(\frac{\beta}{\rho-\alpha}\right). \tag{44b}$$

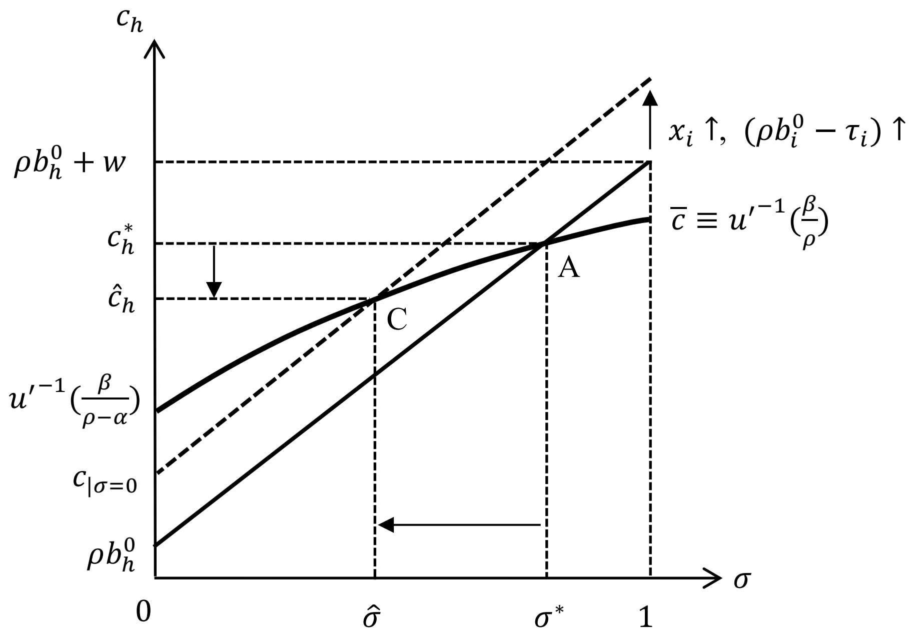

Figure 3:

Pre- and post-immigration equilibria exhibiting unemployment

(second case)

To understand the conditions in (44), notice that an immigrant consumes $c_i = c_h = w\sigma + \rho b_h^0$ by (14). On the other hand, since he sells enough bonds to acquire more real balances than $\overline{m}$, his income cannot exceed $\rho(b_i^0 - \overline{m}) - \tau_i + w\sigma$. Since all individuals consume their entire incomes, we have

$$w\sigma + \rho b_h^0 \leq \rho(b_i^0 - \overline{m}) - \tau_i + w\sigma,$$

which simplifies to

$$\rho b_h^0 < \rho b_i^0 - \tau_i. \tag{45}$$

(45) implies that $c_{|\sigma=1}$ in (44) increases with the number of immigrants x_i. If the country suffers from unemployment before immigration, (19) and (20) also hold. Therefore, (44a) necessarily holds while (44b) holds for a sufficiently small x_i.

Observe that in both types of equilibria with unemployment, an influx of immigrants worsens unemployment and reduces natives' consumption. To see this, note that as $x_i \to 0$, both equations (39) and (43) converge to equation (14). In term of Figure 2, this convergence implies that the broken curve approaches the solid line.[19] Similarly in Figure 3, because (45) holds, the broken line converges from above to the solid line. Since point A indicates the pre-immigration values c_h^* and σ^*, taken from Figure 1, the convergence results imply that when the country has unemployment initially, an influx of immigrants moves the equilibrium from A to B in Figure 2 and from A to C in Figure 3. In either case, immigration decreases the employment rate and natives' consumption level.

It is easy to see that changes in an immigrant's net worth also have similar effects in both types of equilibria. An increase in $(\rho b_i^0 - \tau_i)$ shifts up the broken curve in Figure 2 and the broken line in Figure 3.[20] Thus, in both figures the employment rate and the native's consumption fall as an immigrant's net worth increases. The next proposition records these results.

Proposition 5: Suppose that (17) is violated so the country suffers from unemployment without immigration. Then, an influx of immigrants causes the country's economy to jump to a new steady state, where the rate of employment and natives' consumption are lower. Further,

(a) the richer are immigrants (in terms of international bond holdings), the lower the employment

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[19] This guarantees the existence of a unique equilibrium for sufficiently small x_i.

[20] This is evident from (43).

rate and natives' consumption; and

(b) the more remittances immigrants make, the higher the employment rate and natives' consumption.

Note that Proposition 5 contrast sharply with Proposition 4, obtained under full employment.

7. Immigration and welfare

This section examines the welfare implications of immigration. Consider first the case in which there exists full employment before and after immigration. In this case it is easy to see that natives benefit from immigration because they consume more and hold a greater quantity of real money (Proposition 4). By contrast, when there is unemployment without immigration, an influx of immigrants lowers natives' equilibrium consumption level and unemployment rate (Proposition 5). Despite such economic downturns, however, the welfare impact of immigration is in general ambiguous. On the one hand, lower consumption reduces natives' lifetime welfare. On the other, however, a higher unemployment rate implies a faster price fall, which implies a more rapid expansion in m_h, implying a rise in the present value of the lifetime utility from money holdings. It can be show that natives' lifetime welfare depends on $m_h(0)$, the real balances they hold at the time of immigration. If $m_h(0)$ is sufficiently small compared with the equilibrium consumption level, then natives are harmed by immigration. This result is consistent with the previous discussion because if $m_h(0)$ is too small, the beneficial expansion of real balances explicated earlier is insufficient to countervail the harm done by falling consumption. The next proposition sums up these results (see Appendix B for proof).

Proposition 6. (a) If condition (34) holds, i.e., there is full employment before and after immigration, then an influx of immigrants is welfare-improving for natives.

(b) If there is unemployment initially, an influx of immigrants decreases natives' lifetime welfare if and only if $m_h(0) < \frac{c_h^*}{\rho \eta_h}$, where $\eta_h = -u'' c_h / u'$ is the elasticity of marginal utility of consumption as in (6).

Finally, in the preceding section we pointed out that an influx of too many or too rich immigrants can plunge a full employment economy into secular stagnation. The welfare effect of immigration in this case is also ambiguous in general. First, since immigrants sell international bonds in exchange for host-country currency, natives' interest income is augmented by income transfers from the monetary authority. As a consequence, their consumption level may still be greater than before immigration. Second, with unemployment, the price level keeps falling, increasing the utility from real balances. These facts can potentially make natives better off despite the appearance of unemployment.

8. Concluding remarks

In this paper we develop a dynamic model of a small open country, where agents maximize life-time utility over consumption of the aggregate good and real balances they hold. The model has two salient features: (i) the boundedness of marginal utility of real balances above zero and (ii) downward sluggishness of nominal wage adjustment. We find the following. (1) In the absence of immigration, natives are fully employed if they hold quantities of international interest-earning assets below some threshold level. Otherwise, there is unemployment. (2) If full employment prevails initially, an influx of immigrants boosts natives' consumption and improves their lifetime welfare, provided that immigrants are neither too rich nor too numerous. An influx of too rich or too many immigrants, however, may precipitate the host country into stagnation. (3) If the host country suffers from secular unemployment, immigration always worsens the unemployment rate and reduces natives' consumption. In this case, however, the welfare effect is in general ambiguous. (4) Immigrants' remittances also have contrasting effects, depending on the state of the host-country economy. When there is full employment, remittances reduce natives' consumption and welfare. When there is unemployment, remittances increase natives' consumption and employment.

Several extensions manifest themselves. First, although we assumed native and immigrant workers homogeneous, some studies have explored the implications of skill differences between them.[21] If there is a single aggregate good, one way to represent the skill difference is to assume that the immigrant possesses only a fraction of (effective) labor compared with the native. We expect however that this does not qualitatively affect our results. Second, our model can be applied to study the effect of emigration on the source country. More challenging is an extension to the case of two large countries and labor movement between them. This necessarily introduces interdependence both on the real and the monetary side of the two economies. We hope to address these issues in our future research.

[21] See, e.g., Liu (2010), Chassambouilli and Palivos (2014) and Battisti et al. (2018), who have used two-sector models.

Appendix A: Stability, and proof of Lemma 1

In this appendix we investigate the stability of our model and prove Lemma 1 given in the main text. The stability around the full-employment steady state is standard so we focus on the case with unemployment. As mentioned in deriving (11), c_j stays constant over time in all cases. Having this property in mind, we first examine the benchmark model with unemployment. From (18) we have

$$\rho + \alpha(\sigma - 1) = \frac{\beta}{u'(c_h)} \implies c_h = c_h(\sigma). \tag{A1}$$

Substituting this c_h to (13) gives us

$$\dot{b}_h = \rho b_h + w\sigma - c_h(\sigma),$$

$$\frac{\partial \dot{b}_h}{\partial \sigma} = w + \left(\frac{\alpha}{\beta}\right)\frac{(u')^2}{u''} \equiv \Omega > 0,$$

where Ω is given in (21). These two equations indicate that the dynamics of b_h is unstable, implying that σ and $c_h(\sigma)$ immediately jump to the levels that make $\dot{b}_h = 0$ and stay there. This implies that b_h remains fixed at the initial level b_h^0.

Turning next to the post-immigration steady state with unemployment, consider the second scenario from the text, in which both natives and immigrants hold real balances above $\overline{m}$; i.e., $v'(m_j) = \beta$ for $j = h, i$. In this case, the post-immigration dynamics is given by (42) and the analysis goes through as above, mutatis mutandis, with b_h being replaced by $b_h + x_i(b_i - \tau_i/\rho)$. Consider next the first scenario, in which $m_h > \overline{m}$ while $m_i < \overline{m}$. In this case, $c_h(\sigma)$ is given by (A1) while (35) yields

$$\frac{v'(m_i)}{u'(c_i)} - \alpha(\sigma - 1) = \rho \implies m_i = m_i(c_i, \sigma).$$

Applying these $c_h(\sigma)$ and $m_i(c_i, \sigma)$ to the dynamics of the immigrant's asset holdings in (24) and the current account in (25), we obtain

$$\dot{a}_i = \rho b_i^0 - \tau_i + w\sigma - [\rho + \alpha(\sigma - 1)]m_i(c_i, \sigma) - c_i,$$

$$\dot{b} = \rho(b_h^0 + x_i b_i^0) + w\sigma(1 + x_i) - (c_h(\sigma) + x_i c_i) - x_i \tau_i,$$

where σ and c_i stay constant over time. If they jump so that $\dot{a}_i$ and/or $\dot{b}$ are non-zero, either the feasibility condition or the non-Ponzi game condition is violated. Thus, they initially jump to the levels that make $\dot{a}_i$ and $\dot{b}$ zero and stay invariant thereafter, i.e., (35) and (36) hold. This proves Lemma 1.

Appendix B: Proof of Proposition 6.

To prove result (a), note that under full employment without immigration natives consume c_h^F of output and hold $m_h^F = m$ in real balances. As these quantities remain constant over time, the typical native's lifetime welfare starting from $t = 0$, say, equals $U^F = [u(c_h^F) + v(m_h^F)]/\rho$. As the native consumes a greater quantity and holds more real balances after an arrival of immigrants (Proposition 4), his lifetime welfare clearly increases.

To prove (b) of Proposition 6, suppose that at $t = 0$ the native consumes c_h^* and holds real balances $m_h(0)$ without immigration. Since there is unemployment, the price falls at the rate $\pi^*\big(= \alpha(\sigma^* - 1)\big)$ and hence his real balances $m_h(t)$ increase at the rate $-\pi^*$; that is, $m_h(t) = m_h(0)\exp(-\pi^* t)$. As a result,

$$v\big(m_h(t)\big) = v\big(m_h(0)\big) + \beta m_h(0)[\exp(-\pi^* t) - 1].$$

By contrast, the native's consumption remains c_h^* over time. Therefore, his lifetime welfare under stagnation without immigration is given by

$$U_h^* = \int_0^\infty e^{-\rho t}[u(c_h^*) + v(m_h(t))]dt$$

$$= \frac{u(c_h^*) + v\big(m_h(0)\big) - \beta m_h(0)}{\rho} + \frac{\beta m_h(0)}{\rho + \pi^*}$$

$$= \frac{u(c_h^*) + v\big(m_h(0)\big) - \beta m_h(0)}{\rho} + m_h(0) u'(c_h^*), \qquad \text{(B1)}$$

where the first equality follows from integration while the second follows from the optimality condition, i.e., $\rho + \pi^* = \beta / u'(c^*)$.[22] If immigration occurs at $t = 0$, consumption instantaenously falls to the new equilibrium level $\hat{c}_h$. Since $m(0)$ is unaffected, differentiating (B1) yields:

$$\frac{dU_h^*}{dc_h^*} = \frac{u'(c_h^*)}{\rho} + m_h(0)u''(c_h^*) = \frac{\eta u'(c_h^*)}{\rho c_h^*}[\frac{c_h^*}{\rho \eta_h} - m_h(0)],$$

where $\eta_h \equiv -u'' c_h / u' > 0$. Result (b) of Proposition 6 immediately follows.

[22] This result is the same as in Ono (1994, Chap. 6).

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