

# Illegal immigration, unemployment, and multiple destinations\*

Kaz Miyagiwa<sup>†</sup>    Yasuhiro Sato<sup>‡</sup>

February 13, 2018

## Abstract

We develop a multi-country model of illegal immigration with equilibrium unemployment. Two geographic cases are considered. One has two destinations adjacent to the source country while the other has just one destination country adjacent to it. In both cases, the equilibrium border control proves insufficient compared with the joint optimum, calling for enforcement by federal authorities. Absent such authorities, delegating border control to the country with a larger native labor force can improve each destination country's welfare. In contrast, the equilibrium internal enforcement policy is efficient, obviating enforcement by supranational authorities.

*Keywords:* illegal immigration, immigration policy competition, equilibrium unemployment, multiple destinations, job search

*JEL classification:* F22, F66, H77, J61, J64

---

\*This study was conducted as a part of the Project “Spatial Economic Analysis on Regional Growth” undertaken at the Research Institute of Economy, Trade and Industry (RIETI). This work was also supported by JSPS KAKENHI Grant Number 15H03348, 15H03344, 16H03615, and 17H02519. We thank the editor, two anonymous referees, and seminar participants at Kobe, Hitotsubashi, Kyoto, Osaka, Osaka Prefectural, and Tohoku Universities, NARSC, and RIETI for helpful comments.

<sup>†</sup>Florida International University; *e-mail*: kmiyagiwa@gmail.com

<sup>‡</sup>University of Tokyo, *e-mail*: ysato@e.u-tokyo.ac.jp

# 1 Introduction

Illegal immigration is a perennial problem facing many advanced economies today. In 2014, for example, more than 276,000 immigrants entered the European Union illegally, a figure likely to soar in the coming years.<sup>1</sup> Across the Atlantic, a conservative estimate is that there are 11.5 million illegal immigrants living in the United States.<sup>2</sup> Concerned that such large flows of illegal immigrants may strain their economies, take jobs away from workers, and threaten national security, many destination countries have begun to reassess their immigration policy.<sup>3</sup>

Destination country governments have two instruments at their disposal to control illegal immigration. First, they can stop illegal immigrants at the borders before they gain entry. In addition, they can apprehend and deport illegal immigrants living within their borders. Following Ethier (1986), we call these two instruments external and internal enforcement policy, respectively.

There is much empirical literature examining the efficacy of such enforcement policies. For example, Hanson and Spilimbergo (1999) and Hanson et al. (2002) analyze the effects of external enforcement policy on local labor markets around the U.S.-Mexico border. Bohn et al. (2014) and Hoekstra and Orozco-Aleman (2014) examine the effects of recent changes in Arizona’s internal enforcement policy. While these works study a single destination country/region in isolation, today’s immigration problems often span multiple destination countries/regions. For example, most immigrants to Europe arrive in Greece or Italy but the majority of them eventually settle down in Germany or other Northern European countries. As a result, the “entry” countries and the “settlement” countries do not necessarily agree on E. U. immigration policy; see, e.g., Casarico et al. (2015). Likewise, there are considerable differences in attitude towards illegal immigrants among U.S. states.<sup>4</sup>

Given such environments, the present paper aims at two objectives. The primary objective is to characterize the effects of immigration policies in the presence of multiple destination countries. Since immigrants are often blamed for taking jobs away from native workers, our secondary objective is to explore the nexus

---

<sup>1</sup><http://humanevents.com/2015/01/19/illegal-immigration-is-europe-losing-control-of-its-borders/>

<sup>2</sup><http://www.pewresearch.org/fact-tank/2015/11/19/5-facts-about-illegal-immigration-in-the-u-s/>

<sup>3</sup>For instance, the European Commission gives immigration issues a central priority (see the European Agenda on Migration, published on 13 May 2015).

<sup>4</sup>For example, see the New York Times article by R. C. Archibold entitled “Arizona enacts stronger law on immigration” (April 23, 2010).

between immigration policy and unemployment. For this purpose, we first develop a single-country model of search unemployment featuring illegal immigration, and then extend this “baseline” model to cases with multiple destination countries.

In the “baseline” model we incorporate illegal immigration into the standard model of equilibrium unemployment, where all active firms randomly experience job separations due to idiosyncratic adverse shocks. In our setting, firms employing illegal immigrants can lose workers for another reason: internal enforcement policy. We find that stronger external enforcement policy curtails immigration flows and lowers unemployment rates for both native and immigrant workers. Internal enforcement has similar effects, so the two instruments can be combined optimally to reduce the overall enforcement costs, as in Ethier (1986).

With multiple countries, we face new issues since immigrants can choose where to immigrate. As a more stringent immigration policy in one country can turn away immigrants towards other destination countries, there can be policy spillovers among destination countries, giving rise to policy conflict.<sup>5</sup> It turns out that the precise nature of policy spillovers depend on geographical configurations of destination countries with respect to the source country. While many such configurations are conceivable, we find it useful to focus on two prototypical cases. In one case, all the destination countries border the (common) source country (we still assume one source country).<sup>6</sup> In such a setting, immigrants can cross the border wherever border control is the weakest and then move to the preferred destinations, assuming there are no mobility barriers across destination countries. We call this the “common-border” case. In the second case, only one destination country borders the source country so that all immigrants must enter this unique border country to reach other destination countries. We call this the “single-border” case. These two prototypical cases exhibit distinct properties but also share common features. Once such properties are understood, our analysis can readily be applied to more realistic geographies involving multiple border and non-border countries.

We now outline the equilibrium outcome in each case. In the common-border case, since the country with the weakest border control determines flows of immigrants into all the destination countries, we can apply Hirshleifer’s weakest-link model (Hirshleifer 1983, 1985) to show that the external enforcement (border

---

<sup>5</sup>It is the standard result in the tax competition literature that any policy towards internationally mobile factors has spillovers; see e.g. Wilson (1999) and Cremer and Pestieau (2004) for a survey on this literature.

<sup>6</sup>Actually, many illegal immigrants are known to traverse a number of countries before reaching the U.S. or European borders. Here we ignore the effect of such intermediate countries’ policy towards transitory immigrants.

control) policy game has a continuum of symmetric Nash equilibria.<sup>7</sup> If all the destination countries are symmetric, the equilibrium set contains their most preferred policy vector, and hence there need not be policy conflict among the destination countries. In contrast, if the destination countries are asymmetric, their preferred external enforcement levels diverge, resulting in policy conflict. Further, with asymmetric countries, the Nash equilibrium external enforcement levels fall short of the jointly optimal level, calling for policy coordination.

In contrast, in the single-border case, where one border country controls flows of immigrants into all the other destination countries, the game has a unique Nash equilibrium. In this equilibrium, the border country implements its optimal external enforcement policy. However, the other countries prefer tighter border control. Consequently, we find the equilibrium external enforcement level to be below the joint optimum.

Although external enforcement policy is inefficient in both prototypical cases, the inefficiencies arise for different reasons. In the common-border case, external enforcement is akin to a public good, with the weakest-link structure giving rise to underprovisions of the public good. By contrast, the inefficiency in the single-border case is ascribed to the standard externalities; the unique border country fails to take into account the other destination countries' welfare when implementing its external enforcement policy.

In contrast to external enforcement policy, internal enforcement policy has no spillover effects. Intuitively, if one country strengthens its internal enforcement policy, some immigrants may flee to other destination countries, but the resultant increases in immigrant populations make other destination countries less desirable for future immigrants. As a result, in steady state the number of illegal immigrants remain unaffected in other destination countries. Our analysis thus implies that external enforcement policy should best be handled by the federal or supranational government, whereas internal enforcement can be left up to each destination country's discretion.

Given the inefficiency of external enforcement policy, we next ask how the supranational government should allocate the cost of external enforcement policy among the asymmetric destination countries. Our analysis shows that the country with a larger native labor force should bear a greater share of the enforcement cost. Absent a supranational authority, delegating external enforcement to the larger country (without side

---

<sup>7</sup>Bryant (1983) shows that, if final good production depends on the minimum of the intermediate goods produced, then the model also has a continuum of equilibria.

payments) increases the joint welfare. These findings hold both in the common-border and the single-border case. Intuitively, because of constant returns to scale, the aggregate welfare effect of border enforcement is greater at the margin, the greater the country's native labor force. This implies that a larger country prefers to enforce border control more tightly, given symmetric enforcement costs. Hence, the joint welfare would be greater if border control is delegated to a larger country instead of a smaller country.

We now relate this paper to literature. First, this paper contributes to the line of research initiated by Ethier (1986). Ethier assumed a small destination country to remove its monopsonistic influence against the source country so as to focus on the pure effect of immigration policy. Subsequent research expanded Ethier's work to other immigration-related issues; see Bond and Chen (1987), Djajić (1987), Yoshida and Woodland (2005), and Woodland and Yoshida (2006), among others.<sup>8</sup> This literature usually assumes unemployment due to wage rigidity (e.g., wages set by minimum wage legislation or labor unions). The rigid-wage assumption is analytically convenient but is often criticized for its failure to explain how the wage is set in the first place. In contrast, the wages and the unemployment rates are determined endogenously in our analysis.<sup>9</sup>

Mayr et al. (2012), Giordani and Ruta (2013), and Bandyopadhyay and Pinto (2017) considered multiple destination countries. Mayr et al. (2012), focusing on external enforcement and amnesty policies in the single-border case, show that total enforcement spending is likely to be inefficiently low, which is consistent with our results from the single-border case. Giordani and Ruta (2013) analyze the common-border case, limiting their attention, however, only to external enforcement policy by symmetric countries. Further, their government objective function is not derived from a general equilibrium structure.<sup>10</sup> Lastly, Bandyopadhyay and Pinto (2017) adopt a setting similar to that of Giordani and Ruta (2013), but consider internal enforcement policy (as well as the provision of local public goods). They find that, for a given external enforcement policy level, internal enforcement policy levels are too high and public good provisions are too low relative to the social optima. In their analysis, the inefficiency of internal enforcement policy

---

<sup>8</sup>These models, including ours, focus on economic incentives to migrate and hence are not suited to analyze recent refugee problems triggered by wars, political, and religious persecution.

<sup>9</sup>For models of equilibrium unemployment, see Pissarides (2000) and Rogerson, Shimer and Wright (2005). Carter (1999) introduces unemployment based on the Stiglitz-Shapiro (1984) efficiency-wage hypothesis into the Ethier model.

<sup>10</sup>On the other hand, Giordani and Ruta (2013) discuss in detail how to use risk dominance to select a unique equilibrium from the continuum of Nash equilibria, to which we have nothing to add.

stems from the assumption that immigrant populations are fixed or that destination countries are large to exert their monopolistic power against the source country. If we impose either assumption in our setting, the equilibrium internal enforcement policy also becomes inefficient. For example, with large destination countries, implementation of stronger internal enforcement policy in one country depresses the welfare in the source country due to its monopsonistic power, as pointed out in Ethier (1986), thereby prompting more immigration to other destinations. Thus, one contribution of our analysis to this literature is to identify the channels through which internal enforcement generates spillover effects.

We note, in passing, that the above works have significantly different model structures. Giordani and Ruta (2013) and Mayr et al. (2012) assume standard perfect competitive frameworks, and Bandyopadhyay and Pinto (2017) consider a discrete choice model, while we employ a model of search unemployment. Despite these differences, (in)efficiency results are surprisingly robust

In a nutshell, then, our paper subsumes these recent analyses in a unified framework and provides consistent characterizations of efficient immigration policies in the presence of unemployment. Moreover, our analysis focuses more on asymmetric cases and addresses the question of who should bear the enforcement cost. Our results on the optimal allocation of enforcement costs provide new insights to the cost sharing rule among destination countries. For instance, although the Lisbon Treaty of the European Union requires the fair sharing of responsibility and cost of implementing immigration policies among member states, “it does not provide any rule on how to share these costs in practice” (Russo and Senatore 2013). While Russo and Senatore (2013) analyzed the incentives to share the external enforcement costs, we approach it from the welfare point of view, studying how cost sharing rule should be.<sup>11</sup>

The remainder of the paper proceeds as follows. The baseline model of Section 2 extends the Ethier model to incorporate equilibrium unemployment. Section 3 examines the properties of external and internal enforcement policy in the baseline model. Section 4 characterizes the optimal immigration policy for a single small destination country in the baseline model. Section 5 extends the analysis to cases with two destinations. Section 6 extends the analysis to three countries and to consider mobility of native workers. Section 7 concludes.

---

<sup>11</sup>Chassanboulli and Palivos (2014) place equilibrium unemployment at center stage of their analysis but focus on economic growth with legal immigration only.

## 2 Baseline model: a single destination country

### 2.1 Model structure

In this section we develop the baseline model, where there is one destination country and one source country. Let there be  $L_n$  native workers and  $L_m$  illegal immigrant workers residing in the destination country.  $L_n$  is taken as exogenous while  $L_m$  is determined in the equilibrium. If there are legal immigrants in the destination country, they are considered part of  $L_n$ .<sup>12</sup>

The model features equilibrium unemployment. To keep focus on immigration issues, we adopt the simplest model structure that gives rise to equilibrium unemployment. The basic tenet of the search-theoretic approach to unemployment is the absence of organized labor markets. Thus, unemployed workers and firms with vacancies engage in search activity. Search is uncoordinated, costly, and time-consuming. To cut through the complexity of search activity, the literature utilizes the matching function, which embodies the notion that workers and firms are matched randomly in a Poisson process. The matching function works like a neoclassical production function. First, it exhibits diminishing returns to a “factor.” Holding the number of job seekers constant, an increase in the number of vacancies makes it more difficult for a firm to fill its vacancy while making it easier for a job seeker to find a job. This implies that,  $\theta$ , denoting the ratio of vacant jobs over unemployed workers, is negatively related to  $q$ , the rate at which vacancies are filled per unit of time, and positively related to  $s$ , the rate at which an unemployed worker finds a job per unit of time. That is, we can write these relations as  $q = q(\theta)$  and  $s = s(\theta)$  with the first derivatives given by  $q'(\theta) < 0$  and  $s'(\theta) > 0$ .<sup>13</sup> Second, the matching function exhibits constant returns to scale. This implies that  $s$  and  $q$  are related by the equation  $s(\theta) = \theta q(\theta)$ .<sup>14</sup>

The unemployment pool in the destination country contains both native workers and illegal immigrants. Let  $\alpha$  denote the proportion of illegal immigrants in the pool. A firm looking for a worker is matched randomly with a native ( $n$ ) or an immigrant ( $m$ ). After a match, a firm finds whether the new employee is a native or not and chooses whether to hire that worker or keep searching. In equilibrium, however, a firm always hires the worker that shows up because of costly search.<sup>15</sup>

---

<sup>12</sup>We do not ask who will be illegal/legal immigrants. Mangin and Zenou (2016) develop a model wherein workers decide whether or not to illegally migrate.

<sup>13</sup>Primes denote differentiation.

<sup>14</sup>See Pissarides (2000).

<sup>15</sup>This result that firms have incentive to hire illegal immigrants is consistent with empirical findings. In fact, Brown et

Let  $V$  denote the value of a firm in search of a worker and  $J_i$  the value of a firm employing a worker of type  $i = n, m$ . These values are related in the following equation:

$$rV = -c + q\alpha(J_m - V) + q(1 - \alpha)(J_n - V), \quad (1)$$

where  $c > 0$  is a flow search cost and  $r$  is the rate of interest, both exogenously given. The right-hand side of this equation has the following interpretation. A firm that spends search cost  $c$  is matched with an immigrant at rate  $q\alpha$ , and with a native worker at rate  $q(1 - \alpha)$ , per unit of time. A match changes a firm's asset value by  $J_m - V$  if it is matched with an immigrant and by  $J_n - V$  if it is matched with a native. Thus, the right-hand side of (1) represents the flow value of search. In equilibrium this is just equal to  $rV$ . Further, free entry drives  $V$  to zero in steady state. Setting  $V = 0$  in (1) yields

$$\alpha J_m + (1 - \alpha)J_n = \frac{c}{q}. \quad (2)$$

Native and immigrant workers are assumed equally productive.<sup>16</sup> As a result, every active firm produces  $y$  units of the aggregate good, the numéraire, and pays the wage  $w_i$  to a worker of type  $i (= n, m)$ . Further, all active firms are hit by idiosyncratic adverse shocks, resulting in job separations. Separated firms and workers engage in search activities once again. Suppose that job separations follow a Poisson process with rate  $\lambda$ , which is exogenous and common to all active firms. Consider a firm employing a native worker. It earns the net profit  $y - w_n$  per unit of time and experiences a job separation at rate  $\lambda$  per unit of time, resulting in a loss of its value  $J_n - V = J_n$ . The right-hand side of the following equation thus measures the flow value of a firm employing a native worker. In equilibrium, this flow value equals  $rJ_n$ . Hence,

$$rJ_n = y - w_n - \lambda J_n.$$

Collecting terms yields the firm value:

$$J_n = \frac{y - w_n}{r + \lambda}. \quad (3)$$

A firm employing an immigrant faces the additional risk of losing its worker due to internal enforcement policy. If an immigrant worker is apprehended at rate  $\delta$  per unit of time, the value of a firm employing an immigrant satisfies this asset-value equation:

$$rJ_m = y - w_m - (\lambda + \delta)J_m.$$

---

al (2013) showed by using administrative data from the state of Georgia that employing illegal immigrants can even yield a competitive advantage of firms.

<sup>16</sup>Asymmetry in productivity between natives and immigrants can be introduced but does not affect our qualitative results.

Collecting terms yields:

$$J_m = \frac{y - w_m}{r + \lambda + \delta}. \quad (4)$$

Now, substituting the firm values from (3) and (4) into (2), we can rewrite the free-entry condition as:

$$\frac{\alpha(y - w_m)}{r + \lambda + \delta} + \frac{(1 - \alpha)(y - w_n)}{r + \lambda} = \frac{c}{q}. \quad (5)$$

This equation relates the wages to  $q$ , the rate at which a firm fills its vacancy.

Following the literature, we assume that the wages are set through the Nash bargaining between a worker and a firm after a match. The equilibrium wage  $w_i$  thus maximizes the Nash product  $(W_i - U_i)^p (J_i - V)^{1-p}$ , where  $W_i$  and  $U_i$  denote, respectively, the lifetime welfare value of employment and unemployment for worker type  $i = n, m$ , while the parameter  $p$  measures a worker's relative bargaining power. To keep things simple, we assume equal bargaining power between a firm and a worker, i.e.,  $p = 1/2$ .<sup>17</sup> This implies that a firm and a native worker evenly split the joint surplus created by a match, i.e.,

$$W_n - U_n = J_n. \quad (6)$$

We have already calculated  $J_n$  as a function of  $w_n$ . To calculate  $W_n - U_n$ , note that, when unemployed, a native worker finds a job at rate  $s$  per unit time, which increases her welfare by  $W_n - U_n$ . Thus, an unemployed native worker faces the asset-value function:

$$rU_n = s(W_n - U_n).$$

Similarly, when employed, a native earns the wage  $w_n$ , but can lose the job at rate  $\lambda$ . As a job loss causes the welfare loss  $U_n - W_n$ , the corresponding asset-value equation is given by:

$$rW_n = w_n + \lambda(U_n - W_n).$$

The two asset-value equations can be arranged to yield:

$$W_n - U_n = \frac{w_n}{r + \lambda + s}. \quad (7)$$

Finally, substituting from (3) and (7) into (6), we can express the equilibrium wage for a native as a function of  $s$ , the job-finding rate:

$$w_n = \frac{y(r + \lambda + s)}{2(r + \lambda) + s}. \quad (8)$$

---

<sup>17</sup>It may be more realistic to assume that immigrant workers have weaker bargaining power compared with native workers.

Introducing such an asymmetry does not affect our key results qualitatively, however.

The lifetime welfare of an immigrant can be calculated analogously, except that an immigrant faces deportation risks at rate  $\delta$ . If we let  $W_0$  denote the value of staying in the source country, an immigrant's asset value equations when unemployed and when employed, respectively, are written:

$$\begin{aligned} rU_m &= s(W_m - U_m) + \delta(W_0 - U_m), \\ rW_m &= w_m + \lambda(U_m - W_m) + \delta(W_0 - W_m). \end{aligned}$$

We assume that the destination country is small relative to the source country. This assumption, according to Ethier (1986), prevents the destination country from using immigration policy to extract the monopsony rents from the source country, allowing us to focus on the domestic effects of immigration policy. The small-country assumption implies that  $W_0$  is independent of the destination country's policy. To keep calculations simple, we choose the utility unit so as to set  $W_0 = 0$ . With this "normalization," an immigrant's asset-value equations above yield

$$\begin{aligned} W_m &= \frac{(r + \delta + s)w_m}{(r + \delta)(r + \delta + \lambda + s)}, \\ U_m &= \frac{sW_m}{r + \delta + s}, \end{aligned} \tag{9}$$

and hence

$$W_m - U_m = \frac{w_m}{r + \delta + \lambda + s}. \tag{10}$$

Substituting from (4) and (10), we can write the Nash bargaining solution  $W_m - U_m = J_m$  as:

$$\frac{w_m}{r + \delta + \lambda + s} = \frac{y - w_m}{r + \lambda + \delta},$$

which can be arranged to yield:

$$w_m = \frac{y(r + \delta + \lambda + s)}{2(r + \lambda + \delta) + s}. \tag{11}$$

We next discuss an immigrant's decision to migrate.<sup>18</sup> Let  $b$  denote the disutility an immigrant incurs in an attempt to cross the border. Suppose that all those who try to enter the destination country incurs the disutility  $b$  and get apprehended at the border with probability  $\phi$ . Apprehended immigrants are sent back home, while ones who evade apprehension enter the unemployment pool in the destination country. Thus, an immigrant's expected welfare from trying to cross the border equals:

$$-b + (1 - \phi)U_m + \phi W_0.$$

---

<sup>18</sup>We do not consider immigrants' location choices within the destination country. For a search theoretic analysis of location choice within a country/region/city, see Zenou (2009), among others.

In an interior equilibrium, this must be equal to  $W_0$ , the value of staying home. With the normalization  $W_0 = 0$ , this equilibrium condition is given by

$$U_m = \frac{b}{1 - \phi}. \quad (12)$$

We next describe the relationships that must hold in steady state. First, the total number of jobs destroyed per unit of time must equal the total number of jobs created per unit of time. For natives, we thus have  $\lambda(1 - u_n)L_n = su_nL_n$ , which yields the equilibrium unemployment rate for natives:

$$u_n = \frac{\lambda}{\lambda + s}. \quad (13)$$

With the risk of deportation, the corresponding steady state condition for immigrants is given by  $(\lambda + \delta)(1 - u_m)L_m = su_mL_m$ , and hence immigrants are unemployed at rate:

$$u_m = \frac{\lambda + \delta}{\lambda + \delta + s}. \quad (14)$$

Finally, by the law of large numbers,  $\alpha$  equals the actual proportion of immigrants in the unemployment pool:

$$\alpha = \frac{u_m L_m}{u_m L_m + u_n L_n}. \quad (15)$$

This completes the description of the baseline model.

## 2.2 Solving the baseline model

We now solve the model. First, substituting from (9) into (12) yields

$$U_m = \frac{sW_m}{r + \delta + s} = \frac{b}{1 - \phi}.$$

Substitution for  $W_m$  from (9) turns this equation into

$$\frac{sw_m}{(r + \delta)(r + \delta + \lambda + s)} = \frac{b}{1 - \phi}. \quad (16)$$

Substituting from (11), we can rewrite (16) as

$$\frac{sy}{(r + \delta)[2(r + \delta + \lambda) + s]} = \frac{b}{1 - \phi}, \quad (17)$$

which can be solved for the equilibrium  $s$ :

$$s = \frac{2b(r + \delta)(r + \delta + \lambda)}{(1 - \phi)y - b(r + \delta)}. \quad (18)$$

To ensure that  $s > 0$ , we impose the following two conditions. First, enforcement policy variables  $\phi$  and  $\delta$  are bounded above, having domains  $[0, \bar{\phi}]$  and  $[0, \bar{\delta}]$ , respectively, as illustrated in Figure 1.

[Figure 1 around here]

Second, a worker's productivity  $y$  is large enough to satisfy.

**Assumption 1:**  $(1 - \bar{\phi})y > b(r + \bar{\delta})$ .

Assumption 1 guarantees that the denominator on the right-hand side of (18) is positive, and hence  $s > 0$ , for all relevant values of  $\phi$  and  $\delta$ .

Once the equilibrium value of  $s$  is calculated, the remainder of the model can be solved recursively. Substituting the equilibrium  $s$  from (18) into (8) and (11) determines the equilibrium wages  $w_n$  and  $w_m$ , respectively, while substituting  $s$  into (13) and (14) determines the equilibrium unemployment rates  $u_n$  and  $u_m$ , respectively. Inverting the function  $s(\theta)$  yields the equilibrium  $\theta$ , determining the equilibrium  $q = q(\theta)$ . Then, the equilibrium  $q$ ,  $u_n$  and  $u_m$  can be substituted into (5) for the equilibrium  $\alpha$ , the proportion of unemployed immigrants in the unemployment pool. With the values of  $\alpha$  and the two unemployment rates  $u_n$  and  $u_m$  given, (15) can be used to compute the number of illegal immigrants  $L_m$  in the destination country. Thus, our first result is:

**Proposition 1** *Under Assumption 1 the model has a unique equilibrium.*

Substitution of the equilibrium wages into (3) and (4) yields the equilibrium firm values:

$$\begin{aligned} J_n &= \frac{y}{2(r + \lambda) + s}, \\ J_m &= \frac{y}{2(r + \delta + \lambda) + s}. \end{aligned} \tag{19}$$

A comparison shows that  $J_n > J_m$  for all  $\delta > 0$ ; a firm hiring a native has a higher firm value than one hiring an immigrant, although all workers are equally productive. Similarly, all  $\delta > 0$ ,  $w_n > w_m$  by (8) and (11) and  $u_m > u_n$  by (13) and (14).<sup>19</sup> Intuitively, since immigrant workers are apprehended and deported, they lose jobs at a higher rate than natives. And yet, natives and immigrants find jobs at the same rate,

---

<sup>19</sup>The firm values, wages and unemployment rates are equalized if and only if  $\delta = 0$ .

s. Thus, in steady state, there are proportionately more unemployed immigrants relative to native workers. Further, a higher probability of job separation generates a smaller surplus in a match, resulting in a lower wage for an immigrant and a lower value for a firm that employs an immigrant. These findings are noted in

**Proposition 2** *For all  $\delta > 0$ , (i)  $J_n > J_m$ ; (ii)  $w_n > w_m$ ; (iii)  $u_m > u_n$ .*

We next examine how the size of the native labor force affects the size of the immigrant population in the destination country. To this end, we plug (19) into (2) to obtain

$$\frac{\alpha y}{2(r + \delta + \lambda) + s} + \frac{(1 - \alpha)y}{2(r + \lambda) + s} = \frac{c}{q}.$$

This equation, combined with (2), shows that the equilibrium value of  $\alpha$  is independent of  $L_n$ . Moreover,  $L_n$  does not affect  $u_m$  and  $u_n$ , either. Then, equation (15) implies that an increase in  $L_n$  only raises  $L_m$ . Thus,

**Proposition 3** *A larger destination country (in terms of its native labor force  $L_n$ ) tends to have more illegal immigrants than a smaller destination country, other things being equal.*

Since the native labor force  $L_n$  includes the country's legal immigrants, a destination country that admits more legal immigrants has a larger  $L_n$  compared with a country that admits fewer legal immigrants, other things being equal. Thus, one implication of Proposition 3 is that illegal immigrants tend to migrate to a country already having a large legal immigrant population. This phenomenon is usually explained in cultural and linguistic terms, but here we offer a purely economic explanation for it.

### 3 Policy experiments

In this section we experiment with external and internal enforcement policy. Begin with external enforcement. Tighter border control increases the probability  $\phi$  of apprehension at the border. Differentiating (18) shows the effect on the job-finding rate  $s$ :

$$\frac{\partial s}{\partial \phi} = \frac{2yb(r + \delta)(r + \lambda + \delta)}{[(1 - \phi)y - b(r + \delta)]^2} > 0. \quad (20)$$

Intuitively, tighter border control makes entry into the destination more difficult, reducing the number of unemployed immigrants and raising the job-finding rate  $s$ . By (8) and (11), an increase in  $s$  implies higher

wages  $w_n$  and  $w_m$ , decreasing the firm values  $J_n$  and  $J_m$ . Thus, external enforcement policy has the effect of redistributing income from firm owners to workers of both types. Further, as jobs becomes relatively plentiful for everyone, the unemployment rates  $u_n$  and  $u_m$  fall by (13) and (14). However, a calculation shows that  $\partial(u_n/u_m)/\partial s < 0$ , i.e., the unemployment rate falls more for natives than for immigrants. This is just the consequence of the fact that immigrants initially faced a higher rate of unemployment; see in Proposition 2. Finally, using (3), (4), (5) and (15), we can show that  $\partial L_m/\partial \phi < 0$ ; stronger border controls reduce an immigrant population in the destination country.<sup>20</sup>

We summarize these findings in

**Proposition 4** *An increase in  $\phi$  (tighter border control) has the following results.*

- (A) *The wages increase for both natives and immigrants.*
- (B) *The unemployment rate falls for both types of workers but relatively more for natives than for immigrants.*
- (C) *The values of firms employing workers of either type fall.*
- (D) *The number of immigrants residing in the destination country declines.*

In sum, tighter border control benefits native workers, as they can find jobs in greater numbers at higher wages. Rising wages decrease firm values and redistributes income from firm owners to workers. Interestingly, since their wage rises and unemployment rate falls, immigrants residing in the destination country also benefit from tighter border control.<sup>21</sup>

---

<sup>20</sup>See Appendix A for the proof.

<sup>21</sup>Some authors have empirically investigated the effect of immigration on wages in the destination country. However, empirical results are mixed. For example, Hanson et al. (2002) and Card (2005) argue that the effects are negligibly small in the U.S. On the other hand, Borjas (2003) finds a significant negative wage effect. In more recent studies, D'Amuri et al. (2010) and Ottaviano and Peri (2012) report that new immigrants cast negative effects on previous immigrants' wages. It is to be noted, however, that empirical work does not usually distinguish between legal and illegal immigrants; an exception is Hanson et al. (2002).

The effect of internal enforcement policy is similarly derived. Differentiating (18) yields

$$\begin{aligned} \frac{\partial s}{\partial \delta} &= \frac{2b \left\{ (1 - \phi)y [2(r + \delta) + \lambda] - b(r + \delta)^2 \right\}}{[(1 - \phi)y - b(r + \delta)]^2} \\ &> \frac{2b(r + \delta) [(1 - \phi)y - b(r + \delta)]}{[(1 - \phi)y - b(r + \delta)]^2} \\ &> 0, \end{aligned} \tag{21}$$

where the last inequality comes from Assumption 1. This, by (8), implies a higher wage  $w_n$  for natives. The result is similar for an immigrant. Differentiation of  $w_m$  (i.e., (11)) yields:

$$\frac{\partial w_m}{\partial \delta} = \frac{b}{2(1 - \phi)} > 0.$$

Higher wages imply lower firm values, as can be verified by differentiating (19). The unemployment rates fall:  $\partial u_n / \partial \delta < 0$  and  $\partial u_m / \partial \delta < 0$ , but more for natives than for immigrants since  $\partial(u_n / u_m) / \partial \delta < 0$ . Further, since  $\partial w_m / \partial \delta > 0$ , a procedure analogous to the one used for external enforcement shows that  $\partial \alpha / \partial \delta < 0$  and  $\partial L_m / \partial \delta < 0$ .

**Proposition 5** *An increase in  $\delta$  (internal enforcement policy) has the same qualitative effects as an increase in  $\phi$  (external enforcement policy) as summarized in (A) through (D) of Proposition 4.*

Thus, external and internal enforcement policy are substitutes and can be used jointly to minimize the enforcement cost, as shown by Ethier (1986).

## 4 The optimal immigration policy

In this section we examine the nature of optimal immigration policy for the destination country. Before we proceed, it should be noted that there is no consensus in the literature as to what objective the destination country government maximizes with respect to immigration policy.<sup>22</sup> First, the standard objective of welfare maximization cannot be applied straightforwardly without resolving the question whether to include or exclude immigrants in the national welfare calculus. Although the literature usually takes an exclusionist approach, in dynamic contexts immigrants can be naturalized or have offsprings who are natives. In such cases, it is not obvious how to define national welfare. Second, since restrictive immigration policy redistributes income from firm owners to workers, Ethier (1986) points out that income redistribution itself can

---

<sup>22</sup>See Ethier (1986) for more on this point.

be an objective of immigration policy. Third, reducing the unemployment rate for natives may also be the government's policy objective.

With no lack of consensus, we are somewhat at liberty to define the policy objective of the destination country. In this study we suppose that the destination country maximizes the total surplus created by all the matches involving native workers less the costs of enforcement policy. Excluding the surpluses generated by "illegal matches" in our objective is not inconsistent with the presence of internal enforcement policy.<sup>23</sup> Further, it can be shown that our objective is qualitatively identical to maximizing the employment of native workers.

To define our objective function, note that each match with a native worker generates the surplus  $J_n + W_n - U_n$ . Since there are  $(1 - u_n)L_n$  employed native workers, we define the total gross surplus by

$$\Gamma \equiv L_n(1 - u_n)(J_n + W_n - U_n) = 2L_n(1 - u_n)J_n,$$

(the equality follows from the fact that  $W_n - U_n = J_n$  under the Nash bargaining). Substituting for  $J_n$  and  $u_n$  from (13) and (19), we can rewrite  $\Gamma$  as

$$\Gamma = \frac{2syL_n}{(\lambda + s)[2(r + \lambda) + s]}. \quad (22)$$

Note that  $\Gamma$  does not depend directly on the policy variables.

Turning to enforcement costs, let  $g(\phi)$  and  $h(\delta)$  denote the costs of external and internal enforcement policy, respectively.<sup>24</sup> By Assumption 1,  $g(\phi)$  and  $h(\delta)$  are defined over the closed intervals  $[0, \bar{\phi}]$  and  $[0, \bar{\delta}]$ , respectively. We assume that  $g(\phi)$  and  $h(\delta)$  are convex and increase without bounds over their respective domains.<sup>25</sup>

---

<sup>23</sup>In Appendix B, we explore an alternative objective, which include the values of firms employing illegal immigrants. Although this makes the calculations more tedious, our qualitative results do not change much.

<sup>24</sup>It is assumed that  $g(\phi)$  is independent of the number of immigrants trying to cross the border illegally. This assumption keeps the analysis simple and can be justified as follows. Suppose that the border has length of 1 and immigrants can enter any point on it with uniform probability. Then, without border control illegal entry is 100 % successful. Now, assume that each border patrol agent can secure an interval  $\epsilon > 0$ . Then, with  $z$  agents, at most the fraction  $\epsilon z$  of the border is secured. Thus,  $\phi = \epsilon z$  equals the probability of apprehension on the border and hence its cost  $g(\phi)$  is independent of the number of immigrants trying to enter the country illegally.

<sup>25</sup>Alternatively, one can assume that the government faces a budget constraint  $h(\delta) + g(\phi) \leq B$ , where  $B$  is fixed exogenously and define  $\bar{\delta}$  and  $\bar{\phi}$  by  $h(\bar{\delta}) = g(\bar{\phi}) = B$ . In such a case, our results are unaltered as long as we focus on interior solutions.

**Assumption 2:** (i)  $g(\phi)$  and  $h(\delta)$  are twice continuously differentiable on  $(0, \bar{\delta})$  and  $(0, \bar{\phi})$ , the interiors of their domains. (ii)  $g(0) = h(0) = 0$ ,  $g(\bar{\phi}) = h(\bar{\delta}) = \infty$  and  $\lim_{\phi \rightarrow 0} g' = \lim_{\delta \rightarrow 0} h' = 0$ .<sup>26</sup>

We now define social welfare by

$$SW \equiv \Gamma - g(\phi) - h(\delta).$$

The government chooses  $\phi$  and  $\delta$  to maximize  $SW$  under Assumption 2.<sup>27</sup> The optimal external enforcement policy satisfies the first-order condition:

$$\frac{\partial SW}{\partial \phi} = \frac{\partial \Gamma}{\partial s} \frac{\partial s}{\partial \phi} - g'(\phi) = 0,$$

where

$$\frac{\partial \Gamma}{\partial s} = \frac{2[2\lambda(r + \lambda) - s^2]yL_n}{(\lambda + s)^2 [2(r + \lambda) + s]^2}.$$

From the preceding section we have  $\partial s / \partial \phi > 0$ . Since  $g' > 0$ , an interior optimum requires that  $\partial \Gamma / \partial s > 0$ . This condition is satisfied if  $2\lambda(r + \lambda) - s^2 > 0$ . Substituting from (18) for the equilibrium  $s$ , we express this condition as

$$\mathbf{Assumption 3:} \quad 2\lambda(r + \lambda) > \{2b(r + \bar{\delta})(r + \bar{\delta} + \lambda) / [(1 - \bar{\phi})y - (r + \bar{\delta})]\}^2.$$

Assumption 3 is a sufficient condition for  $\partial \Gamma / \partial s > 0$  and is likely to be met if  $y$ , output per worker, is sufficiently large. The existence of the optimal policy is guaranteed in the next lemma (proved in Appendix C).

**Lemma 1.** *Under Assumptions 1-3, (i)  $SW$  is strictly concave in  $\phi \in [0, \bar{\phi}]$  and  $\delta \in [0, \bar{\delta}]$ . (ii)  $SW$  has the unique interior maximizers (denoted by  $\phi^*$  and  $\delta^*$ ).*

Then, computation leads to

**Proposition 6** *Under Assumptions 1-3,  $\partial \phi^* / \partial L_n > 0$  and  $\partial \delta^* / \partial L_n > 0$ ; the larger  $L_n$ , the greater is the destination country's effort to intercept illegal immigrants internally and externally.*

The intuition is straightforward. Note that  $s$  in (22) is independent of  $L_n$ . Thus, an increase in  $L_n$  raises the gross social surplus multiplicatively at the margin, and hence the first-order optimality condition

<sup>26</sup>Primes denote derivatives.

<sup>27</sup>We suppose that the destination country can choose  $\phi$  and  $\delta$  without budgetary constraints.

calls for an appropriate increase in enforcement. As an immediate application of the result, suppose that a destination country admits more legal immigrants and consequently has a greater “native” work force. Then, Proposition 6 implies that such a country tends to impose more stringent enforcement policy compared with a similar country that admits a fewer legal immigrants. This is consistent with the recent shift in some European countries towards tougher anti-immigration policy.

## 5 Multiple destination countries

We are finally in a position to consider cases with multiple destination countries. Before proceeding, we introduce additional assumptions that keep the analysis relatively simple. First, we suppose there are only two destination countries, indexed by  $i = 1, 2$ . Having more destination countries complicates the analysis without additional insight. However, in Subsection 6.2 we briefly consider a three-country setting. Second, immigrants face no mobility barriers across destination countries. Third, job search is still localized; to find a job in any destination country, an immigrant has to be in that country’s unemployment pool. Fourth, natives do not migrate. This assumption however is relaxed in Subsection 6.1. Under these assumptions we examine the two geographical cases.

### 5.1 The common-border case

Begin with the common-border case, in which both destination countries are contiguous with the source country. Immigrants cross the border where border control is the weaker of the two and settle in the country that gives them higher welfare. Thus, the effective level of border control for both destination countries equals  $\phi = \min(\phi_1, \phi_2)$ , where  $\phi_i$  denotes the level of external enforcement policy of country  $i$  ( $= 1, 2$ ).<sup>28</sup> Since the values of unemployment  $U_{mi}$  for an immigrant are equalized in the two countries in steady state, the following condition holds in equilibrium

$$U_{m1} = U_{m2} = \frac{b}{1 - \phi}. \quad (23)$$

The equilibrium values of  $J_{mi}$ ,  $U_{mi}$  and  $W_{mi}$  are determined independently for each country  $i$  as in the baseline model. In particular, an immigrant’s wage in country  $i$  is given by

$$w_{mi} = \frac{y(r + \delta_i + \lambda + s_i)}{2(r + \delta_i + \lambda) + s_i},$$

---

<sup>28</sup>Throughout this section we use the country subscript  $i$  to distinguish between the destination countries.

as in (11). Note that the parameters  $y$ ,  $r$ , and  $\lambda$  are assumed common in two destination countries, whereas policy variable  $\delta_i$  is country- $i$  specific. Therefore, the equilibrium condition is analogous to (17):

$$\frac{s_i y}{(r + \delta_i) [2(r + \delta_i + \lambda) + s_i]} = \frac{b}{1 - \phi},$$

which determines the job-finding rate  $s_i$  in each country  $i$ , given  $\delta_i$  and  $\phi$ ; i.e.,  $s_i = s_i(\phi, \delta_i)$ .<sup>29</sup> The equilibrium values of other variables can be solved as in the baseline model.

Suppose that our two destination countries pursue their immigration policies simultaneously and independently. Then, country  $i$  chooses the policy vector  $(\phi_i, \delta_i)$ , given country  $j$ 's policy choice  $(\phi_j, \delta_j)$ , to maximize its welfare

$$SW_i = \Gamma_i - g(\phi_i) - h(\delta_i),$$

subject to the constraint  $\phi = \min(\phi_1, \phi_2)$ . Recall that country  $i$ 's gross welfare

$$\Gamma_i = 2J_{ni}(1 - u_{ni})L_{ni}. \quad (24)$$

depends on the policy vector indirectly through  $s_i$ ; that is  $\Gamma_i = \Gamma_i(s_i(\phi, \delta_i))$ . Therefore, given  $\phi$ , each country's welfare depends on its internal enforcement policy  $\delta_i$  but is independent of the other destination country's internal enforcement policy. Thus, internal enforcement policy has no spillover effects. To understand this result intuitively, suppose that country 1, say, raises  $\delta_1$  for given  $\delta_2$ . Since (23) holds in steady state,  $U_{m2} = b/(1 - \phi)$  implies that the equilibrium in country 2 is undisturbed by changes in  $\delta_1$ .<sup>30</sup>

The independence of internal enforcement policy implies that the first-order condition

$$\frac{\partial \Gamma_i}{\partial s_i} \frac{\partial s_i}{\partial \delta_i} - h'(\delta_i) = 0$$

determines the optimal  $\delta_i$  independently of  $\delta_j$  ( $i \neq j$ ). In other words, each country can implement internal enforcement policy efficiently, given the border policy  $\phi$ . This makes it possible to solve the model in two stages; the destination countries first choose  $\phi_i$  and then, given  $\phi = \min(\phi_1, \phi_2)$ , they choose  $\delta_i$ . Since the internal enforcement policy choices are efficient conditional on  $\phi$ , the remainder of this subsection focuses on the external enforcement policy game.

<sup>29</sup>We impose the conditions similar to the one in Assumption 1 to keep  $s_i$  positive.

<sup>30</sup>The interdependency result stems from our assumption of small destination countries. If destination countries are large, internal enforcement policies have well-known spillover effects. For example, an increase  $\delta_1$  reduces the illegal immigrant population in country 1, which depresses the welfare of being in the source country and results in an increase in immigration into country 2. Allowing native labor to migrate also gives rise to spillovers; see Subsection 6.1.

Let  $\phi_1^*$  denote country 1's optimal external enforcement policy derived in the baseline model; that is, when it is the only destination country. In this section we call the  $\phi_1^*$  country 1's unconstrained optimum. Then, country 1's best-response function,  $BR_1(\phi_2)$ , is derived as follows. First, if  $\phi_2 \leq \phi_1^*$ , then  $BR_1(\phi_2) = \phi_2$ . To see this, note that raising  $\phi_1$  above  $\phi_2$  is costly but has no effect on the flow of immigration into country 1 because all immigrants come through country 2. Lowering  $\phi_1$  below  $\phi_2$  can control the flow of immigrants into country 1's territory but it is a move away from its unconstrained optimum and hence is welfare decreasing. This proves the claim. Next, if  $\phi_2 > \phi_1^*$ , country 1 controls immigration flows at any  $\phi_1 < \phi_2$ , and hence it chooses its unconstrained optimum  $\phi_1^*$ . Thus,  $BR_1(\phi_2) = \phi_1^*$ . Country 2's best-response function can be derived similarly; namely,  $BR_2(\phi_1) = \phi_1$  for  $\phi_1 \leq \phi_2^*$  and  $BR_2(\phi_1) = \phi_2^*$  for  $\phi_1 > \phi_2^*$ , where  $\phi_2^*$  denotes country 2's unconstrained optimum external enforcement level.

Suppose that two destination countries are symmetric (i.e.,  $L_{n1} = L_{n2} = L_n$ ). Then,  $\phi_1^* = \phi_2^* = \phi^*$  and hence the two countries' best-response functions are also symmetric as shown in Figure 2, where  $\phi_1$  is on the horizontal axis and  $\phi_2$  on the vertical axis.

[Figure 2 around here]

Country 1's best-response function  $BR_1$  comprises the segment 0A of the 45-degree line and the vertical line at point A =  $(\phi^*, \phi^*)$ . Similarly,  $BR_2$  comprises the segment 0A and the horizontal line at point A. Therefore, any point on the segment 0A is a Nash equilibrium. Although there is a continuum of Nash equilibria, the game is supermodular, so all the equilibria are uniquely welfare-ranked, with point A representing the Pareto-dominant one. It is evident that this Pareto-dominant equilibrium corresponds to the unconstrained optimal policy vector  $(\phi_i^*, \delta_i^*)$ . It is also straightforward to show that the Pareto-dominant equilibrium maximizes the joint welfare  $SW_1 + SW_2$ . As noted by Giordani and Ruta (2013), if the equilibrium is not at A, both countries have the incentive to move jointly to point A. Thus, with symmetric countries, there may be coordination failures but no policy conflict. The next proposition restates what we have found so far.

**Proposition 7** (i) *The common-border case with symmetric destination countries has a continuum of equilibria in external enforcement policy, which includes the one that maximizes the joint welfare of the two destination countries; (ii) in all other equilibria, each country's external enforcement level is too low relative*

to the jointly optimum level, (iii) the equilibrium internal enforcement policy is uniquely determined and efficiently (given  $\phi$ ).

The above results change dramatically when destination countries are asymmetric. To illustrate this point, assume  $L_{n1} > L_{n2}$ . Then, Proposition 6 implies that  $\phi_1^* > \phi_2^*$ ; the larger country (country 1) prefers tighter border control. This makes the best-response functions asymmetric as shown in Figure 3.

[Figure 3 around here]

In Figure 3,  $BR_1$  comprises the segment 0B of the 45-degree line and the vertical line at point B.  $BR_2$  is the same as in Figure 2. Thus, the equilibrium set is the same as in the symmetry case, comprising all the points on the segment 0A, with point A still representing the Pareto-dominant one. Hence, in equilibrium both countries choose the common policy level. In particular, the Pareto-dominant equilibrium at point A implements the unconstrained optimum policy for country 2. However, for country 1 it is too low relative to its unconstrained optimum level, which is at point B. Thus, if destination countries are asymmetric, there necessarily arises policy conflict as regards external enforcement policy.

Further, if both countries raise the level of external enforcement slightly above the equilibrium level  $\phi_2^*$ , there is no first-order welfare loss for country 2 but country 1 welfare increases. Clearly, there is no Nash equilibrium that maximizes the joint welfare. To further explore this point, consider the joint welfare maximization problem:

$$\begin{aligned} & \max_{\phi_1, \phi_2, \phi} SW_1 + SW_2 \\ & \text{s.t. } \phi_1 = \phi_2 = \phi. \end{aligned}$$

The objective function is concave under Assumption 3 and differentiable. The first-order condition is

$$\left\{ \frac{\partial \Gamma_1}{\partial s_1} \frac{\partial s_1}{\partial \phi} - g'(\phi) \right\} + \left\{ \frac{\partial \Gamma_2}{\partial s_2} \frac{\partial s_2}{\partial \phi} - g'(\phi) \right\} = 0.$$

Let  $\phi^J$  denote the jointly optimal external enforcement policy. Evaluated at  $\phi = \phi_1^* > \phi_2^*$  the first expression in braces on the left-hand side vanishes while the second is negative since country 2 is smaller. Thus,  $\phi^J < \phi_1^*$ . At  $\phi = \phi_2^*$ , the second expression vanishes while the first is positive. Thus,  $\phi^J > \phi_2^*$ . Therefore,  $\phi^J \in (\phi_2^*, \phi_1^*)$ ; the jointly optimal policy  $\phi^J$  lies in the interior of line segment AB in Figure 3. Thus,

with size asymmetry, the destination countries face both coordination failures (when the equilibrium is not Pareto-dominant) and policy conflict (when the equilibrium is Pareto-dominant). We have established the following results.

**Proposition 8** *In the common-border case with asymmetric destination countries:*

(A) *There is a continuum of Nash equilibria.*

(B) *The Pareto-dominant external enforcement level coincides with the smaller country's unconstrained optimum level but is lower than the larger country's unconstrained optimum level.*

(C) *The jointly optimal external enforcement level is higher than the smaller country's unconstrained optimum level but lower than the larger country's.*

Suppose next that the two destination countries agree to create, and delegate external enforcement to, a federal or supranational government, whose task is to maximize the joint welfare. Then, a supranational government sets  $\phi_i = \phi^J$  and must devise a lump-sum tax scheme  $t_i$  for each country to defray the cost of enforcement. We consider how the tax burden should be distributed between the two countries. When countries are symmetric, simply setting  $t_i = g(\phi^J)$ ,  $i = 1, 2$  suffices. That is, tax rates should be equalized. When countries are asymmetric, however, taxes cannot remain equalized because country 2, the smaller of the two, would refuse to delegate external enforcement policy to the supranational government. In fact, we show, in Appendix D, that  $\hat{t}_1 > \hat{t}_2$ , i.e., the larger country needs to bear a greater share of the enforcement cost than the smaller country so as to keep country 2 indifferent between the jointly optimal external enforcement policy and the Pareto-dominant Nash equilibrium policy. A tax scheme that makes country 2 better off increases the tax burden for the larger country.

**Proposition 9** *Asymmetric destination countries can attain the jointly optimum external enforcement level by creating and delegating external enforcement policy to a supranational government only under a tax scheme that assigns a greater share of the enforcement cost to the larger country.*

We next consider a second-best situation. The question is whether, in the absence of a supranational government, it is possible to delegate control of the two borders to one country without side payments and, if so, which country should be assigned the responsibility. The responsible country  $i$  chooses  $\phi$  to maximize

$\Gamma_i - 2g(\phi)$  as it controls the two borders. The first-order condition is

$$\partial\Gamma_i/\partial\phi - 2g'(\phi) = 0.$$

Let  $\phi_i^D$  satisfy this first-order condition of this delegation problem. It is evident that  $\phi_i^D < \phi_i^*$  since now country  $i$  incurs the cost of patrolling both borders. It is also obvious that  $\phi_i^D < \phi^J$  because country  $i$  is not concerned about the other country's welfare. Also, since  $\partial\Gamma_i/\partial\phi$  is increasing in  $L_i$ , we have  $\phi_1^D > \phi_2^D$ . The last result implies that the joint net surplus is greater if border control is delegated to the larger country than to the small country.

Now, suppose that the initial Nash equilibrium is the Pareto-dominant one ( $\phi_2^*$ ) at point A. Then,  $\phi_2^D < \phi_2^*$ , so delegating border control to country 2 cannot improve the joint welfare. On the other hand, delegation to country 1 can increase the joint net welfare, if and only if  $\phi_1^D > \phi_2^*$ , which requires that country 1 be sufficiently large; otherwise, delegation of external enforcement to an individual country will not succeed. By contrast, if the initial Nash equilibrium is not the Pareto-dominant outcome, even delegation to the smaller country 2 may increase the joint net welfare if (and only if)  $\phi_2^D > \phi^N$ .

**Proposition 10** *In the absence of a supranational government, (i) delegating border control to the larger country yields greater joint welfare than doing so to the smaller country, and (ii) in the presence of substantial asymmetries, each country can benefit from delegation of border control to the large country.*

## 5.2 The single-border case

We now turn to the single-border case. Assume without loss of generality that only country 1 is contiguous with the source country so immigrants must travel through country 1 to reach country 2. Then, only country 1 can enforce border control, so in equilibrium

$$U_{m1} = \frac{b}{1 - \phi_1}.$$

Since immigrants can move freely between countries 1 and 2,  $U_{m1} = U_{m2}$  also holds in equilibrium. Therefore,

$$U_{m1} = U_{m2} = \frac{b}{1 - \phi_1}.$$

In the present case, country 1 maximizes  $SW_1 = \Gamma_1 - g(\phi_1) - h(\delta_1)$  with respect to  $\phi_1$  and  $\delta_1$  whereas country 2 maximizes  $SW_2^E = \Gamma_2 - h(\delta_2)$  with respect to  $\delta_2$ , given  $\phi_1$ . Given concavity of the welfare

functions, this game has a unique equilibrium, in which country 1 implements its unconstrained optimum policy vector, namely  $(\phi_1^*, \delta_1^*)$ , while country 2's internal enforcement policy is efficient, given  $\phi_1^*$ . Note that this result does not depend on the relative size of the destination countries.

**Proposition 11** *In the single-border case, the border country implements its external and internal enforcement policy at the unconstrained optimal levels, while the non-border country implements its internal enforcement policy efficiently, given the former's border policy.*

While the two countries' equilibrium policy levels are efficient from an individual point of view, they do not maximize joint welfare. To show it, consider maximization of the jointly optimal policy:

$$SW_1 + SW_2^E = \Gamma_1 + \Gamma_2 - g(\phi_1) - h(\delta_1) - h(\delta_2).$$

The optimal external enforcement policy satisfies the first-order condition:

$$\frac{\partial \Gamma_1}{\partial s_1} \frac{\partial s_1}{\partial \phi_1} - g'(\phi_1) + \frac{\partial \Gamma_2}{\partial s_2} \frac{\partial s_2}{\partial \phi_2} \Big|_{\phi_2=\phi_1} = 0.$$

Given the concavity of the objective function, there is a unique jointly optimum external enforcement policy,  $\tilde{\phi}^J$ .

When evaluated at the equilibrium policy  $\phi_1^*$ , the first two terms on the left-hand side of the first-order condition vanish but the last term is positive. That is, the joint welfare is increasing at the equilibrium policy choice ( $\phi_1 = \phi_1^*$ ), implying that  $\tilde{\phi}^J > \phi_1^*$ ; country 1's external enforcement policy is too low for a joint optimum.

**Proposition 12** *In the single-border case, the jointly optimal external enforcement level exceeds the Nash equilibrium level.*

Proposition 12 has the standard externality-based explanation: when implementing external enforcement policy, the border country does not take into account the impact of its policy on the other country's surplus, and hence the equilibrium external enforcement is below the joint optimum. This result holds independently of the relative size of the two countries.

We now turn to the question of how to allocate the border enforcement cost between the two countries for the joint optimum. Since  $\tilde{\phi}^J > \phi_1^*$ , we have  $\Gamma_1(\tilde{\phi}^J) > \Gamma_1(\phi_1^*)$ . That is, the jointly optimum border control

policy raises country 1's gross surplus. However, it also means a greater enforcement cost. Therefore, it is necessary to tax country 2 to defray the enforcement cost for country 1. To be more specific, there exists  $\tilde{\phi}$  satisfying  $\Gamma_1(\tilde{\phi}^J) - g(\tilde{\phi}) = \Gamma_1(\phi_1^*) - g(\phi_1^*)$ . That is,  $\tilde{\phi}$  makes country 1 indifferent between the jointly optimal policy and the Nash equilibrium policy. Then, taxing countries 1 and 2 at the rates  $\tilde{t}_1 = g(\tilde{\phi}) > g(\phi_1^*)$  and  $\tilde{t}_2 = g(\tilde{\phi}^J) - g(\tilde{\phi}) > 0$ , respectively, yields the joint optimum, while keeping country 1 indifferent. If country 1 is to be made strictly better off, country 2 would have to bear a greater tax burden than  $\tilde{t}_2$ .

**Proposition 13** *Delegating external enforcement policy to a supranational government attains the joint optimum, which dominates the Nash equilibrium in the Pareto sense, if the two countries are taxed at the rate  $\tilde{t}_1$  and  $\tilde{t}_2$  defined above.*

Finally, supposing there is no supranational authority, consider delegation of border control to country 2 without side payments. The argument we used to obtain Proposition 12 can be employed to show that  $\tilde{\phi}^J > \phi_2^*$  and hence  $\tilde{\phi}^J > \max(\phi_1^*, \phi_2^*)$ ; the jointly optimal border control level exceeds each country's unconstrained optimum. Therefore, the joint surplus function  $\Gamma_1(\phi) + \Gamma_2(\phi) - g(\phi)$  is increasing at  $\phi < \tilde{\phi}^J$ . Then, since a larger country would choose a higher border control level in equilibrium, delegation of external enforcement to a larger country yields greater joint welfare.

**Proposition 14** *In the single-border case, absent the supranational government, delegation of border enforcement policy to the larger country yields a greater joint net surplus.*

## 6 Extensions

This section presents three extensions. The first extension relaxes the assumption that native workers do not migrate to the other destination country. The second extension combines the two prototypical cases in a model with three destination countries. The third extension considers general cases of inter-country asymmetry and shows that policy conflicts arises from the presence of any such asymmetry.

### 6.1 Mobile native workers

In this subsection we relax the assumption that natives do not migrate between destination countries. Since the basic model is linear in the size of the native population, mobility of native workers results either in

indeterminacy of their distribution between two destination countries or in corner solutions (all natives move to one country). To avoid such undesirable consequences, assume that natives attach extra utility values  $a > 0$  to staying in their native countries. Assume that  $a$  is drawn independently for each native worker from common distribution  $F(a)$ , with continuous and positive density  $f(a) = dF(a)$  over positive support. With this modification, the asset-value equations for native workers staying in their home countries  $i$  are rewritten as

$$\begin{aligned} rU_{ni} &= a + s(W_{ni} - U_{ni}), \\ rW_{ni} &= w_{ni} + a + \lambda(U_{ni} - W_{ni}), \\ W_{ni} - U_{ni} &= \frac{w_{ni}}{r + \lambda + s_i}. \end{aligned}$$

Because the values of firms are unaffected by native labor mobility, the wage for native workers satisfies (8).

Hence,  $U_{ni}$  is expressed as

$$U_{ni} = \frac{a}{r} + \frac{s_i y}{r [2(r + \lambda) + s_i]}. \quad (25)$$

Assuming that native workers move only when they are unemployed, focus on country-two natives. If some of them move to country 1, the following threshold condition holds for those (i.e.,  $\hat{a}_2$ ) who are indifferent between staying in country 2 and migrating to country 1:

$$U_{n1}|_{a=0} = U_{n2}|_{a=\hat{a}_2}. \quad (26)$$

Substituting, we can rewrite this condition as

$$\frac{s_1 y}{2(r + \lambda) + s_1} = \hat{a}_2 + \frac{s_2 y}{2(r + \lambda) + s_2}.$$

The left-hand side represents the welfare from migrating to country 1, while the right-hand side represents the the welfare from staying in country 2. Thus, all country-two natives with  $a < \hat{a}_2$  move to country 1 while those with  $a > \hat{a}_2$  remain in country 2.

Assume that each destination country government counts all the “native” resident workers in its native labor force, regardless of where they come from. Then, when choosing its enforcement policy, the country-1 government takes its effect on  $\hat{a}_2$  into consideration (while still taking the wage and job-finding rate in country 2,  $w_{n2}$  and  $s_2$ , as given). From (26), we have

$$\frac{\partial \hat{a}_2}{\partial \delta_1} = r \frac{\partial U_{n1}}{\partial s_1} \frac{\partial s_1}{\partial \delta_1} > 0,$$

and hence a stronger internal enforcement policy in country 1 increases that country's native labor force by

$$\frac{\partial L_{n1}}{\partial \delta_1} = F'(\hat{a}_2)(1 - u_{n2})L_{n2} \frac{\partial \hat{a}_2}{\partial \delta_1} > 0.$$

Similar arguments hold for country 2.

By Equation (24) country  $i$ 's welfare increases in  $L_{ni}$ , implying that both countries prefer to have a greater native labor force. More specifically, we know from (21) that an increase in  $\delta_i$  raises  $U_{ni}$ , so two governments compete to attract native workers from each other with stronger internal enforcement. However, there cannot be two-way native migration in equilibrium.<sup>31</sup> Thus, only two types of equilibria are conceivable. In one, there is no migration of native labor but internal enforcement levels are set higher than in the analysis of Section 5 to keep each other's native worker from leaving. In the other equilibrium, there is one-way immigration; say, there is migration of country 2 workers to country 1, but not the other way around. In this case, country 1 sets its internal enforcement level sufficiently higher than country 2 so that these conditions hold in equilibrium:

$$\frac{s_1 y}{2(r + \lambda) + s_1} = \hat{a}_2 + \frac{s_2 y}{2(r + \lambda) + s_2} \quad \text{and} \quad \frac{s_2 y}{2(r + \lambda) + s_2} < \underline{a} + \frac{s_1 y}{2(r + \lambda) + s_1},$$

where  $\underline{a}$  denotes the minimum of support of  $F(a)$ . In this case, even if two destination countries are initially identical in the sizes of their native populations, country 1 becomes the larger of the two. The above analysis is valid both for the common-border and the single-border case.

Turning to the effect of native mobility on external enforcement, we can show that changes in  $\phi_i$  has similar effects as in  $\delta_i$ . Namely, from (26), we obtain

$$\frac{\partial \hat{a}_2}{\partial \phi_1} = r \frac{\partial U_{n1}}{\partial s_1} \frac{\partial s_1}{\partial \phi_1} > 0.$$

Hence, the country-1 government expects its native labor force to change by

$$\frac{\partial L_{n1}}{\partial \phi_1} = F'(\hat{a}_2)(1 - u_{n2})L_{n2} \frac{\partial \hat{a}_2}{\partial \phi_1} > 0.$$

Similar results hold for country 2.

Recall, however, that in the common-border case, an uncoordinated increase in  $\phi_i$  above the common rate  $\phi = \min\{\phi_1, \phi_2\}$  has no effect on  $s_i$ . On the other hand, if welfare increases with more stringent border control, the proceeding argument implies that the set of continuum of Nash equilibria is larger than the one

---

<sup>31</sup>Proof is by contradiction. Assuming that (26) and its counterpart for country-one immigrants produces a contradiction.

given in Section 5. In the single-border case, this consideration prompts the border country to implement stronger external enforcement than in the analysis of Section 5.

In short, when natives move between two destination countries, the destination country governments choose higher levels of internal enforcement policy and possibly higher levels of external enforcement, as well, compared with when natives do not move. In contrast, if two countries delegate immigration to a supragovernment, the joint optimum is unaffected by mobility of natives.

## 6.2 Three destination countries

Returning to the case of non-migrating native labor, we next extend our analysis to a slightly more complicated geography than in Section 5. Suppose there are two border countries (say, Greece and Italy) and one non-border country (say, Germany). This extension combines elements from our two prototypical cases. The equilibrium flows of immigration into all three destinations depend on the weaker of the two border countries' external enforcement policy levels. If Germany has no say on Greek and Italian enforcement policy, we also know that the external enforcement levels are suboptimal to all three countries due to the both types of externalities highlighted in our two prototypical cases. The suboptimality calls for implementation of external enforcement policy at a federal (E.U.) level. Failing that, delegating the decision to the largest country (Germany) can solve the coordination failure problem between Greece and Italy.

## 6.3 General inter-country asymmetry

When considering inter-country differences in the preceding analysis, we assumed that two destination countries had unequal sizes of native populations. In this subsection we consider other types of asymmetries, and show that any asymmetry can result in conflict in external enforcement policy. Let  $x$  denote a parameter representing an asymmetry. A destination country's objective function is now written  $\Gamma(s(\phi, x), x) - g(\phi)$ . Its optimal external enforcement policy is given by

$$\frac{\partial \Gamma(s(\phi, x), x)}{\partial s} \frac{\partial s(\phi, x)}{\partial \phi} - g'(\phi) = 0. \quad (27)$$

If  $x$  affects the first term on the left, there is policy conflict between destination countries. For example, if  $x$  raises the first term, a country having a higher  $x$  prefers more stringent external enforcement than a country having a lower  $x$ , other things being equal. When  $x = L_n$ , the analysis is particular simple, since

$\Gamma(s(\phi, x), x) = \Gamma(s(\phi))L_n$  so the first term in (27) is linear in  $L_n$ .

As an example of alternative inter-country asymmetry, suppose there is a sizable difference in productivity between two destination countries. This can be interpreted in two ways. If productivity differences are agent-independent, the derivative of the first term of (27) with respect to  $y$  may not be positive for all relevant values, although it is positive if the curvature of  $\Gamma$  with respect to  $s$  (i.e.,  $-(\partial^2\Gamma/\partial s^2) / (\partial\Gamma/\partial s)$ ) is sufficiently large.<sup>32</sup> That is, if this condition holds, a more productive country prefers more stringent external enforcement than a less productive one. On the other hand, suppose that productivity differences refer only to native labor, that is, a native worker is more productive, say, in country 1 than one in country 2, while immigrants' productivity  $y_m$  is identical regardless of where they work. In this case, the first-term of (27) becomes  $(\partial\Gamma(s(\phi, y_m), y_n)/\partial s)(\partial s(s(\phi, y_m)/\partial\phi)$ ; i.e.,  $s$  is again independent of native labor productivity  $y_n$  and it can be shown that a country with more productive native labor prefers more stringency in external enforcement policy than a country with less productive native labor. In either case, we have policy conflict in the way we described in Section 5.

## 7 Conclusions

In this paper we examine the effect of external and internal enforcement policy to combat illegal immigration in the presence of unemployment. Our analysis extends the literature on illegal immigration that started with Ethier (1986) on two separate fronts. First, our model is based on search-theoretic unemployment, while the literature typically assumed unemployment due to wage rigidity. Second, we extend the literature to cases of multiple destination countries and examine both policy conflict and policy coordination problems they face.

Assuming there are two destination countries, we consider two prototypical geographic cases. In the common-border case, both destination are contiguous with the source country so that immigrants cross the border where external enforcement is the weaker and settle in their preferred destinations. In such a case, unless destinations are symmetric, there is at least one country that prefers tighter border control than the equilibrium level, thereby giving rise to policy conflict. In the single-border case, where only one destination is contiguous with the source country, the border country implements its unconstrained optimum policy (as

---

<sup>32</sup>The proof is given in Appendix E.

in the baseline single-destination model) while the other country implements its internal enforcement policy efficiently. Although the equilibrium policies are efficient from a single-country point of view, however, the equilibrium external enforcement is too low relative to the joint optimum because the border country does not take into account the effect of its policy on the non-border country. Although policy conflict occurs with respect to external enforcement policy, in both cases internal enforcement is always implemented efficiently and hence gives rise to no policy conflict. Thus, our analysis has the clear policy implication: external enforcement policy should best be handled by the federal or supranational government, but the implementation of internal enforcement should be left up to each destination country's discretion.

We then analyze two follow-up questions. One is how the supranational government should allocate its external enforcement cost between the destination countries. Another is whether to delegate border control to one country, if there is no such supranational authority to enforce border control. With respect to the first question, we find that the joint optimum requires the larger country to bear a greater tax burden for border control. With respect to the second, we find that delegating border control to the larger country increases joint welfare. These results hold both in the common-border and single-border cases.

Several extensions manifest themselves. Firstly, our approach to modeling equilibrium unemployment was minimalist. We can incorporate richer features into our analysis to examine issues not considered in this paper; e.g., unemployment benefits, public goods provisions, possibility of amnesty, etc. A more challenging extension is introduction of capital and capital mobility between two destination countries. As shown by Kessler et al. (2002), the properties of policy competition may change if another mobile factor is introduced. Secondly, we treated native and immigrant labor as homogenous, but this assumption can be relaxed in a number of ways; for example, natives may be more productive than immigrants (however, we showed in Subsection 6.3 that this makes no qualitative difference on our results). Another extension is to treat native and immigrant labor as complementary to each other. This can make immigration beneficial to the destination countries; see e.g., Liu (2010). If so, however, there arises a different puzzle: why do destination countries try to stop illegal immigration? We leave such extensions and applications for future research.

## References

- [1] **Bandyopadhyay, Subhayu, and Santiago Pinto.** 2017. “Illegal Immigration and Fiscal Competition.” *European Economic Review* 92, 283-305.
- [2] **Bohn, Sarah, Magnus Lofstrom, and Steven Raphael.** 2014. “Did the 2007 Legal Arizona Workers Act Reduce the State’s Unauthorized Immigrant Population?” *Review of Economics and Statistics* 96: 258-269.
- [3] **Bond, Eric W., and Tain-J. Chen.** 1987. “The Welfare Effect of Illegal Migration.” *Journal of International Economics.* 23: 315-328.
- [4] **Borjas, George J.** 2003. “The Labor Demand Curve is Downward Sloping; Reexamining the Impact of Immigration on the Labor Market.” *NBER Working Paper No. 9755.*
- [5] **Brown, J. David, Julie L. Hotchkiss, and Myriam Quispe-Agnoli.** 2013. “Does Employing Undocumented Workers Give Firms a Competitive Advantage?” *Journal of Regional Science* 53, 158-170.
- [6] **Bryant, John.** 1983. “A Simple Rational Expectations Keynes-Type Model.” *Quarterly Journal of Economics* 98: 525-528.
- [7] **Card, David.** 2005. “Is the New Immigration Really so Bad?” *Economic Journal.* 115: F300-F323.
- [8] **Carter, Thomas J.** 1999. “Illegal Immigration in an Efficiency Wage Model.” *Journal of International Economics.* 49: 385-401.
- [9] **Casarico, Alessandra, Giovanni Facchini, and Tommaso Frattini.** 2015. “Illegal Immigration: Policy Perspectives and Challenges.” *CESifo Economic Studies* 61: 673-700.
- [10] **Chassamboulli, Andri, and Theodore Palivos.** 2014. “A Search-Equilibrium Approach to the Effects of Immigration on Labor Market Outcomes.” *International Economic Review.* 55: 111-129.
- [11] **Cremer, Helmuth, and Pierre Pestieau.** 2004. “Factor Mobility and Redistribution.” in: J.V. Henderson and J.F. Thisse (ed.), *Handbook of Regional and Urban Economics.* 4: 2529-2560, Elsevier.

- [12] **D'Amuri, Francesco, Gianmarco I.P. Ottaviano, and Giovanni Peri.** 2010. "The Labor Market Impact of Immigration in Western Germany in the 1990's." *European Economic Review*. 54: 550-570.
- [13] **Djajić, Slobodan.** 1987. "Illegal Aliens, Unemployment and Immigration Policy." *Journal of Development Economics*. 25: 235-249.
- [14] **Ethier, Wilfred J.** 1986. "The Illegal Immigration: The Host Country Problem." *American Economic Review*. 76: 56-71.
- [15] **Giordani, Paolo E., and Michele Ruta.** 2013. "Coordination Failures in Immigration Policy." *Journal of International Economics*. 89: 55-67.
- [16] **Hanson, Gordon H., and Antonio Spilimbergo.** 1999. "Illegal Immigration, Border Enforcement, and Relative Wages: Evidence from Apprehensions at the U.S.-Mexico Border." *American Economic Review* 89: 1337-1357.
- [17] **Hanson, Gordon H., Raymond Robertson, and Antonio Spilimbergo.** 2002. "Does Border Enforcement Protect U.S. Workers?" *Review of Economics and Statistics*. 84: 73-92.
- [18] **Hirshleifer, Jack.** 1983. "From Weakest-Link to Best-Shot: The Voluntary Provision of Public Goods." *Public Choice*. 41: 371-386.
- [19] **Hirshleifer, Jack.** 1985. "From Weakest-Link to Best-Shot: Correction." *Public Choice*. 46: 221-223.
- [20] **Hoekstra Mark, and Sandra Orozco-Aleman.** 2014. "Illegal Immigration, State Law, and Deterrence." *NBER Working Paper* 20801.
- [21] **Kessler, Anke S., Christoph Lüfessmann, and Gordon M. Myers.** 2002. "Redistribution, Fiscal Competition, and the Politics of Economic Integration." *Review of Economic Studies*. 69: 899-923.
- [22] **Liu, Xiangbo.** 2010. "On the Macroeconomic and Welfare Effects of Illegal Immigration." *Journal of Economic Dynamics and Control*. 34: 2547-2567.
- [23] **Mangin, Sephorah, and Yves Zenou.** 2016. "Illegal Migration and Policy Enforcement." *Economics Letters* 148: 83-86.

- [24] **Mayr, Karin, Steffen Minter, and Tim Krieger.** 2012. “Policies on Illegal Immigration in a Federation.” *Regional Science and Urban Economics* 42: 153-165.
- [25] **Ottaviano, Gianmarco I. P., and Giovanni Peri.** 2012. “Rethinking the Effect of Immigration on Wages.” *Journal of the European Economic Association*. 10: 152-197.
- [26] **Pissarides, Christopher A.** 2000. *Equilibrium unemployment theory*. The MIT Press: Cambridge.
- [27] **Rogerson, Richard, Robert Shimer, and Randall Wright.** 2005. “Search-Theoretic Models of the Labor Market: A Survey.” *Journal of Economic Literature*. XLIII: 959-988.
- [28] **Russo, Giuseppe, and Luigi Senatore.** 2013. “Who Contributes? A Strategic Approach to a European Immigration Policy.” *IZA Journal of Migration* 2, DOI: 10.1186/2193-9039-2-16.
- [29] **Yoshida, Chisato, and Alan D. Woodland.** 2005. *The economics of illegal immigration*. Palgrave Macmillan.
- [30] **Wilson, John D.** 1986. “A Theory of Inter-Regional Tax Competition.” *Journal of Urban Economics*. 19: 296-315.
- [31] **Wilson, John D.** 1999. “Theories of Tax Competition.” *National Tax Journal*. 52: 269-304.
- [32] **Woodland, Alan D., and Chisato Yoshida.** 2006. “Risk Preference, Immigration Policy and Illegal Immigration.” *Journal of Development Economics*. 81: 500-513.
- [33] **Zenou, Yves.** 2009. *Urban Labor Economics*. Cambridge University Press: Cambridge.
- [34] **Zodrow, George R., and Peter Mieszkowski.** 1986. “Pigou, Tiebout, Property Taxation, and the Underprovision of Local Public Goods.” *Journal of Urban Economics*. 19: 356-370.

## Appendices

### Appendix A: Proof of $\partial L_m / \partial \phi < 0$

To find the effect on the number of immigrants in the destination country, differentiate (5) and substitute from (3) and (4) into the resulting equation to derive

$$(J_n - J_m) \frac{\partial \alpha}{\partial \phi} + \frac{\alpha}{r + \lambda + \delta} \frac{\partial w_m}{\partial \phi} + \frac{1 - \alpha}{r + \lambda} \frac{\partial w_m}{\partial \phi} = \frac{c}{q^2} \frac{\partial q}{\partial \phi}. \quad (28)$$

Given  $s'(\theta) > 0$ ,  $\partial s/\partial \phi > 0$  implies  $\partial \theta/\partial \phi > 0$ , and hence  $\partial q/\partial \phi < 0$ , given  $q'(\theta) < 0$ . This makes the right-hand side of (28) negative. On the left-hand side of the equation, since  $\partial w_i/\partial \phi > 0$ , the second and the third terms are positive. Hence, (28) holds only if the first term on the left-hand side is negative. Since  $J_n - J_m > 0$  by Proposition 2, we conclude that  $\partial \alpha/\partial \phi < 0$ ; i.e., tighter border control (an increase in  $\phi$ ) decreases the proportion of immigrants in the unemployment pool. Finally, (15) is arranged to yield

$$L_m = \frac{\alpha}{1 - \alpha} \frac{u_n}{u_m} L_n.$$

Since tighter border control lowers the two ratios  $\alpha/(1 - \alpha)$  and  $(u_n/u_m)$ , the total number of immigrants in the destination country must fall.

## Appendix B: An Alternative Welfare Function

If we include the surplus of firms employing illegal workers, the total (gross) surplus is given by

$$G \equiv (1 - u_n)L_n(J_n + W_n - U_n) + (1 - u_m)L_m J_m = 2L_n(1 - u_n)J_n + (1 - u_m)L_m J_m.$$

Then, substituting for  $J_n$ ,  $J_m$ ,  $u_n$ , and  $u_m$  from (13), (14), and (19) yields an extended measure of gross welfare

$$G = \frac{2syL_n}{(\lambda + s)[2(r + \lambda) + s]} + \Theta L_m,$$

where  $\Theta$  is defined as

$$\Theta \equiv \frac{rys}{(r + \delta + s)\{r[2(r + \lambda) + s] + \delta(2r + s)\}}.$$

The government now chooses  $\phi$  and  $\delta$  to maximize

$$SW = G - g(\phi) - h(\delta).$$

The optimal external enforcement policy satisfies the first-order condition:

$$\frac{\partial SW}{\partial \phi} = \frac{\partial G}{\partial \phi} - g'(\phi) = 0,$$

and the optimal internal enforcement policy satisfies the first-order condition:

$$\frac{\partial SW}{\partial \delta} = \frac{\partial G}{\partial \delta} - h'(\delta) = 0.$$

Note that the first term of  $G$  is a function of  $s$  only but the second term is not. Hence, the policy effects on  $G$  becomes somewhat complicated:

$$\begin{aligned}\frac{\partial G}{\partial \phi} &= \frac{2[2\lambda(r+\lambda) - s^2]yL_n}{(\lambda+s)^2 [2(r+\lambda) + s]^2} \frac{\partial s}{\partial \phi} \\ &\quad + \frac{ry(r+\delta) [2r(r+\delta+\lambda) - s^2] L_m}{(r+\delta+s)^2 \{2r^2 + \delta s + r [2(\delta+\lambda) + s]\}^2} \frac{\partial s}{\partial \phi} + \Theta \frac{\partial L_m}{\partial \phi}, \\ \frac{\partial G}{\partial \delta} &= \frac{2[2\lambda(r+\lambda) - s^2]yL_n}{(\lambda+s)^2 [2(r+\lambda) + s]^2} \frac{\partial s}{\partial \delta} \\ &\quad + \frac{ry(r+\delta) [2r(r+\delta+\lambda) - s^2] L_m}{(r+\delta+s)^2 \{2r^2 + \delta s + r [2(\delta+\lambda) + s]\}^2} \frac{\partial s}{\partial \delta} \\ &\quad - \frac{rsy (4r^2 + 2r(2\delta + \lambda + 2s) + s(2\delta + s)) L_m}{(\delta+r+s)^2 (2r^2 + r(2(\delta+\lambda) + s) + \delta s)^2} + \Theta \frac{\partial L_m}{\partial \delta}.\end{aligned}$$

The first terms of the above two equations are the same as in the case described in the main text, while the other terms capture the effect through changes in the surpluses accruing to illegitimate firms. Especially, the last terms represent the losses from decreasing illegitimate employment. If such losses dominate the other effects, governments have no incentive to conduct enforcement policies. If not, governments exert enforcement policies, which is the focus of our paper. In such a case, we can conduct the same analysis as the one described in the main text.

### Appendix C: Proof of Lemma 1

We have  $\partial s / \partial \phi > 0$  and

$$\frac{\partial^2 s}{\partial \phi^2} = \frac{4y^2 b (r+\delta) (r+\delta+\lambda)}{[(1-\phi)y - (r+\delta)]^3} > 0,$$

Thus,  $s(\phi)$  is convex and strictly increasing and hence invertible. Since  $\Gamma$  depends on  $s$  but not directly on the policy variables, we can write the objective function as  $\Gamma(s) - g(\phi(s))$ . Maximizing the latter with respect to  $s$  yields the first-order condition

$$\Gamma'(s) - g'(\phi(s))\phi'(s) = 0,$$

where

$$\Gamma'(s) = \frac{2[2\lambda(r+\lambda) - s^2]yL_n}{(\lambda+s)^2 [2(r+\lambda) + s]^2}.$$

The second-order condition requires

$$\Gamma''(s) - g''(\phi(s))[\phi'^2 - g'(\phi(s))\phi''(s)] < 0$$

at the optimum. Here, we know that  $g'(\phi(s)) > 0$ ,  $g''(\phi(s)) > 0$ , and  $\phi''(s) > 0$ . Moreover, differentiation shows that  $\Gamma''(s) < 0$  under Assumptions 1 and 2. Thus, the second-order condition holds globally. Finally,  $\lim_{s \downarrow 0} \Gamma'(s) > 0$  implies that  $s > 0$  at the optimum.

#### Appendix D: Proof of $\hat{t}_1 > \hat{t}_2$

Note first that  $\phi^J > \phi_2^*$  implies that  $\Gamma_2(\phi^J) - g(\phi^J) < \Gamma_2(\phi_2^*) - g(\phi_2^*)$  so country 2 is made worse off by the policy  $\phi^J$ . However, (20) and Assumption 3 imply that  $\Gamma_i$  is increasing in  $\phi$ . Then,  $\Gamma_2(\phi^J) - g(\phi_2^*) > \Gamma_2(\phi_2^*) - g(\phi_2^*)$ . Therefore, there exists  $\hat{\phi} \in (\phi_2^*, \phi^J)$  that makes country 2 indifferent in the sense that  $\Gamma_2(\phi^J) - g(\hat{\phi}) = \Gamma_2(\phi_2^*) - g(\phi_2^*)$ . Hence, if the supranational government imposes lump-sum taxes  $\hat{t}_1 = 2g(\phi^J) - g(\hat{\phi}) > 0$  and  $\hat{t}_2 = g(\hat{\phi}) > 0$  on countries 1 and 2, respectively,

$$\begin{aligned} \Gamma_1(\phi^J) - \hat{t}_1 &= \Gamma_1(\phi^J) - \left(2g(\phi^J) - g(\hat{\phi})\right) \\ &= \Gamma_1(\phi^J) + \Gamma_2(\phi^J) - 2g(\phi^J) - (\Gamma_2(\phi_2^*) - g(\phi_2^*)) \\ &> \Gamma_1(\phi_2^*) - g(\phi_2^*), \end{aligned}$$

where the inequality comes from the property of the joint optimum. Thus, the above tax scheme makes country 1, the larger country, better off than in the Nash equilibrium. Since  $g(\phi^J) > g(\hat{\phi})$ , we have that  $\hat{t}_1 > \hat{t}_2$ .

#### Appendix E: Inter-country asymmetry in productivity

Replace  $x$  with  $y$  in the first term on the left of (27) and differentiate the result with respect to  $y$  to obtain

$$\left(\frac{\partial^2 \Gamma}{\partial s^2} \frac{\partial s}{\partial y} + \frac{\partial^2 \Gamma}{\partial s \partial y}\right) \frac{\partial s}{\partial \phi} + \frac{\partial \Gamma}{\partial s} \frac{\partial^2 s}{\partial \phi \partial y},$$

which captures the effect on the marginal value of external enforcement policy from a change in productivity.

From (18) and (20), we obtain

$$\begin{aligned} \frac{\partial s}{\partial y} &= -\frac{2b(1-\phi)(\delta+r)(\delta+\lambda+r)}{[(1-\phi)y - b(\delta+r)]^2}, \\ \frac{\partial^2 s}{\partial \phi \partial y} &= -\frac{2b(\delta+r)(\delta+\lambda+r)[(1-\phi)y + b(\delta+r)]}{[(1-\phi)y - b(\delta+r)]^3}, \end{aligned}$$

which, combined with (20) and the properties of  $\Gamma$ , shows that in equilibrium

$$\frac{\partial^2 \Gamma}{\partial s^2} < 0, \quad \frac{\partial s}{\partial y} < 0, \quad \frac{\partial^2 \Gamma}{\partial s \partial y} > 0, \quad \frac{\partial s}{\partial \phi} > 0, \quad \frac{\partial \Gamma}{\partial s} > 0, \quad \text{and} \quad \frac{\partial^2 s}{\partial \phi \partial y} < 0.$$

Hence, the effect of  $y$  is positive if

$$\frac{\partial^2 \Gamma}{\partial s^2} \frac{\partial s}{\partial y} \frac{\partial s}{\partial \phi} + \frac{\partial \Gamma}{\partial s} \frac{\partial^2 s}{\partial \phi \partial y} > 0. \quad (29)$$

We can use these expressions and (20) to rewrite (29) as

$$-\frac{\partial^2 \Gamma / \partial s^2}{\partial \Gamma / \partial s} > \frac{[(1 - \phi)y]^2 - [b(\delta + r)]^2}{2by(1 - \phi)(\delta + r)(\delta + \lambda + r)}.$$

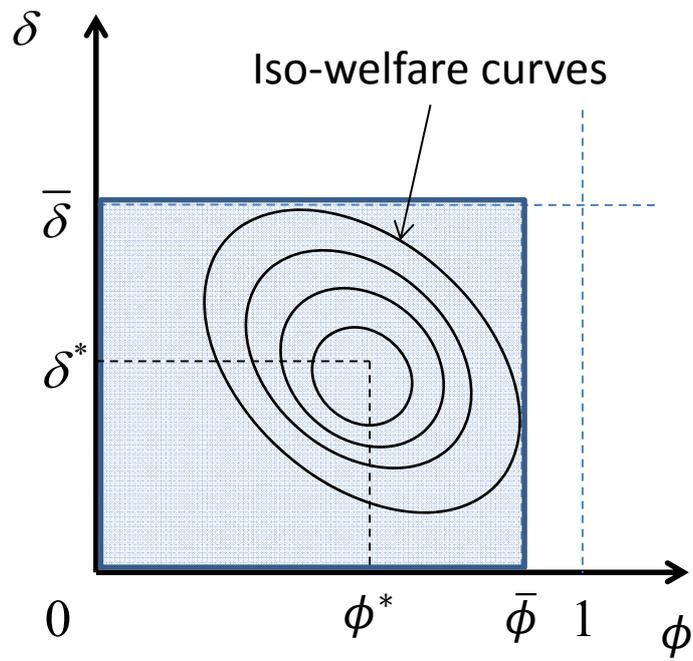


Figure 1. Ranges of  $\phi$  and  $\delta$

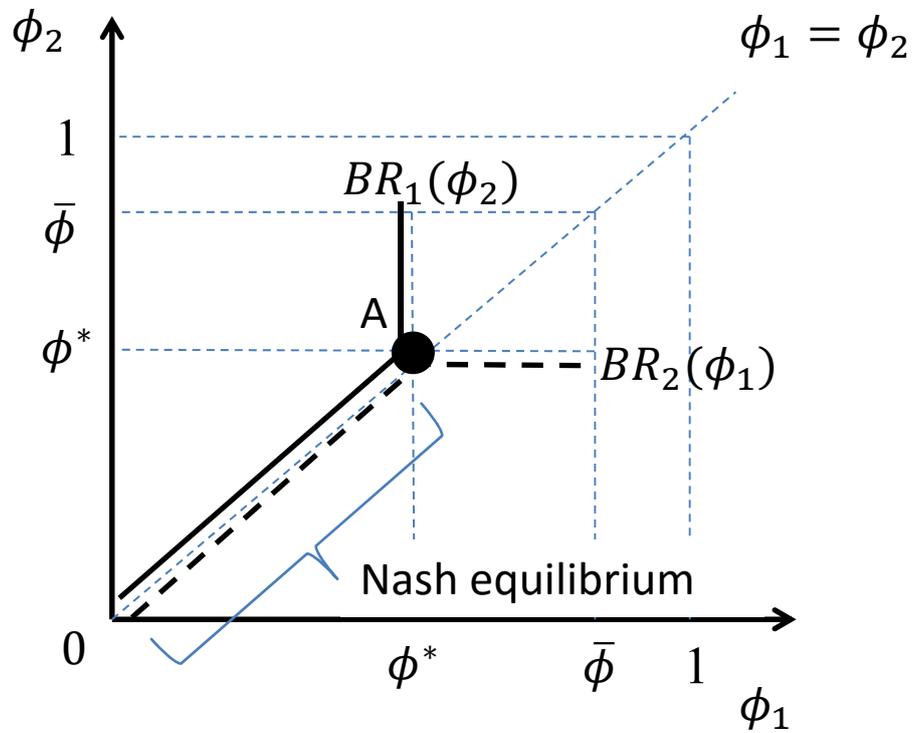


Figure 2. Symmetric destination countries with common border

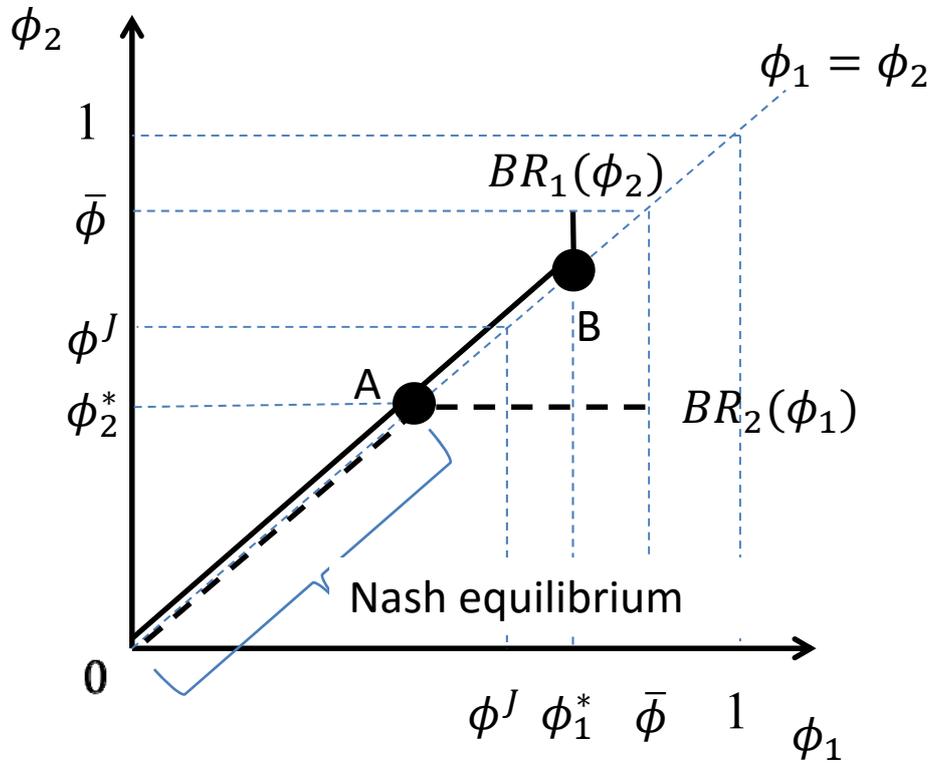


Figure 3. Asymmetric destination countries with common border