Too Small To Protect? The Role of Firm Size in Trade Agreements*

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Abstract

This paper develops a model of a trade agreement that puts at centre stage the competing interests between firms within a sector: Larger firms tend to be pro-trade liberalization whereas smaller firms favor protection. This set-up contrasts with the prior literature in economics that has tended to focus on competing interests between sectors, and further develops a literature in political science. The paper determines the set of circumstances under which a trade agreement will be reached, and the extent of trade liberalization, in an environment where firms can lobby the government for or against a proposed agreement. Lobbying is modelled as an all-pay auction, incorporating the feature that binding contracts over contributions for policies cannot be written. This approach gives rise to the possibility that a trade agreement may not go ahead even if it would increase efficiency. In this set-up, if a proposed agreement is over non-tariff barriers and if it goes ahead then it always entails free trade. On the other hand, if a proposed agreement is over tariffs and if it goes ahead then it either entails free trade or the tariff revenue maximizing tariff.

Keywords: All-pay auction, firm heterogeneity, non-tariff barriers, tariffs, trade agreement.

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1 Introduction

It is generally understood that the outcome of a trade agreement is determined by a tension between competing interests. In real-world commentaries of trade agreements, this tension is usually identified between export sectors on the one hand who favor greater openness and import-competing sectors on the other who favor greater protection. For example, the UK government's Department for Trade and Industry report to Parliament on the outcome of the Uruguay Round focused first and foremost on the benefits to UK export sectors (DTI 1994, 10). (The Uruguay Round was the last successfully completed round of trade talks under the General Agreement on Tariffs and Trade, or GATT). The phasing out of the Multi-Fibre Agreement also received attention because this lowered tariffs in industrialized countries on textiles and clothing, which the UK imports largely from developing countries but where it still maintains some competing interests. This basic approach of considering 'export sectors versus import competing sectors' is mirrored in literature on the economics of trade agreements as well (Grossman and Helpman 1995, Maggi and Rodriguez-Clare 1998, Bagwell and Staiger 1999).

At the same time there appears to be an awareness that, even within a sector, firms' interests over a trade agreement may conflict. Perhaps surprisingly, given the present focus on the behavior of firms in economics, it has been political scientists rather than economists who have emphasized this dimension of conflicting interests between firms. This division is expressed most clearly by Thacker (1998, 2000) in his study of Mexican business interests and their relationship with government and trade liberalization: "Where oligopolistic markets exist and economies of scale give competitive advantages to large firms, size will also affect a firm's trade policy preferences. Under these conditions, larger firms will be more competitive internationally and therefore more likely to favor free trade. Smaller firms will be more protectionist" (Thacker 1998, 3). This view is also at the heart of work on the GATT and World Trade Organization (WTO) by Cowles (1998) and Aaronson (2001) for example. And this view is represented albeit more subtlely in real-world accounts of trade agreements as well. For example, DTI (1994, 10, para. 5.13) in the assessment of the Uruguay round discussed above, also draws attention to the fact that UK firms both export industrial electronics and compete with foreign imports in the same sector. The present paper develops a model of a trade agreement that puts at centre stage the competing interests not of sectors but of firms. It determines the set of circumstances under which a trade agreement will be reached and the extent of trade liberalization in an environment where firms can lobby the government for the policy that they would prefer.

The economic model is essentially that of Melitz (2003): in each of two countries, a continuum of monopolistically competitive firms produces a horizontally differentiated product at varying degrees of productivity. One of the main predictions of Melitz (2003) is that larger firms tend to export while smaller firms tend only to serve the domestic market. Throughout our analysis, we make the standard assumption that the variation in productivity across firms is approximated by the Pareto distribution. Trade restrictions are captured by one of two possible instruments: iceberg transport costs or ad valorem tariffs.

Starting from autarky, a trade agreement is reached according to the following procedure. The two governments announce a proposed agreement to lower trade restrictions, to the same level, across the two countries. The proposed level can lie anywhere below autarky and is bounded from below at free trade. Each firm then has one of two options with regard to lobbying. It can contribute to a lobby group that will lobby the government in support of the proposed trade agreement. Or it can contribute an amount to a lobby group that will lobby the government in support of the status quo at autarky. This captures the well known feature of real-world trade policy-making that firms form lobby groups to lobby for and against trade agreements. For example, the 'Alliance for GATT Now' group was formed to support the Uruguay Round vote whereas the 'Labor/Industry Coalition for International Trade' was (someone incongruously) formed to lobby against it (Dam 2001, 14). It is through this set of policy interactions that the equilibrium trade agreement configuration will be determined. Our framework abstracts entirely from the role of consumers in order to focus on interactions between firms and their national government in lobbying over trade policy. This approach is tractable and approximates a more general framework that incorporates consumers but where there is a relatively large weight in the government's payoff function on contributions made by firms.

We will make the assumption that once contributions have been made they cannot be retrieved regardless of the outcome. This assumption is reasonable given that in many national legislatures including the US Congress it is illegal for lobby groups to 'pay by results'. Based on the assumption that contributions are irretrievable, we model the lobbying process as an all-pay auction (with common knowledge). An all-pay auction is like a regular first-price auction except that everyone must pay their bids. Use of an all-pay auction to model lobbying over trade policy was first proposed by Hillman (1988) and has been used to model a wide variety of political contests where contributions are sunk.

We begin our analysis of trade agreement outcomes by focusing on the case where trade restrictions are approximated by iceberg transport costs. This reflects the standard approach in the literature on heterogenous firms, and is the most tractable approach to the modeling of trade restrictions in this setting. We will refer to iceberg transport costs as 'non-tariff barriers' (NTBs) to clarify that they are determined by the government (as opposed to being determined by geography) but raise no revenue. A surprising feature of this set-up is that the contributions made by firms who favor autarky is identical to that for the trade agreement. This result depends on the assumption that firm productivity is distributed Pareto and holds regardless of the extent of trade liberalization under the agreement. In equilibrium, the proposed trade agreement entails free trade because this extracts the largest average contributions from firms.

A novel feature of the equilibrium is that the trade agreement does not go ahead with certainty. This outcome rests on the underlying all-pay auction structure to the process of lobbying. An environment where everyone forfeits their contribution regardless of the outcome creates an incentive to increase contributions more aggressively. But once one lobby group contributes its full valuation for the outcome then the other lobby group has an incentive to contribute nothing. The first lobby group, knowing this, now only wants to contribute a small amount, but then the incentive to raise contributions aggressively arises once again. So there can be no Nash equilibrium in pure strategies. Consequently, lobbies randomize their contributions and this in turn introduces a degree of uncertainty into the outcome. This uncertainty is appealing in that it stems from the 'real world' feature that it is not possible for a lobby group to write a contract with the government that is enforceable in a court of law.

We then redo the analysis of the trade agreement formation but instead of NTBs we assume that trade restrictions are modelled as (ad valorem) tariffs. Tariffs alter the analysis from NTBs in two ways. Tariffs generate revenue for the government and, as Besedeš and Cole (2013), Cole (2011), Schröder and Sørensen (2011), and others have shown, ad valorem tariffs affect the extensive margin differently than iceberg transport costs in models of monopolistic competition. With this set-up, starting from autarky, unlike for NTBs a move toward (but not including) free trade would yield lower aggregate profits because a proportion of exporters' profits are shifted abroad by foreign tariffs. However, with tariffs, the variation in the level of openness across possible agreements also implies variation in the level of tariff revenue. Hence, when combined with lobbying revenues, one possible agreement may be more attractive to the government than others. In this setting, we are able to show that the government will choose between two possible outcomes (depending on parameter values). One is free trade while the other is the tariff that maximizes tariff revenue. If free trade is chosen it is because the forces to liberalize illustrated by the NTB case dominate, whereby maximizing lobby revenues push the government towards free trade. If an agreement arises where tariff revenue is maximized then, interestingly, the lobbies choose not to contribute but the agreement goes ahead with certainty.

Following Grossman and Helpman (1994), the use of a menu auction to model the process of lobbying over trade policy has become standard in the literature on the political economy of tariff setting. Therefore, it is important to explain how the outcome of a trade agreement where lobbying by heterogeneous firms takes place according to a menu auction, the approach adopted by Chang and Willman (2014), would differ and why use of an all-pay auction approach is preferable in our context. The first difference to note is that the standard (conceptual) starting point for a menu auction to determine trade policy is free trade. In our setting, adopting free trade as a starting point would omit the the influence of firms that would prefer autarky once it had been adopted but who are unprofitable at free trade and so do not produce, let alone engage in the lobbying process. Using our analysis of NTBs as a reference point, under free trade whereby a number of firms that would lobby for autarky do not produce, firms that prefer trade would always have sufficient resources to ensure through lobbying that the equilibrium remained at free trade. For this reason, it makes more sense to follow Melitz (2003) and assume that the starting point is autarky and not free trade.

There remains the question of whether we could start the lobbying process at autarky but still use a menu auction to replace the all-pay auction process that we have adopted. This would be straight forward, but yield significantly less interesting results than when lobbying is modelled as an all-pay auction. For the agreement over NTBs, the outcome under a menu auction is that the total contribution level by the firms that favor free trade is the same as by those that favor autarky, with the outcome being determined by a tie-breaking rule. For the agreement over tariffs, the outcome always goes in the direction of the firms that prefer free trade because, with the help of tariff revenue, they are always able to dominate the lobbying process and swing the outcome in their favor. So we lose the interesting feature that arises under the all-pay auction approach whereby an agreement that would raise efficiency may fail to go ahead.

There are currently two main strands to the literature on trade agreements. The first strand, associated with Bagwell and Staiger (1999), focuses on trade agreements under the institutional arrangements of the General Agreement on Tariffs and Trade (GATT)/World Trade Organization (WTO). Under this approach, interest groups may lobby governments who may also be subject to distributional concerns. Bagwell and Staiger show that a trade agreement is motivated by the terms-of-trade externality that arises when countries have power on world markets. When setting tariffs, each country fails to internalize the negative terms-of-trade externality that its tariff imposes. The two GATT/WTO pillars of reciprocity and nondiscrimination allow countries to escape the terms-of-trade externality and reach an efficient agreement. In Bagwell and Staiger's framework, if the import-competing interests in each country are equally opposed, and if each government does not have any distributional concerns of its own, then the equilibrium outcome of a trade agreement would be free trade. In our framework, countries are large in our framework and can improve their terms of trade through tariff setting. And interest groups can lobby the government to try to sway the outcome of an agreement in the direction that they would like to see it go. But, in contrast to previous contributions to this strand of the literature, trade agreements in our paper are not driven by changes in the terms of trade so much as by gains that arise from variation in firm productivity.

Under the second branch, that highlights a commitment problem faced by governments wishing to liberalize, terms of trade losses through trade liberalization are small relative to efficiency gains, and the case for trade liberalization is essentially a unilateral one (Staiger and Tabellini 1987, Staiger 1995, Maggi and Rodriguez-Clare 1998). The argument is that trade agreements can serve as a means through which a government can commit to trade liberalization in the face of opposition from protectionist forces, say industrial interest groups, within its own nation. There is a sense in which this motivation for a trade agreement exists within our framework as well. In our model no firm would agree to unilateral trade liberalization as they would lose domestic market share without gaining a greater share of export markets. Hence trade liberalization must be reciprocated in order for it to take place at all. However, reciprocal trade liberalization is an assumption in our framework and we do not explore the role of trade agreement as commitment device in this paper.

The paper proceeds as follows. Section 2 sets out the model. It begins with a brief recapitulation of the Melitz model and then goes on to explain how lobbying works and how the trade agreement is determined. Section 3 then examines the scope for reaching a trade agreement, first for NTBs and then for tariffs. It is here that we characterize the trade agreements that may arise under each of the two trade policy instruments. Conclusions are drawn in Section 4.

2 The Model

There are two countries in the model, Home and Foreign. Variables pertaining to Foreign are denoted with a superscript $*$. There are two goods. Good X comprises a continuum of differentiated varieties, each of which is indexed by i. As is standard in the Melitz model, these varieties are produced by a continuum of monopolistically competitive firms, correspondingly indexed by i, each using an increasing returns to scale technology. There is free entry in sector X but entry into the sector is costly, so profits are zero in expectation but positive ex post. Each variety of good X may face a trade barrier as it crosses the border between the countries. Good Y is a homogeneous good that is produced under constant returns to scale, perfect competition and free trade. We will choose good Y as the numeriare and, since it is freely traded, world prices and domestic prices of this good can be normalized to 1. Each unit of good Y is produced from just one unit of labor, and so the wage is normalized to 1 as well. The role of good Y in the model is to capture the general equilibrium effects of trade agreement formation. For this purpose, we will assume that each country is endowed with a sufficient quantity of labor that enough of good Y is produced to clear the trade account in equilibrium. In terms of economic structure, the two countries are identical to one another but will produce different varieties in a trading equilibrium.

There are two time periods. In period 1 there is autarky, with trade restrictions set at prohibitive levels. Producers of good X undertake entry decisions and production. Then markets clear for period 1 and consumption takes place. In period 2, the governments may announce a proposed trade agreement, which entails a symmetrical reduction in trade restrictions to a level that would allow at least some trade, possibly free trade, between the countries. Governments are able to communicate with each other over the proposed trade agreement before announcing it, and each has veto power over the agreement prior to announcement (but not afterwards). Once the proposed trade agreement is announced, each firm predicts (perfectly) the amount of profit it would earn under the alternatives of autarky and the trade agreement and hence which policy it would prefer. There are two lobbyists, one named L_A that supports the status quo of autarky in sector X and one named L_T that supports the trade agreement. Each firm makes a contribution to one of the lobby groups, depending on the regime under which its profits would be maximized. We will assume that within the group of firms making contributions to a particular lobbyist the collective action problem is resolved. Aggregate contributions made by all firms in support of autarky to L_A total l_A and aggregate contributions in support of the trade agreement to L_T total l_T. Once collected, these contributions are passed by the lobby groups to the government and cannot be retrieved thereafter. The policy that receives the greatest financial support, autarky or the trade agreement, is then enacted. If the trade agreement goes ahead, the government resets tariffs in accordance with the agreement. Finally, conditional on tariffs, consumption takes place and markets clear.

Although we will undertake separate treatments of trade agreements over NTBs and tariffs, our development of the model will incorporate both instruments simultaneously. This will provide a parsimonious representation of the model and makes it possible to compare the effects of the two restrictions. The remainder of this section specifies the specific details of the model.

2.1 Consumers

Let the utility function for the representative agent in Home take the following form:

$$U = \mu \ln(X) + Y \tag{1}$$

where

$$X = \left(\int_{i \in \Omega} x(i)^{\alpha} di \right)^{\frac{1}{\alpha}},$$

where $0 < \alpha < 1$, and $\varepsilon = 1/(1-\alpha)$ is the elasticity of substitution between varieties of X, and Ω is the set of varieties available to the consumer. Demand for each good by a consumer in Home is

$$x(i) = \frac{p(i)^{-\varepsilon} \mu}{\mathcal{P}^{1-\varepsilon}}$$

where $p(i)$ is the price of variety i sold in home, and

$$\mathcal{P}^{1-\varepsilon} = \int_{i \in \Omega} p(i)^{1-\varepsilon} di$$

is the aggregate price index in Home. An analogous set of equations holds for the foreign country where, by assumption, $\mu^* = \mu$.

2.2 Heterogeneous Firms

Firms considering entry to sector X face a one time sunk market entry cost f_E (measured in units of labor). If this cost is paid, the firm then draws a constant marginal cost coefficient a from the Pareto distribution

$$G(a) = \left(\frac{a}{a_U} \right)^k, \ 0 < a < a_U$$

where the shape parameter $k > (\varepsilon - 1)$.We will denote by a_i the marginal cost drawn by firm i. Once this is observed, a firm decides whether or not to undertake production. If it chooses to produce, it must incur an additional fixed cost f_D paid each period. If trade restrictions are at a level that allows for trade, a firm must pay an additional fixed cost $f_X = \gamma f_D > f_D$ in order to serve the foreign market. Production exhibits constant returns to scale with labor as the only factor of production.

The decision of whether or not to undertake production and whether to export depends on the associated profits. Fixing the wage equal to 1, the per-period operating profit of firm i facing marginal cost a_i and selling only domestically is

$$\begin{aligned} \pi_D(i) &= [p(i) - a_i]\, Q_D(i) - f_D \\ &= \left[\frac{[p(i) - a_i]\mu}{\mathcal{P}^{1-\varepsilon}}\right] p(i)^{-\varepsilon} - f_D. \end{aligned}$$

A firm selling domestically will charge a price equal to a constant markup over marginal cost, $p(i) = \frac{a_i}{\alpha}$. Therefore, the operating profit function for a purely domestic firm is

$$\pi_D(i) = a_i^{1-\varepsilon} B - f_D \tag{2}$$

where

$$B = \frac{1}{\varepsilon \alpha^{1-\varepsilon}} \left(\frac{\mu}{\mathcal{P}^{1-\varepsilon}}\right).$$

In order to reach the foreign market, in addition to the fixed cost γf_D, firm i incurs a per-unit cost arising from either an NTB or a tariff: modelled as an iceberg transport cost, $\tau > 1$, and an ad valorem tariff, $t > 1$, respectively. For tractability we assume that any tariff revenue is used by the government to consume only the numeriare.[1] Thus the additional

[1] Tariff revenue can only affect demand for good X if: (1) The government's preferences are not, in fact, encompassed by the representative consumer and the government demands some of good X; (2) Tariffs are not symmetric across countries and thus income is being shifted from one country to the other (however this is not an issue with quasi-linear utility) or; (3) The government simply throws the revenue away (this is not an issue with quasi-linear utility as long as a positive amount of the numeriare is produced and consumed). We do not allow for these three possibilities in our model.

operating profit from exporting for a firm that exports is

$$\pi_X = \frac{(t\tau a_i)^{1-\varepsilon} B^*}{t} - \gamma f_D, \tag{3}$$

where

$$B^* = \frac{1}{\varepsilon \alpha^{1-\varepsilon}} \left(\frac{\mu}{\mathcal{P}^{*1-\varepsilon}} \right).$$

The set-up is standard in the literature on heterogenous firms.

2.3 Government

The government's objective is to maximize income, which is derived entirely from tariff revenue and contributions by lobbyists. The government consumes only the numeriare good. Therefore, the only effect of government policy on sector X comes through the distortions created by t and τ. In period 2 when governments may propose a trade agreement the Home government's problem is formalized as follows:

$$\max_{t,\tau} \{l_A(t,\tau), l_T(t,\tau) + TR(t,\tau)\}, \tag{4}$$

where $TR(t,\tau)$ represents tariff revenue for given t and τ. (Recall that in period 1 when there is autarky the government earns no income). The Foreign government's problem is analogous. The government chooses whichever policy receives the greatest financial support, autarky or the trade agreement. If $l_A(t,\tau) = l_T(t,\tau) + TR(t,\tau)$ then we will assume that a coin toss determines whether or not the trade agreement goes ahead.

Although the problem here is defined over NTBs and tariffs, τ and t, by assumption governments will make an agreement over only one instrument. That is, in the next section tariffs will be set to zero ($t = 1$) while we first consider an agreement over NTBs, and after that NTBs will be set to zero ($\tau = 1$) while we consider an agreement over tariffs. Until we consider each type of agreement in turn, we will continue to develop the model in terms of both t and τ together.

2.4 Lobby Groups

Based on the assumption that contributions to the government cannot be retrieved once they have been made, we can model the lobbying process as a first price all-pay auction (with complete information). Lobbies contest the government's decision over whether or not to adopt a trade agreement. The contribution that each lobby group makes to the government may be regarded as a (non-negative) sealed bid. The 'value' to L_A of maintaining autarky, denoted v_A, is given by

$$v_A = N_E \int_{\varphi}^{a_A} \left(\pi_A(a) - \pi_T(a)\right) dG(a), \tag{5}$$

where N_E is the number (mass) of entrepreneurs taking a draw from the productivity distribution, φ is the firm that is just indifferent between autarky and the trade agreement, a_A is the least efficient firm in autarky, and $\pi_A(a)$ and $\pi_T(a)$ are the respective profit levels of firm $a \in [\varphi, a_A]$ under autarky and the trade agreement.[2] Similarly, the 'value' to L_T of the trade agreement being adopted is

$$v_T = N_E \int_{0}^{\varphi} \left(\pi_T(a) - \pi_A(a)\right) dG(a). \tag{6}$$

On this basis, the value of each lobby group to swaying the government's decision in its favor depends on the underlying profits made under the respective policies of autarky and the trade agreement. The payoff to lobby L_A is given by

$$u_A(t,\tau) = \begin{cases} -l_A(t,\tau) & & l_A(t,\tau) < l_T(t,\tau) + TR(t,\tau) \quad \text{(A)} \\ \frac{v_A}{2} - l_A(t,\tau) & \text{if} & l_A(t,\tau) = l_T(t,\tau) + TR(t,\tau) \quad \text{(B)} \\ v_A - l_A(t,\tau) & & l_A(t,\tau) > l_T(t,\tau) + TR(t,\tau) \quad \text{(C)} \end{cases} . \tag{7}$$

[2] These terms will be defined explicitly when we characterize the equilibrium.

The payoff function for L_T is analogous. This specification of the payoff function makes clear that each lobby group contributes regardless of the outcome.

3 Trade Agreements with Heterogenous Firms

Now we calculate whether or not the governments of Home and Foreign would reach an agreement to move away from autarky and, if so, whether or not it would be characterized by free trade. We begin by setting out the framework for analyzing the outcome of the lobbying process with an examination of autarky equilibrium and then proceed to characterize a trade agreement.

3.1 Autarky

Use subscript-A to denote autarky. There exists a cutoff marginal cost a_A which represents the productivity of the firm that is indifferent, under autarky, between supplying the domestic market and exiting. Using equation (2), this is characterized by:

$$f_D = \frac{\mu}{\varepsilon}\left(\frac{a_A}{\alpha \mathcal{P}_A}\right)^{1-\varepsilon}. \tag{8}$$

There is free entry, so that an entrepreneur will pay to take a draw from the productivity distribution as long as the present value of average profits $\bar{\pi}$ is positive. We assume that firms fully discount profits in period 2, and so take into account only the expected profit in the current period when making their entry decision.[3] The free entry condition is

$$V(a_A)B_A - G(a_A)f_D = f_E \tag{9}$$

where

$$V(z) = \int_0^z a^{1-\varepsilon} dG(a) = \frac{k}{k-\varepsilon+1}\left(\frac{z}{a_U}\right)^k z^{1-\varepsilon}.$$

[3]This assumption does not affect the qualitative results, but affects the level of firms taking a draw, N_E.

Conditions (8) and (9) close the model and pin down a_A and the number (mass) of entrepreneurs taking a draw from the productivity distribution, N_E.

We restrict entry to period 1 only, and our only equilibrium conditions are the nonnegative profit conditions. This set of assumptions would only be restrictive if more entrepreneurs wanted to take a productivity draw under the trade agreement than under autarky. In that case we would need to account for these new entrants and how they might lobby.[4] Our approach of fixing the mass of entrants is not unlike others in the literature such as Do and Levchenko (2009), Eaton and Kortum (2005), Chaney (2008), and Arkolakis (2008). Since we have assumed firm productivity is distributed Pareto, we can find closed form solutions for these two variables:

$$\begin{aligned} a_A &= \left[\frac{\psi f_E}{f_D}\right]^{\frac{1}{k}} a_U, \\ N_E &= \frac{\alpha\mu}{k f_E}, \end{aligned}$$

where

$$\psi \equiv \frac{[k-(\varepsilon-1)]}{(\varepsilon-1)} > 0.$$

Note that all parameters are identical across countries, so it follows that $a_A = a_A^*$. Furthermore, in order to rule out corner solutions ($a_A \leq a_U$), we make the simplifying assumption that the shape parameter k is bounded from above as well: $k \leq (\varepsilon-1)[f_E + f_D]$. Finally, we are left with ex post aggregate per period industry profits in autarky equal to

$$\begin{aligned} \Pi_A &= N_E \int_0^{a_A} \pi_A(a)\, dG(a) \\ &= N_E \left[V(a_A)B_A - G(a_A) f_D\right] = \frac{\alpha\mu}{k}. \end{aligned} \tag{10}$$

[4] Since productivity is distributed Pareto, this is not an issue as it can be shown that

$$V(a_D)B_T - G(a_D)f_D + \frac{V(a_X)(t\tau)^{1-\varepsilon}B_T}{t} - G(a_X)f_X \leq f_E.$$

This solution for Π_A gives us a simple parametric expression that will be used to underpin the values to L_A and to L_T of swinging the government's decision in their respective favors.

3.2 Basics of a Trade Agreement

To develop the framework needed to analyze equilibrium under a trade agreement, begin by assuming that a trade agreement proposed by the two governments has been adopted. We can now derive the expressions we need to evaluate the payoffs entailed. Under the trade agreement, a given firm will continue to produce domestically if it makes nonnegative profits from doing so. There exists a cutoff productivity level a_D which represents the productivity of the firm that is indifferent under the trade agreement between supplying the domestic market and exiting:

$$\frac{\mu}{\varepsilon}\left(\frac{a_D}{\alpha \mathcal{P}_T}\right)^{1-\varepsilon} = f_D. \tag{11}$$

A firm will export if the profits from doing so are positive. The cutoff productivity level a_X for becoming an exporter is given by the following condition:

$$\frac{\mu}{\varepsilon t}\left(\frac{\tau t a_X}{\alpha \mathcal{P}_T^*}\right)^{1-\varepsilon} = \gamma f_D. \tag{12}$$

Since we are considering a symmetric trade agreement in which trade restrictions are at the same level in both countries, with aggregate expenditure on good X identical across both countries and equal to μ, we will have a symmetric equilibrium under an agreement and henceforth drop the $*$ superscripts. Thus, these two conditions (11) and (12) close the model under the trade agreement. Since firm productivity is distributed Pareto, we can find closed form solutions for the two cutoffs:

$$a_D = \left[\frac{\gamma^{\psi}\tau^k t^{\frac{k}{\alpha}}}{\gamma^{\psi}\tau^k t^{\frac{k}{\alpha}} + t}\frac{\psi f_E}{f_D}\right]^{\frac{1}{k}} a_U;$$

$$a_X = \left[\frac{1}{\gamma^{\psi}\tau^k t^{\frac{k}{\alpha}} + t}\frac{\psi f_E}{\gamma f_D}\right]^{\frac{1}{k}} a_U.$$

Using these cut-offs, we are able to calculate ex-post aggregate industry profits in an equilibrium under the trade agreement, Π_T:

$$\begin{aligned}\Pi_T &= N_E \int_0^{a_D} \pi_T(a)\, dG(a) \\ &= \frac{\alpha\mu}{k}\left[\frac{\gamma^{\psi}\tau^k t^{\frac{k}{\alpha}}+1}{\gamma^{\psi}\tau^k t^{\frac{k}{\alpha}}+t}\right]\end{aligned} \tag{13}$$

Note that since by (10) $\Pi_A = \alpha\mu/k$, we can write Π_T as

$$\Pi_T = \Pi_A\left[\frac{\gamma^{\psi}\tau^k t^{\frac{k}{\alpha}}+1}{\gamma^{\psi}\tau^k t^{\frac{k}{\alpha}}+t}\right]. \tag{14}$$

For the firm φ that is just indifferent between autarky and the trade agreement,

$$\Delta\pi(\varphi) = \pi_T(\varphi) - \pi_A(\varphi) = 0.$$

Explicitly, φ is determined as follows:

$$\varphi = \left(\frac{1}{\gamma}\left[(1+\tau^{1-\varepsilon}t^{-\varepsilon}) - F^{1-\varepsilon}\right]\right)^{\frac{1}{\varepsilon-1}} a_D = \lambda a_D \tag{15}$$

where

$$F \equiv \left[\frac{\gamma^{\psi}\tau^k t^{\frac{k}{\alpha}}}{\gamma^{\psi}\tau^k t^{\frac{k}{\alpha}}+t}\right]^{\frac{1}{k}},$$

and

$$\lambda = \left(\frac{1}{\gamma}\left[(1+\tau^{1-\varepsilon}t^{-\varepsilon}) - F^{1-\varepsilon}\right]\right)^{\frac{1}{\varepsilon-1}}.$$

We can use the expressions for aggregate profits and aggregate contributions that we have derived to determine how the outcome of the lobbying process is related to aggregate profits.

For given tariff revenue, the greater is v_A compared to v_T, the more likely autarky is to be the outcome. So it will be useful to be able to relate the difference in lobbying revenues to the difference in aggregate profits. Using (10), (14), (5), and (6),[5]

$$\begin{aligned} v_A - v_T &= N_E \int_0^{a_A} (\pi_A(a) - \pi_T(a))\, dG(a) \\ &= \Pi_A - \Pi_T. \end{aligned} \tag{16}$$

Therefore, the difference in the values to L_A and L_T is determined by the difference in aggregate profits under the two outcomes. This insight will be useful in determining the outcome of a proposed agreement.

3.3 Trade Agreement Over NTBs

In this setting we will consider an agreement over NTBs, τ, while setting $t = 1$. In this setting any agreement that the governments reach raises no revenue; $TR(1, \tau) = 0$. So the home government's problem (4) may be simplified to

$$\max_{\tau} \{l_A(1, \tau), l_T(1, \tau)\}.$$

It can be seen by inspection of (14) that if we set tariffs equal to zero in that expression (i.e. $t = 1$) and focus on τ as the only form of trade restriction, ex-post aggregate industry profits in autarky and the trade agreement are always equal for any proposed agreement, i.e. $\Pi_T = \Pi_A = \alpha\mu/k$ for any τ. Therefore, by (16), $v_A = v_T$.[6] Intuitively, this means that the aggregate gains from adopting the trade agreement exactly offset the aggregate losses; for any given proposed level of τ, the value to L_A of maintaining autarky is the same as the value to L_T of adopting the trade agreement. So we can say that in an agreement over NTBs the lobby groups have 'homogenous valuations'. Taking the agreement NTB level as given at τ, we can then analyze the interaction between the lobby groups as a game that takes the

[5] This derivation uses the fact that $\pi_T(a) = 0$ for $a \in [a_D, a_A]$ to rewrite equation (14) as $\Pi_T = N_E \int_0^{a_A} \pi_T(a)\, dG(a)$.

[6] It should be noted that this result is driven by the unique properties of both the Pareto distribution and the iceberg transport cost assumptions. However, the tensions described are present in general.

form of an all-pay auction. The Nash equilibrium outcome of a two-player all-pay auction with homogeneous valuations is known from Hillman and Riley (1989) and Baye, Kovenock and de Vries (1996). Their characterization of equilibrium is adapted to the present context in the following lemma.

Lemma 1. *In an agreement over NTBs, taking the agreed NTB τ as given, the Nash equilibrium is unique and symmetric. The value to lobby group L_A of remaining in autarky is the same as the value to L_T of moving to a proposed trade agreement for any τ between (and including) autarky and free trade. Therefore, the equilibrium outcome is one in which both L_A and L_T randomize their respective contributions l_A and l_T continuously on the interval $[0, v_A]$. Since the lobby groups randomize continuously, the highest contribution wins with probability one. Each lobby group earns an expected payoff of zero in equilibrium. The expected payoff to the government is $v_A = v_T$.*

This equilibrium characterization has a number of interesting features. First, there can be no equilibrium in pure strategies. To see this note from (7) that if one lobby group adopts the pure strategy of contributing their value then it is rational for the other lobby group to contribute nothing. Then the first lobby group would have an incentive to deviate to a contribution just above zero, giving rise to a further incentive for the lobby group contributing nothing to adopt a positive contribution level.

This logic also applies to mixed strategies where either lobby group tends to favor a contribution at a particular contribution level in the interval $[0, v_A]$ but does not contribute at that level with certainty. As a benchmark, consider the situation where a given lobby group randomizes continuously over the interval $[0, v_A]$; that is, it does not favor any particular contribution level within the interval. Then with a continuous probability distribution the probability that any particular contribution level is chosen is zero. On the other hand, if a particular contribution level $l_i \in [0, v_A]$ is favored by L_i then we say that L_i 'puts mass' at l_i; that is, it contributes l_i with positive probability. But if L_i puts mass at $l_i \in [0, v_A]$ then L_j, $j \neq i$ has an incentive to shift mass to a point above l_i such that it outbids L_i on average and increases its expected payoff at L_i's expense. Once one group puts mass at v_A, the tendency is for the other group to put mass at zero and so on. Consequently, both lobby groups must randomize continuously on the entire interval $[0, v_A]$, competing each others' expected profits from the contest to zero in the process. The reason the government's expected revenue is v_A is because each lobby group expects to contribute $v_A/2 = v_T/2$ on average. The reason expected payoffs to the lobby groups are zero is because each group expects to win half the time.

In one respect this outcome mirrors the outcome one would obtain in Bagwell and Staiger's framework, where there are two goods, countries are equally sized, and governments do not have distributional concerns over trade policy outcomes. Then free trade would be reached in a trade agreement. The key difference in the outcome we show above, however, is that this feature holds only in expectation, with the actual policy outcome going either way with equal probability and each lobby group having the potential to have positive or negative gains. This means that the outcome of a trade agreement may fail to get off the ground in any given period even though on efficiency grounds it should do so.

So far we have only said that the governments may propose to reduce NTBs from autarky in a symmetrical trade agreement. We can go further and predict the trade agreement that the governments will propose. To do so, first recall that both lobby groups randomize continuously over $[0, v_A]$. So in order to maximize the lobby groups' average contributions, each government needs to choose τ in such a way as to maximize the value of autarky $v_A(1, \tau)$, to L_A. For this, the government needs to choose τ in such a way that it makes the firms who would lobby for autarky as badly off as possible under the trade agreement:[7]

$$\max_{\tau} \; v_A(1, \tau) = \max_{\tau} \; N_E \left[\int_{\varphi}^{a_A} \pi_A(a) dG(a) - \int_{\varphi}^{a_X} \pi_X(a) dG(a) - \int_{\varphi}^{a_D} \pi_D(a) dG(a) \right]. \quad (17)$$

To characterize the problem, first note that when the agreement NTB level changes, the following also change: the indifferent firm between autarky and trade, φ; the least efficient

[7] Identically, the government could maximize v_T and arrive at the same revenue maximizing policy.

exporter, a_X; the profits of an exporting firm, $\pi_X(a)$; the least efficient purely domestic firm, a_D; and the domestic profits of firms that would prefer autarky to trade. By definition, it follows that $\pi_X(a_X) = 0$, $\pi_D(a_D) = 0$ and $\pi_A(\varphi) - \pi_T(\varphi) = 0$. Therefore, the tension we are concerned with is how the decrease in domestic profits that occur when the government reduces τ weigh against the increase in profits from exporting. In formal terms:

$$\frac{dv_A(1,\tau)}{d\tau} = -N_E \left[\int_{\varphi}^{a_X} \frac{d\pi_X(a)}{d\tau} dG(a) + \int_{\varphi}^{a_D} \frac{d\pi_D(a)}{d\tau} dG(a) \right]. \tag{18}$$

The first term in brackets captures the sum of the export profit $\pi_X(a)$ of the group of firms that export but would be better off in autarky, $a \in (\varphi, a_X)$. The second term in brackets captures the domestic profits of those same firms, plus the domestic profits of the firms that only serve the domestic market. As NTBs are lowered, $\pi_D(a)$ is reduced for all firms. But at the same time, as NTBs are lowered, this increases the return that a given firm would make from exports. Lowering NTBs also changes the set of firms that would find it profitable to export. The lower the NTB in the trade agreement, the larger the mass of firms that would prefer the trade agreement to autarky; φ increases. These are the tensions that the government faces in setting τ. The balance of these tensions is revealed in our first lemma.

Lemma 2. *The NTB policy that maximizes lobby revenue for the government is free trade; i.e.* $\tau = 1$.

The proof shows that

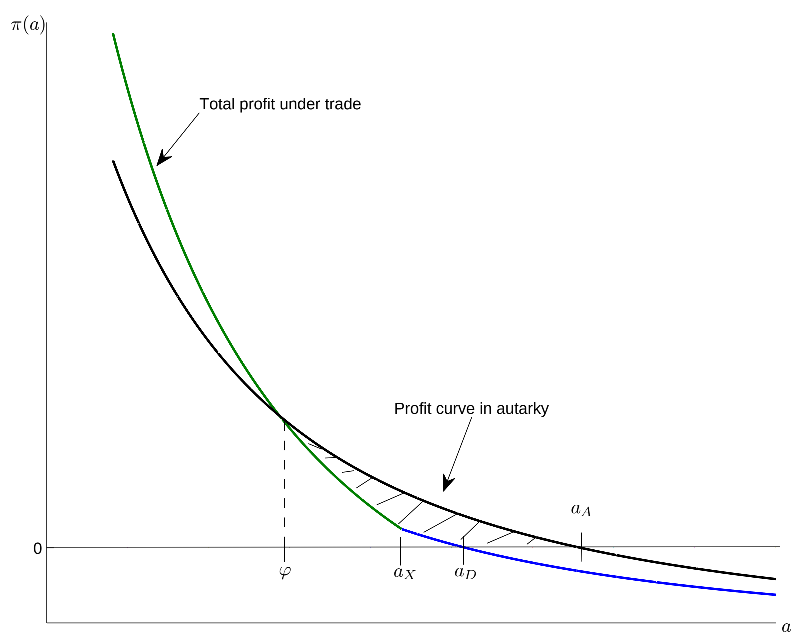
$$\frac{dv_A(1,\tau)}{d\tau} = -\frac{N_E k f_D}{\tau\psi} \left[\gamma^{\psi} \tau^{k+1-\varepsilon} - 1\right] \lambda^{k+1-\varepsilon} \left(\frac{a_D}{a_U}\right)^k \epsilon_{a_D}^{\tau} < 0 \quad \forall\ \tau \geq 1$$

where $\epsilon_{a_D}^{\tau}$ is the elasticity of the marginal purely domestic firm. This is negative because $\gamma > 1$, $\tau \geq 1$, and $k > \varepsilon - 1$. This means that the value of autarky to the firms in L_A monotonically increases as τ is reduced. So by lowering the tariff associated with the agreement each government increases the interval $[0, v_A]$ over which both firms continuously randomize their contributions, hence increasing the size of the (average) contribution that the government expects to receive from each of the lobby groups. By combining Lemmas 1 and 2 we can say that, under an agreement involving NTBs, the governments will propose an agreement that involves free trade: $\tau = 1$.

Proposition 1. *Assume an agreement over NTBs. The governments propose a free trade agreement, wherein $\tau = 1$. Both L_A and L_T randomize their respective contributions l_A and l_T continuously on the interval $[0, v_A]$ and the highest contribution wins with probability one. Each lobby group earns an expected payoff of zero in equilibrium. The expected payoff to the government is $v_A = v_T$.*

The key feature of equilibrium in Proposition 1, which does not feature in Bagwell and Staiger (1999), is that the trade agreement may not go ahead even though it would increase efficiency. If L_A makes a higher contribution than L_T then the agreement stalls and the economy remains in autarky.

Figure 1: Profits Under Two Regimes: Autarky and Trade



Further intuition about the characterization of equilibrium can be gleaned from the graphical illustration in Figure 1. As explained above, the government's problem can be formalized as seeking to maximize aggregate contributions by L_A. Our formalization shows that this is equivalent to maximizing the difference between profits under autarky and the trade agreement, or maximizing the shaded region in Figure 1. With a lower tariff domestic profits decrease, as illustrated by a downward shift of the blue line. However, at the same time profits from exporting increase thereby shifting the green line to the right. Under NTBs, this area is maximized under free trade, $\tau = 1$. The government receives this aggregate contribution from both lobby groups whichever group makes the higher contribution and whether or not the agreement goes ahead.

3.4 Trade Agreement Over Tariffs

Next we consider an agreement over tariffs, t, while setting $\tau = 1$. Tariff revenue is raised by any outcome other than autarky and free trade, and so equation (4) is the appropriate expression of the Home government's problem. For this form of agreement we need to calculate explicitly how much tariff revenue the government would receive for any tariff level t in a proposed agreement:

$$\begin{aligned} TR &= (t-1) \times \text{imports value} \\ &= (t-1) N_E \int_0^{a_X} p(a)x(a)dG(a) \\ &= \left[\frac{(t-1)}{\gamma^{\psi} t^{\frac{k}{\alpha}} + t} \right] \mu. \end{aligned} \tag{19}$$

To calculate aggregate profits of the firms that prefer the trade agreement we set $\tau = 1$ in (13) to obtain

$$\Pi_T = \frac{\alpha\mu}{k} \left[\frac{\gamma^{\psi} t^{\frac{k}{\alpha}} + 1}{\gamma^{\psi} t^{\frac{k}{\alpha}} + t} \right]. \tag{20}$$

We can now calculate, for any trade agreement involving positive but non-prohibitive tariffs $t > 1$, the difference between the aggregate profits of firms that prefer autarky and those that prefer the trade agreement. This is calculated by taking the difference between (14) (with $\tau = 1$) and (20). Interestingly, the aggregate profits of firms that gain from the trade agreement are less than the aggregate profits of those who would prefer autarky:

$$0 \leq \Pi_A - \Pi_T = \left[\frac{(t-1)}{\gamma^{\psi} t^{\frac{k}{\alpha}} + t} \right] \frac{\alpha \mu}{k} = \theta TR \leq TR,$$

where $\theta = \alpha/k < 1$. However, from this expression we can also say that the combined aggregate profits of firms that prefer the trade agreement plus tariff revenue is greater than the aggregate profits of firms that prefer autarky:

$$\Pi_T \leq \Pi_A \leq \Pi_T + TR. \tag{21}$$

Using equation (16), we have that

$$v_A - TR \leq v_T,$$

holding with equality only at autarky and free trade. So although the value of the agreement net of tariff revenue to L_T is less than the value of remaining in autarky is to L_A, the aggregate value of a trade agreement to L_T and the government is greater than the value of autarky to L_A. Since L_T only has to bring the aggregate value of contributions (inclusive of tariff revenue) above the level made by L_A, the presence of tariff revenue will tend to shift the outcome in L_T's favor. The presence of this asymmetry in the agreement over tariffs changes the basic characterization from an all-pay auction with homogenous valuations to one where valuations are heterogeneous. Once again, we can use Hillman and Riley (1989) and Baye, Kovenock and de Vries (1996) to characterize the equilibrium outcome of the trade agreement over tariffs.

We know from the prior literature that the outcome of an equilibrium with heterogeneous valuations differs from that with homogeneous valuations in that under heterogeneous valuations there may be an incentive to put mass at a certain value. We allow for this possibility in our present setting by introducing the following notation: $\alpha_i(z)$ denotes the mass that lobby $i \in \{A, T\}$ puts at the contribution level $l_i = z$;[8] $l_i \sim U[x, y]$ denotes that lobby i randomizes its contribution continuously over the uniform distribution $U[x, y]$.

We can now use similar logic as for a trade agreement with NTBs to determine the ranges over which the respective lobbies will contribute. First let us consider contributions by L_A. As with NTBs, there can be no incentive to bid more than v_A. But, differently from the NTB case, if $TR > 0$ then any $l_A < TR$ will lose for sure. In addition, as in the NTB case, there can be no incentive to contribute a positive amount with positive probability because L_T can win for sure with a slightly higher aggregate (i.e. inclusive of TR) contribution. We can therefore say, without loss of generality, that L_A may put mass at 0, i.e. may choose to set $\alpha_A(0) > 0$. In any case L_A will, with probability $1 - \alpha_A(0)$, randomize its contribution continuously over the interval $l_A \sim U[TR, v_A]$.

Next let us consider contributions by L_T. As with NTBs, there can be no incentive to bid less than 0. But, differently from the NTB case, any contribution above $v_A - TR$ would be wasteful given that the government receives $TR > 0$ as well if the agreement goes ahead. Therefore, without loss of generality, L_T may put mass at 0, setting $\alpha_T(0) > 0$, and with probability $1 - \alpha(0)$ randomize its contribution continuously over the interval $l_T \sim U[0, v_A - TR]$. The characterization of equilibrium then entails determination of equilibrium values for $\alpha_A(0)$ and $\alpha_T(0)$.

As the next step towards characterization, observe that the choices of equilibrium strategies by L_A and L_T, together with the corresponding expected payoffs, are completely characterized in Table 1.

Using the fact that L_A will contribute $l_A = 0$ with probability $\alpha_A(0)$ and $l_A \sim U[TR, v_A]$ with probability $1 - \alpha_A(0)$, while L_T will contribute $l_T = 0$ with probability $\alpha_T(0)$ and $l_T \sim U[0, v_A - TR]$ with probability $1 - \alpha_T(0)$, the payoffs in Table 1 can be translated into

[8] That is, $\alpha_i(z)$ denotes the probability with which L_i contributes the level $l_i = z$.

Table 1: Expected payoffs

Trade Lobby	Autarky Lobby: $l_A = 0$	Autarky Lobby: $l_A \sim U[TR, v_A]$
$l_T = 0$	$A = 0$ $T = v_T$ $G = TR$	$A = v_A - \left(\frac{v_A+TR}{2}\right)$ $T = 0$ $G = \left(\frac{v_A+TR}{2}\right)$
$l_T \sim U[0, (v_A - TR)]$	$A = 0$ $T = v_T - \left(\frac{v_A-TR}{2}\right)$ $G = TR + \left(\frac{v_A-TR}{2}\right)$	$A = \frac{v_A}{2} - \left(\frac{v_A+TR}{2}\right)$ $T = \frac{v_T}{2} - \left(\frac{v_A-TR}{2}\right)$ $G = \frac{TR}{2} + v_A$

expected payoff functions for the respective lobbies and for the government. Writing the expected payoff for lobby L_A as $\mathbb{E}[u_A]$, we can write L_A's expected payoff function as:

$$\begin{aligned}\mathbb{E}[u_A] &= 0\alpha_A(0) + \left[\left(\frac{v_A - TR}{2}\right)\alpha_T(0) - \left(\frac{TR}{2}\right)(1-\alpha_T(0))\right](1-\alpha_A(0)) \\ &= \left[\frac{v_A}{2}\alpha_T(0) - \frac{TR}{2}\right](1-\alpha_A(0)). \end{aligned} \tag{22}$$

For L_T's expected payoff, we have:

$$\begin{aligned}\mathbb{E}[u_T] &= \left[v_T\alpha_A(0) + 0(1-\alpha_A(0))\right]\alpha_T(0) + \\ &\quad \left[\left(v_T - \left(\frac{v_A - TR}{2}\right)\right)\alpha_A(0) + \left(\frac{v_T}{2} - \left(\frac{v_A - TR}{2}\right)\right)(1-\alpha_A(0))\right](1-\alpha_T(0)) \\ &= v_T\alpha_A(0)\alpha_T(0) + \left[\frac{v_T}{2}\alpha_A(0) + \frac{v_T}{2} - \left(\frac{v_A - TR}{2}\right)\right](1-\alpha_T(0)) \end{aligned} \tag{23}$$

Finally, for the government's expected payoff we have:

$$\begin{aligned}\mathbb{E}[u_G] &= \alpha_T(0)\left[\alpha_A(0)\left(\frac{TR - v_A}{2}\right) + \left(\frac{v_A + TR}{2}\right)\right] + (1-\alpha_T(0))\left[\left[\frac{2v_A + TR}{2}\right] - \frac{\alpha_A(0)v_A}{2}\right] \\ &= \alpha_T(0)\left(\frac{\alpha_A(0)TR - v_A}{2}\right) + \left[\frac{2v_A + TR}{2}\right] - \frac{\alpha_A(0)v_A}{2} \end{aligned} \tag{24}$$

We now use these payoff functions in the characterization of equilibrium.

3.4.1 Characterizing Equilibrium of an Agreement over Tariffs

Following the same approach as for NTBs, we begin by taking the agreement tariff level as given at t and analyze the interaction between the lobby groups as a game that takes the form of an all-pay auction. First consider the situation where $t = 1$ so that the outcome corresponds to free trade, wherein $TR = 0$. In that case, $v_A = v_T$ and the game is one of homogeneous valuations as in the NTB case. This is analyzed exactly as in the NTB case, so that in equilibrium the expected payoff to L_A and L_T are both equal to zero and the payoff to the government equal to $v_A = v_T$, which is the same as characterized in Lemma 1.

Next consider the situation where $t > 1$ and, by assumption for now, both lobbies make positive contributions. We will characterize below the circumstances under which the lobbies make a contribution. As before, the equilibrium involves both lobbies playing a mixed strategy. As a result of the advantage given to L_T by the fact that $v_T > v_A - TA$, L_T can always outbid L_A. As a result, L_A can do no better in equilibrium than if it did not contribute, i.e. $\mathbb{E}[u_A] = 0$. But if L_A did not contribute at all then L_T could win with a very small contribution, inviting L_A to make a slightly larger contribution itself. So both lobby groups contribute at positive levels in expectation in equilibrium and, following the same logic as for the NTB game, contribution at a positive level must involve continuous randomization of contribution levels for both lobby groups. Moreover, L_A cannot make a lower return than 0 in equilibrium because it could obtain a higher payoff from playing a pure strategy $l_A = 0$. And it cannot make a higher return than 0 because this would mean that L_T would make a return lower than $v_T - (v_A - TR)$, inducing L_T to secure a return of $v_T - (v_A - TR) - \epsilon$ by playing a pure strategy $l_T = v_A - TR + \epsilon$. So in equilibrium L_A chooses $\alpha_A(0)$ such that $\mathbb{E}[u_A] = 0$.

Staying with the case where $t > 1$ and both lobbies make positive contributions, and turning to lobby L_T, it can always win for sure by playing a pure strategy $v_T = v_A - TR + \epsilon$, obtaining a payoff of $v_T - (v_A - TR) - \epsilon$ for sure. But it can obtain a higher expected payoff $v_T - (v_A - TR)$ by randomizing continuously over the interval $[0, v_A - TR]$ and making a zero contribution with probability $(v_T - (v_A - TR)) / (v_T)$. L_T cannot obtain a higher expected payoff than $v_T - (v_A - TR)$, because this would mean that L_A would make a lower expected return than 0, inducing L_A simply to refrain from making a contribution. So in equilibrium L_T chooses $\alpha_T(0)$ such that $\mathbb{E}[u_T] = v_T - (v_A - TR)$.

The foregoing observations about equilibrium enable us to solve for $\alpha_A(0)$ and $\alpha_T(0)$. First, we can use the fact that in equilibrium $\mathbb{E}[u_A] = 0$ in (22) to obtain $\alpha_T(0) = \frac{TR}{v_A}$. In the same way, using the fact that in equilibrium $\mathbb{E}[u_T] = v_T - (v_A - TR)$, plus the fact that $\alpha_T(0) = \frac{TR}{v_A}$, to obtain $\alpha_A(0) = 1 - \frac{(v_A - TR)}{v_T}$.

We can now characterize the circumstances under which the lobbies will make a contribution in equilibrium. We can see from the solutions just obtained that at $TR = v_A$ we have that $\alpha_A(0) = \alpha_T(0) = 1$, implying that neither L_A nor L_T will make a positive contribution in equilibrium at this level of TR or above. The reason is that the high level of TR gives v_T such an advantage that it does not have to contribute towards the agreement to persuade the government to choose it, and there is nothing L_A can do to stop the agreement from going ahead. As we will see, $TR \geq v_A$ can arise for some tariff levels. Therefore, we must allow for the possibility that neither L_A nor L_T make positive contributions given $TR \geq v_A$.

In order to fully characterize the equilibrium outcome, we need to establish the relationship between TR and v_A. To begin with, we can show that

$$v_A = (1 + \chi)\theta TR, \tag{25}$$

$$v_T = \chi\theta TR \tag{26}$$

where

$$\chi = \frac{\lambda^k \gamma^{\frac{k}{\varepsilon - 1}} t^{\frac{k}{\alpha}}}{(t-1)}.$$

So

$$\frac{v_A}{v_T} = 1 + \frac{1}{\chi},$$

and $\frac{1}{\chi}$ is a measure of the wedge between v_A and v_T. This wedge primarily depends on the marginal exporting firm indifferent between the trade agreement and autarky ($\varphi = \lambda a_D$), the difference between the fixed cost of entry to a foreign and domestic market ($f_X = \gamma f_D$), and the proposed tariff rate (t). Holding all else equal, as λ increases, the proportion of active firms in the trade equilibrium (a_D) that prefer trade ($a < \varphi$) increases. This lowers the gap between v_A and v_T. Similarly, as γ increases, we see from (15) that φ decreases, but this is countered by the direct effect of $\gamma^{\frac{k}{\varepsilon-1}}$ which increases χ.[9] In other words, though it is more costly to become an exporter, once a firm becomes one there are greater ex post profits, thereby increasing v_T and lowering the gap between v_A and v_T. Finally, as we approach the free trade equilibrium ($t \to 1$), then $\frac{1}{\chi} \to 0$ and we are left with our result that $v_A = v_T$. Moreover, as we approach autarky ($t \to \infty$), then tariff revenue goes to zero again leaving us with our result that $v_A = v_T$.

Next, it will be useful to establish the bounds on the term $F \equiv \left[\frac{\gamma^{\psi} t^{\frac{k}{\alpha}}}{\gamma^{\psi} t^{\frac{k}{\alpha}} + t}\right]^{\frac{1}{k}}$.

Lemma 3. *F is bounded between $\left(\frac{1}{2}\right)^{\frac{1}{k}}$ and 1.*

The proof simply shows that F has well defined endpoints at $\left(\frac{1}{2}\right)^{\frac{1}{k}}$ and ∞ and that it varies monotonically between these endpoints.

Before proceeding to the next lemma, it will be helpful to have some additional notation. Denote by $\bar{t}$ the tariff level at which $v_A = TR$ and by $\hat{t}$ the tariff level at which TR is maximized. Armed with the foregoing results and notation, we can now state the next lemma.

Lemma 4. *$\hat{t} > \bar{t}$.*

Lemma 4 shows that tariff revenue is still increasing at $v_A = TR$, where both lobbies cease to make contributions. Based on this specification of the game, we can now provide a basic

[9] Note that
$$\frac{d\chi}{d\gamma} = \frac{\psi t (F\lambda)^{k+1-\varepsilon}}{(t-1)\gamma} > 0.$$

characterization of equilibrium of the lobbying game, taking as given a non-prohibitive tariff level t.

Lemma 5. *In an agreement over tariffs, there are three possibilities:*
(i) if $t = 1$ then the equilibrium is symmetric and exactly as characterized in Lemma 1.
(ii) If $1 < t \leq \bar{t}$ then $TR > 0$ and the Nash equilibrium is unique and asymmetric. Then in equilibrium, L_T contributes $l_T = 0$ with probability $\alpha_T(0) = TR/v_A$, and randomizes its contribution continuously over $l_T \sim U(0, v_A - TR]$ with probability $1 - \alpha_T(0)$. In equilibrium, L_A contributes $l_A = 0$ with probability $\alpha_A(0) = (v_T - (v_A - TR))/(v_T)$, and randomizes its contribution continuously on $l_T \sim U[TR, v_A]$ with probability $1 - \alpha_T(0)$. L_T's expected payoff in equilibrium is $\mathbb{E}[u_T] = v_T - (v_A - TR)$, while L_A's expected payoff in equilibrium is $\mathbb{E}[u_A] = 0$, and the government's expected payoff is $\mathbb{E}[u_G] = \alpha_A(0)\left(\frac{TR^2 - v_A^2}{2v_A}\right) + v_A$.
(iii) If $t > \bar{t}$, both lobby groups contribute zero and the only payoff to the government is through tariff revenue. The expected payoff to the government inclusive of lobby revenue and tariff revenue is $\mathbb{E}[u_G] = TR$.

Lemma 5 shows that the determination of government expected revenues through strategic interaction between the lobbies will be different depending on whether $t = 1$, $1 < t \leq \bar{t}$, or $t > \bar{t}$. The government's objective is therefore to choose whichever tariff level from among these three brings about the highest expected revenues.

Differentiating the government's expected payoff yields the following:

$$\frac{d\mathbb{E}[u_G]}{dt} = \begin{cases} \left(\frac{TR^2 - v_A^2}{2v_A}\right)\frac{d\alpha_A(0)}{dt} + \frac{\alpha_A(0)TR}{v_A}\frac{dTR}{dt} + \left(1 - \frac{\alpha_A(0)(TR^2 + v_A^2)}{2v_A^2}\right)\frac{dv_A}{dt} & \text{if} \quad t \in [1, \bar{t}] \\ \frac{dTR}{dt} & \text{if} \quad \bar{t} < t \end{cases} \tag{27}$$

where

$$\begin{aligned} \frac{d\alpha_A(0)}{dt} &= \frac{1}{v_T}\left[\frac{dTR}{dt} - \frac{dv_A}{dt} - \left(\frac{TR - v_A}{v_T}\right)\frac{dv_T}{dt}\right] \\ &= \frac{1}{v_T}\left[(1-\theta)\frac{dTR}{dt} - \alpha_A(0)\frac{dv_T}{dt}\right] \end{aligned}$$

Thus the government must weigh how the tariff affects both its potential tariff revenue and the contributions of both lobbies. Although this is a complicated problem, the solution will actually involve one of two possible outcomes: the government chooses either free trade or a tariff that maximizes tariff revenue. To show this, we break the problem down into parts. First we introduce an assumption to ensure that the government's first order condition (27) is negative at free trade: $k \geq \varepsilon - \alpha$[10]

Lemma 6. *If $k \geq \varepsilon - \alpha$ then the government's expected payoff is decreasing at free trade.*

By Lemma 4, we know that $\mathbb{E}[u_G]$ is increasing at $\bar{t}$ and there is a local maximum at $\hat{t} = \arg\max TR$.[11] Therefore the government will choose between free trade and $\hat{t}$. We characterize this choice and the equilibrium under tariffs in the following proposition.

Proposition 2. *Assume an agreement over tariffs. The governments propose a free trade agreement if*

$$v_A(1,1) > \max TR$$
$$\Leftrightarrow \left[2 - \left[\frac{1+\gamma^{\psi}}{\gamma^{\psi}}\right]^{\frac{\varepsilon-1}{k}}\right]^{\frac{k}{\varepsilon-1}} > \left[\frac{k(1+\gamma^{\psi})}{\left((k-\alpha)\gamma^{\psi}\hat{t}^{\frac{k}{\alpha}}\right)}\right]$$

and propose tariffs to maximize TR otherwise. In the case where free trade is proposed, both L_A and L_T randomize their respective contributions l_A and l_T continuously on the interval $[0, v_A]$ and the highest contribution wins with probability one. Each lobby group earns an expected payoff of zero in equilibrium. The expected payoff to the government is $v_A = v_T$. In

[10]This additional assumption is for brevity and is in line with the empirical literature (See for example Del Gatto et al. 2008 and Crozet and Koenig 2010). Allowing for the entire parameter space (i.e. $k > \varepsilon - 1$) would result in ambiguity:

$$\left.\frac{dE[U_G]}{dt}\right|_{t\to 1,\ k=\varepsilon-1} = \left[\frac{2-4\alpha-\alpha^2}{2(1-\alpha)}\right]\frac{\mu}{2k} \leq 0 \Leftrightarrow \alpha \geq \sqrt{6}-2 \approx 0.45.$$

[11]In the appendix, we confirm that there is unique extremum for the parameter space $t \in [1, \bar{t}]$ that is a minimum.

the case where $\hat{t}$ is proposed, both L_A and L_T contribute zero and the payoff to the government is $TR(\hat{t})$ with certainty.

This result shows that, out of the three possible types of outcome characiterized in Lemma 5, only one of two will arise. Either the government will choose free trade or it will choose the revenue maximizing tariff. The forces brought to light in the agreement over NTBs may dominate in equilibrium, in that proposing free trade induces the lobbies to maximize their contributions and these are greater than the maximum possible tariff revenue. Or the conventional forces of tariff revenue maximization dominate, in which case contributions by the lobbies could not sway the government decision. We will now undertake comaprative statics to explore the parameter values under which each respective possible equilibrium outcome is more likely to prevail.

3.4.2 Comparative Statics

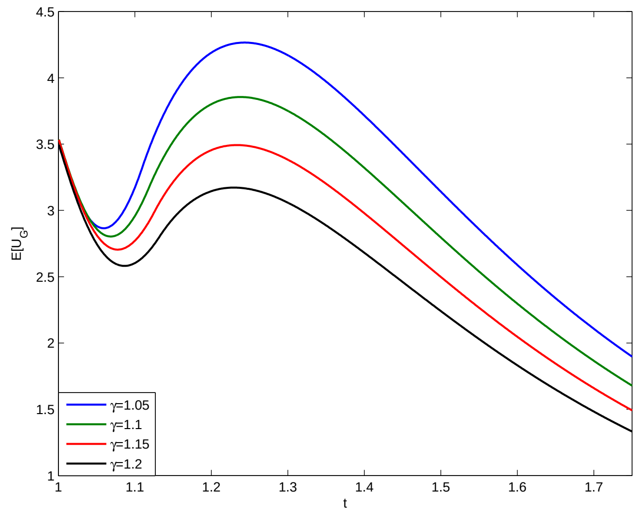
We first present a comparative statics result for the parameter γ:

Lemma 7. *The government is more likely to choose free trade the greater the fixed cost to exporting is relative to the fixed cost of serving the domestic market, i.e. the greater is γ.*

Intuitively, as the fixed cost of exporting, γ, increases there are less exporters which means there is a smaller tax base on which to generate tariff revenue. However, matters are less straight forward in consideration of contributions made by L_A. On the one hand there is less foreign competition as γ is increased, which implies each firm that favors autarky is willing to contribute less to see that it is maintained. But on the other hand, there are more firms that prefer autarky (i.e. $d\varphi/d\gamma < 0$ and $da_D/d\gamma > 0$) which means more firms contribute in support of autarky. The proof shows that the negative effect of an increase in γ on tariff revenue will always outweigh the potentially negative effect on lobby contributions, making free trade more likely to arise in equilibrium.

The model properties derived in Lemma 7 are illustrated in Figure 2. For values of $k = 2.5$ and $\alpha = 0.4$, Figure 2 shows that for $\gamma = 1.05$ or $\gamma = 1.1$, the expected government payoff is maximized at the revenue maximizing tariff. On the other hand, when γ increases to 1.15 or 1.2, the expected government payoff is maximized at free trade.

Figure 2: Expected Government payoff when $k = 2.5$ and $\alpha = 0.4$



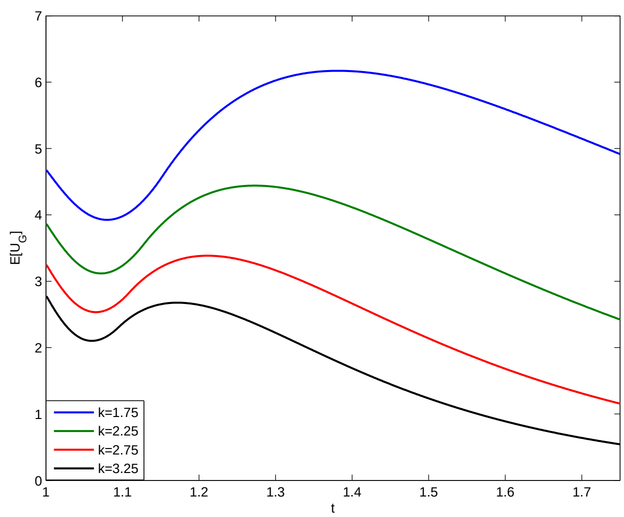
These properties are somewhat surprising. They suggest that when the fixed costs of exporting are relatively high then a trade agreement, if it is struck, is likely to go all the way to free trade, whereas when fixed costs of exporting are relatively low a trade agreement may involve positive tariffs. One might have expected the opposite; that any trade agreement is likely to involve greater liberalization the lower are the fixed costs of exporting. Another interesting feature of this equilibrium is that in the parameter range where a trade agreement involves free trade, the agreement is not guaranteed since L_A may make the higher contribution in equilibrium. However, in the parameter range where a trade agreement involves positive tariffs, the trade agreement will go ahead for sure.

We next turn our attention to comparative statics of the shape parameter k. It can easily be shown that tariff revenue is decreasing in k:

$$\frac{k}{TR}\frac{dTR}{dk} = -\left[\frac{k\left[\varepsilon\log(t)+\log(\gamma)\right]\gamma^{\psi}t^{\frac{k}{\alpha}}}{(\varepsilon-1)\left(\gamma^{\psi}t^{\frac{k}{\alpha}}+t\right)}\right] < 0.$$

As k increases, the distribution becomes more skewed towards less productive firms which decreases the ex ante expected profit and thus lowers the number of firms taking a draw. This in turn reduces the number of large exporting firms, lowering the amount of tariff revenue generated from imports. In terms of lobby contributions ($v_A(1,1)$), when k is sufficiently large relative to $(\varepsilon - 1)$, we have that $dv_A(1,1)/dk < 0$. The intuition here is that as k increases there are relatively more firms at the lower end of the productivity distribution, which lowers aggregate profit (making the black line in Figure 1 flatter). Since the "gains" from autarky are lower, the amount firms are willing to contribute to stay in autarky are correspondingly lower as well. In Figure 3 we provide an example in which, for a sufficiently high value of k, the decrease in tariff revenue dominates the change in lobby contributions and each government will choose free trade as the proposed policy.

Figure 3: Expected Government payoff when $\gamma = 1.1$ and $\alpha = 0.4$

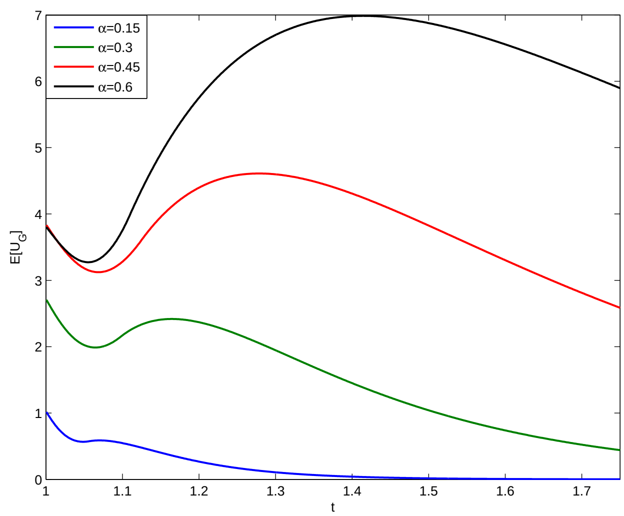


Finally we investigate the effect α (and consequently the elasticity of substitution ε) has on the equilibrium. The effect on tariff revenue is straightforward and positive:

$$\frac{\alpha}{TR}\frac{dTR}{d\alpha} = \left[\frac{k\left[\log(t) + \log(\gamma)\right]\gamma^{\psi} t^{\frac{k}{\alpha}}}{\alpha\left(\gamma^{\psi} t^{\frac{k}{\alpha}} + t\right)^2} \right] > 0.$$

As α increases, so does the elasticity of substitution. This drives down the equilibrium price for each variety but increases the amount sold. Therefore, even though the amount of tax collected per unit is lower (since the price decreases) the number of units increases resulting in higher tariff revenue. Similarly, as long as k is sufficiently large relative to $(\varepsilon - 1)$, the lobby contributions $(v_A(1,1))$ also increases with α. The intuition for this can be gleaned from the fact that aggregate profits in autarky increase (see Figure 1 and note that the black line becomes steeper and shifts up). This means that the profit loss from going to free trade is now higher despite being spread over fewer firms. In Figure 4 we provide an example in which tariff revenue increases more than $v_A(1,1)$ in response to an increase in α. For a sufficiently high α, the government will choose a tariff policy that maximizes tariff revenue $(\hat{t})$ in equilibrium and lobby contributions will be equal to zero.

Figure 4: Expected Government payoff when $\gamma = 1.1$ and $k = 2.5$



We end our discussion with a summary of how the model parameters affect the tariff that maximizes tariff revenue, should $\hat{t}$ be the chosen policy.

Proposition 3. *The tariff that maximizes tariff revenue is:*

1. Decreasing in γ;

2. Decreasing in the shape parameter k;

3. Increasing in α and the elasticity of substitution ε.

4 Conclusion

In this paper we have developed a model of trade agreements where the tension is between larger firms who have an interest in trade liberalization and smaller firms who would prefer autarky. This contrasts with the standard framework wherein the conflict of interest over a trade agreement is between an export sector and an import competing sector. We also considered lobbying over trade agreement formation in a new way by modelling it as an all-pay auction instead of the standard framework's menu-auction approach.

Our model of a trade agreement over NTBs formed a benchmark. In that setting, if an agreement came about then it involved free trade whereby all the surplus generated by the agreement was transferred to the government. This result is surprisingly similar to Example 2 of Grossman and Helpman (1994), which might be taken as a benchmark in that the strengths of the exporter and import-competing lobbies are equally opposed, derived in the standard framework. The key difference is that in our all-pay auction framework the agreement may stall if the autarky lobby makes the bigger contribution, helping to explain why trade liberalization through trade agreement formation has not been even more rapid.

Our examination of agreements over tariffs brought tariff revenues into play. This tilted the outcome of the lobbying process in favor of the pro-trade lobby. In spite of the additional complexities introduced, the equilibrium outcome was surprisingly straight forward. Either the forces characterized by the agreement over the NTBs would dominate and the trade agreement, if it came about, would involve free trade. Or the agreement would be determined entirely by the net tariff revenues raised and there would be no revenues from lobbying in equilibrium. The surprise here was that relatively high costs of exporting were more likely to give rise to an agreement that resulted in free trade, if it went ahead, whereas relatively low costs of exporting would be more likely to give rise to a more conventional looking agreement involving positive tariffs and tariff revenue.

What are the future directions for research? It would be useful in future to consider the role of consumers in our framework. In a qualitative sense, as we argued in the introduction, the role of consumers is likely to be fairly clear cut. Adding a term in the government's payoff function to capture consumer preferences would tilt the outcome in favor of free trade, while the formal approach to analyzing equilibrium would be the same as in the model of an agreement over tariffs. Incorporating consumers into the framework would be useful in terms of trying to get a sense of how much the competing forces matter either as part of an econometric or calibration exercise. It might then be possible to predict based on the underlying characteristics of countries when they would be more likely to reach free trade in a trade agreement, and test these predictions against the data. It might also be possible to gain a sense of which matters more in trade agreement formation: the conflict of interest over trade policy between sectors or within sectors.

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A Appendix

A.1 Proof of Propositions

Lemma 2. *The NTB policy that maximizes lobby revenue for the government is free trade; i.e.* $\tau = 1$.

Proof. First note that

$$\frac{d\pi_X(a)}{d\tau} = \frac{(\varepsilon-1)\gamma f_D}{\tau}\left(\frac{a}{a_X}\right)^{1-\varepsilon}\epsilon^{\tau}_{a_X} = \frac{(1-\varepsilon)\gamma f_D}{\tau}\left(\frac{a}{a_X}\right)^{1-\varepsilon}\left(\frac{\gamma^{\psi}\tau^k}{\gamma^{\psi}\tau^k+1}\right)$$
$$\frac{d\pi_D(a)}{d\tau} = \frac{(\varepsilon-1)f_D}{\tau}\left(\frac{a}{a_D}\right)^{1-\varepsilon}\epsilon^{\tau}_{a_D} = \frac{(\varepsilon-1)f_D}{\tau}\left(\frac{a}{a_D}\right)^{1-\varepsilon}\left(\frac{1}{\gamma^{\psi}\tau^k+1}\right),$$

where ϵ refers to elasticity. So the first derivative, equation (18), reduces to:

$$\begin{aligned}\frac{dv_A(1,\tau)}{d\tau} &= \frac{-N_E(\varepsilon-1)f_D}{\tau}\left[\int_{\varphi}^{a_X}\left(\frac{a}{a_X}\right)^{1-\varepsilon}\gamma\epsilon^{\tau}_{a_X}dG(a) + \int_{\varphi}^{a_D}\left(\frac{a}{a_D}\right)^{1-\varepsilon}\epsilon^{\tau}_{a_D}dG(a)\right]\\ &= -\frac{N_E(\varepsilon-1)f_D}{\tau}\left[\frac{\gamma\epsilon^{\tau}_{a_X}}{a_X^{1-\varepsilon}}\left[V(a_X)-V(\varphi)\right] + \frac{\epsilon^{\tau}_{a_D}}{a_D^{1-\varepsilon}}\left[V(a_D)-V(\varphi)\right]\right]\\ &= -\frac{N_E k f_D}{\tau\psi}\left[\gamma^{\psi}\tau^{k+1-\varepsilon}-1\right]\lambda^{k+1-\varepsilon}\left(\frac{a_D}{a_U}\right)^k\epsilon^{\tau}_{a_D} < 0 \ \ \forall\ \tau \geq 1.\end{aligned}$$

This is negative because $\gamma > 1$, $\tau \geq 1$, and $k > \varepsilon - 1$. Therefore, the government will choose free trade. □

Lemma 3. *F is bound between* $\left(\frac{1}{2}\right)^{\frac{1}{k}}$ *and 1.*

Proof. Observe the following limits

$$\lim_{\gamma\to 1} F = \left[\frac{t^{\frac{k}{\alpha}}}{t^{\frac{k}{\alpha}}+t}\right]^{\frac{1}{k}}, \qquad \lim_{\gamma\to\infty} F = 1,$$

$$\lim_{t\to 1} F = \left[\frac{\gamma^{\psi}}{\gamma^{\psi}+1}\right]^{\frac{1}{k}}, \qquad \lim_{t\to\infty} F = 1.$$

Since $\gamma > 1$, $t > 1$, and $\psi > 0$ the lower bound is $\left(\frac{1}{2}\right)^{\frac{1}{k}}$. □

Lemma 4. $\hat{t} > \bar{t}$.

Proof. Recall that $v_A = (1+\chi)\theta TR$. Define $\bar{t}$ such that $v_A = TR$, then it must be the case that

$$\chi(\bar{t}) = \frac{(k-\alpha)}{\alpha}.$$

Consider $\tilde{t} = \frac{k}{k-\alpha}$, then

$$\frac{dTR(\tilde{t})}{dt} = \frac{\mu}{\left(\gamma^{\psi}\tilde{t}^{\frac{k}{\alpha}} + \tilde{t}\right)^2} > 0.$$

Therefore $\tilde{t} < \hat{t}$ where $\hat{t} = \arg\max TR$. Next, we need to show that $\bar{t} < \tilde{t}$; i.e. $v_A(\tilde{t}) < v_A(\bar{t})$. To do this, first note that v_A evaluated at $\tilde{t}$ is decreasing;

$$\frac{dv_A(\tilde{t})}{dt} = \frac{\mu\tilde{t}^{\varepsilon-1}}{\left(\gamma^{\psi}\tilde{t}^{\frac{k}{\alpha}} + \tilde{t}\right)^2}\left(1-\gamma^{\psi}\tilde{t}^{\varepsilon\psi}\right) < 0.$$

Thus it is sufficient to show that $\chi(\tilde{t}) < \chi(\bar{t})$

$$\begin{aligned}
\chi(\tilde{t}) = \left(\frac{k-\alpha}{\alpha}\right)\lambda^k\gamma^{\psi+1}\tilde{t}^{\frac{k}{\alpha}} &< \left(\frac{(k-\alpha)}{\alpha}\right) = \chi(\bar{t}) \\
\left[(1+\tilde{t}^{-\varepsilon}) - F^{1-\varepsilon}\right]\tilde{t}^{\varepsilon} &< 1 \\
1 &< F^{1-\varepsilon}.
\end{aligned}$$

Since $\left(\frac{1}{2}\right)^{\frac{1}{k}} < F < 1$ by Lemma 3 and $\varepsilon > 1$, it follows that $1 < F^{1-\varepsilon} < 2^{\frac{(\varepsilon-1)}{k}}$. Therefore, $\bar{t} < \tilde{t} < \hat{t}$. □

Lemma 6. *The government's expected payoff is decreasing at free trade if* $k \geq \varepsilon - \alpha$.

Proof. Evaluating the first derivative at free trade yields

$$\left.\frac{dE[U_G]}{dt}\right|_{t\to 1} = \left.\frac{dv_A}{dt}\right|_{t\to 1} - \frac{(1-\theta)}{2}\left.\frac{dTR}{dt}\right|_{t\to 1} \tag{28}$$

which equals

$$\left.\frac{dE[U_G]}{dt}\right|_{t\to 1} = \left[\gamma^{\psi}\lambda^{k-(\varepsilon-1)}\left(1-\frac{2(\alpha+k\gamma^{\psi})}{(\gamma^{\psi}+1)}\right)-\frac{(k-3\alpha)}{2}\right]\frac{\mu}{k(\gamma^{\psi}+1)}.$$

It can easily be shown that this is decreasing in k. Therefore, we evaluate this at $k=\varepsilon-\alpha$ and get

$$\left.\frac{dE[U_G]}{dt}\right|_{t\to 1,\ k=\varepsilon-\alpha} = \left[\gamma^{\psi}\lambda^{\frac{1}{\varepsilon}}\left(\frac{1-\gamma^{\psi}-2\alpha(1+\varepsilon\alpha\gamma^{\psi})}{(\gamma^{\psi}+1)}\right)-\frac{(1-2\alpha)^2}{2}\right]\frac{\mu}{k(\gamma^{\psi}+1)}<0.$$

□

Proposition 2. *Assume an agreement over tariffs. The governments propose a free trade agreement if*

$$v_A(1,1) > \max TR$$

$$\Leftrightarrow \left[2-\left[\frac{1+\gamma^{\psi}}{\gamma^{\psi}}\right]^{\frac{\varepsilon-1}{k}}\right]^{\frac{k}{\varepsilon-1}} > \left[\frac{k(1+\gamma^{\psi})}{\left((k-\alpha)\gamma^{\psi}\hat{t}^{\frac{k}{\alpha}}\right)}\right]$$

and propose tariffs to maximize TR otherwise. In the case where free trade is proposed, both L_A and L_T randomize their respective contributions l_A and l_T continuously on the interval $[0, v_A]$ and the highest contribution wins with probability one. Each lobby group earns an expected payoff of zero in equilibrium. The expected payoff to the government is $v_A = v_T$. In the case where $\hat{t}$ is proposed, both L_A and L_T contribute zero and the payoff to the government is $TR(\hat{t})$ with certainty.

Proof. We know from Lemma 4 that $\hat{t} > \bar{t}$; tariff revenue is maximized at $\hat{t}$ over the range $t \in [\bar{t}, \hat{t}]$). So the government will always choose $\hat{t}$ in preference to any tariff $t \in [\bar{t}, \hat{t})$. We also know from Lemma 6 that the government's expected payoff is decreasing in t at free trade. For the government to choose only between free trade and $\hat{t}$, it is sufficient to show that the government's expected payoff is convex on the interval $[1, \bar{t}]$ and that any extremum is a minimum.

To begin, write the government's objective function in terms of χ and tariff revenue, TR:

$$\begin{aligned} E[u_G] &= \left[\frac{\theta^2(1+\chi)^2\left[\theta(2\chi+1)-1\right]+(1-\theta)}{2\theta^2\chi(1+\chi)}\right]TR \\ &= \Upsilon TR \end{aligned}$$

We will now show that this function is convex over the range $t \in (1, \bar{t})$ by showing that Υ is monotonically decreasing on $t \in (1, \bar{t})$ while TR is monotonically increasing over the same range. We can easily establish from (19) that TR is monotonically increasing over the range $t \in (1, \bar{t})$. To show that Υ is monotonically decreasing on $t \in (1, \bar{t})$, first observe that χ is monotonically decreasing in t for $t \in (1, \bar{t})$. This can be seen from the first derivative:

$$\begin{aligned}
\frac{d\chi}{dt} &= \chi \left(\frac{k}{\lambda}\frac{d\lambda}{dt} + \frac{k}{\alpha t} - \frac{1}{(t-1)} \right) \\
&= \chi \left(\frac{k\left(\frac{(k-\alpha)}{k} \left(\frac{1}{\gamma\left(\gamma^{\psi} t^{\frac{k}{\alpha}} + t\right)} \right)^{\frac{k+1-\varepsilon}{k}} - \frac{1}{t} \right) t^{-\varepsilon}}{\alpha\gamma\lambda^{\varepsilon-1}} + \frac{k}{\alpha t} - \frac{1}{(t-1)} \right) \\
&= \chi \left(\frac{(k-\alpha)\left[\left(\frac{1}{\gamma\left(\gamma^{\psi} t^{\frac{k}{\alpha}} + t\right)} \right)^{\frac{k+1-\varepsilon}{k}} - \frac{k}{t(k-\alpha)} \right]}{\alpha t^{\varepsilon}\gamma\lambda^{\varepsilon-1}} + \left[\frac{(k-\alpha)t - k}{\alpha t(t-1)} \right] \right) < 0
\end{aligned}$$

The inequality follows from the fact that $k > \varepsilon - 1$ and $t < \bar{t} < \tilde{t} = \frac{k}{k-\alpha}$. Note that the restriction on the parameter space is a sufficient but not necessary condition.

We can now show that, over the relevant range $t \in (1, \bar{t})$, i.e. $\chi \in \left(\frac{1-\theta}{\theta}, \infty\right)$, the term Υ is monotonically decreasing in t. Taking the derivative yields:

$$\begin{aligned}
\frac{d\Upsilon}{dt} &= \left[\frac{\left(\theta^3(1+\chi)^2\left(2\chi^2 - 1\right) + \theta^2(1+\chi)^2 + 2\theta\chi + \theta - 2\chi - 1\right)}{2\theta^2\chi^2(1+\chi)^2} \right] \frac{d\chi}{dt} \\
&= \Upsilon_{\chi} \frac{d\chi}{dt}
\end{aligned}$$

Note that

$$\frac{d\Upsilon_{\chi}}{d\chi} = -\frac{(1-\theta)}{\theta^2} \left[\frac{\theta^2 - 1}{\chi^3} + \frac{1}{(\chi+1)^3} \right]$$

Since the bracketed term is monotonic in χ, we just need to check the bounds of χ to determine the sign.

$$\begin{aligned}
\lim_{\chi \to \frac{1-\theta}{\theta}} \frac{d\Upsilon_{\chi}}{d\chi} &= \left[\frac{(3-\theta)\theta^2}{(1-\theta)} \right] > 0 \\
\lim_{\chi \to \infty} \frac{d\Upsilon_{\chi}}{d\chi} &= 0
\end{aligned}$$

Thus $\frac{d\Upsilon_{\chi}}{d\chi} > 0$ and we now just need to check the limit of Υ_{χ} as χ approaches its lower bound, i.e. $\chi \to \frac{1-\theta}{\theta}$:

$$\lim_{\chi \to \frac{1-\theta}{\theta}} \Upsilon_{\chi} = 0.$$

Therefore $\Upsilon_{\chi} > 0$ and $\frac{d\Upsilon}{dt} < 0$ for all relevant values of t and χ.

So the government's payoff function between free trade and $t = \bar{t}$ is convex and so any turning point over the range $t \in (1, \bar{t})$ must be a minimum. Therefore, there can exist no point on the government's payoff function greater than either free trade or $\hat{t}$ and the government will maximize its payoff by choosing whichever yields the higher payoff. □

Lemma 7. *The government is more likely to choose free trade the greater the fixed cost to exporting is relative to the fixed cost of serving the domestic market, i.e. the greater is* γ.

Proof. Let

$$\Gamma \equiv \left(\frac{1}{\hat{t}^{\varepsilon}} \left[\frac{k}{(k-\alpha)} \right]^{\frac{\varepsilon-1}{k}} + 1 \right) \left[\frac{1+\gamma^{\psi}}{\gamma^{\psi}} \right]^{\frac{\varepsilon-1}{k}}$$

so

$$\begin{aligned} \frac{d\Gamma}{d\gamma} &= -\left[\frac{\psi\alpha\varepsilon}{\gamma^{\psi+1}k} \right] \left(\frac{1}{\hat{t}^{\varepsilon}} \left[\frac{k}{(k-\alpha)} \right]^{\frac{\varepsilon-1}{k}} + 1 \right) \left[\frac{1+\gamma^{\psi}}{\gamma^{\psi}} \right]^{\frac{-(k-(\varepsilon-1))}{k}} - \frac{\varepsilon}{\hat{t}^{2\varepsilon}} \left[\frac{k}{(k-\alpha)} \right]^{\frac{\varepsilon-1}{k}} \left[\frac{1+\gamma^{\psi}}{\gamma^{\psi}} \right]^{\frac{\varepsilon-1}{k}} \frac{d\hat{t}}{d\gamma} \\ &= -\left[\frac{\psi\alpha\varepsilon}{\gamma k} \right] \left[\frac{1+\gamma^{\psi}}{\gamma^{\psi}} \right]^{\frac{\varepsilon-1}{k}} \left(1 + \frac{1}{\hat{t}^{\varepsilon}} \left[\frac{k}{(k-\alpha)} \right]^{\frac{\varepsilon-1}{k}} \left[1 - \frac{\alpha \hat{t}^{1-\varepsilon}}{(k-\alpha)(\hat{t}-1)\gamma^{\psi}\hat{t}^{\frac{k-\alpha}{\alpha}}} \right] \right) \end{aligned}$$

We know from Lemma 4 that

$$\hat{t} > \frac{k}{k-\alpha}$$

so

$$\begin{aligned} \left.\frac{d\Gamma}{d\gamma}\right|_{\hat{t}=\tilde{t}} &= -\left[\frac{\psi\alpha\varepsilon}{\gamma k} \right] \left[\frac{1+\gamma^{\psi}}{\gamma^{\psi}} \right]^{\frac{\varepsilon-1}{k}} \left(1 + \frac{1}{\tilde{t}^{\varepsilon}} \left[\frac{k}{(k-\alpha)} \right]^{\frac{\varepsilon-1}{k}} \left[1 - \frac{\alpha \hat{t}^{1-\varepsilon}}{\alpha\gamma^{\psi}\hat{t}^{\frac{k-\alpha}{\alpha}}} \right] \right) \\ &= -\left[\frac{\psi\alpha\varepsilon}{\gamma k} \right] \left[\frac{1+\gamma^{\psi}}{\gamma^{\psi}} \right]^{\frac{\varepsilon-1}{k}} \left(1 + \frac{\tilde{t}^{\frac{\varepsilon-1}{k}}}{\tilde{t}^{\varepsilon}} \left[1 - \frac{\hat{t}^{1-\varepsilon}}{\gamma^{\psi}\hat{t}^{\frac{k-\alpha}{\alpha}}} \right] \right) < 0. \end{aligned}$$

The previous inequality follows from the fact that $\frac{d^2\Gamma}{d\gamma d\hat{t}} < 0$. □

Proposition 3. *The tariff that maximizes tariff revenue is:*

1. *Decreasing in* γ*;*
2. *Decreasing in the shape parameter* k*;*
3. *Increasing in* α *and the elasticity of substitution* ε.

Proof. The tariff that maximizes tariff revenue is defined by $\hat{t}$ such that

$$(k-\alpha)\gamma^{\psi}\hat{t}^{\frac{k}{\alpha}} = \left(k\gamma^{\psi}\hat{t}^{\frac{k}{\alpha}-1} + \alpha \right) \tag{29}$$

Totally differentiating (29) yields:

$$\frac{\gamma}{\hat{t}}\frac{d\hat{t}}{d\gamma} = -\left[\frac{\psi\alpha^2}{k(k-\alpha)(\hat{t}-1)\gamma^{\psi}\hat{t}^{\frac{k-\alpha}{\alpha}}}\right] < 0 \tag{30}$$

$$\frac{k}{\hat{t}}\frac{d\hat{t}}{dk} = -\left[\frac{\log\left(\gamma\hat{t}^{\varepsilon}\right)\left(k(\hat{t}-1)-\alpha\hat{t}\right)}{\varepsilon(k-\alpha)(\hat{t}-1)} + \frac{\alpha}{(k-\alpha)}\right] < 0 \tag{31}$$

$$\frac{\alpha}{\hat{t}}\frac{d\hat{t}}{d\alpha} = \left[\frac{\alpha^2\hat{t}\left(\gamma^{k+1}+\gamma^{\frac{k}{\alpha}}\hat{t}^{\frac{k}{\alpha}}\right)}{k\gamma^{\frac{k}{\alpha}}\hat{t}^{\frac{k}{\alpha}}(k-\alpha)\left(\hat{t}-1\right)} + \frac{\log\left(\hat{t}\gamma\right)\left(k(\hat{t}-1)-\alpha\hat{t}\right)}{(k-\alpha)\left(\hat{t}-1\right)}\right] > 0. \tag{32}$$

These inequalities follow from the fact that $\hat{t} > \frac{k}{k-\alpha}$ (as shown in Lemma 4) and $\gamma > 1$. Recall also that $\frac{d\varepsilon}{d\alpha} = \alpha\varepsilon^2 > 0$. □