Mathematical Methods of Economic Analysis

ECO 6405/7405 (Fall 2022)

Class Times

The class meets on Tuesday and Thursday from 5:00pm to 6:15pm in DM-164.

Cross-listing

This course is cross-listed as 6405/7405. Since they meet concurrently in the same room, the material covered will be the same.

Course Description

This course focuses on mathematical methods used in modern economics. These include:

1. linear methods often used in mathematical modelling,
2. the portions of mathematical analysis relevant for studying optimization problems,
3. the construction and solution of optimization problems, and
4. the solution of difference and differential equations characteristic of modern intertemporal models, as used in both microeconomics and macroeconomics.

The first part of the course develops some basic mathematical tools of analysis which we will use to solve optimization problems. This covers roughly parts II and III of the text, and may include excerpts from parts VI and VII. The second part (part IV of the textbook) covers classical, calculus-based methods of optimization including Lagrange multipliers and the Kuhn-Tucker theorem. The methods of Lagrange and Kuhn-Tucker have been invaluable in solving many of the problems you will typically encounter in economics (consumer and producer choice, social welfare max, etc.). We then cover the solution of difference and differential equations, and their stability properties (part V). If time permits, we will look at dynamic optimization and the Maximum Principle.

Course Objectives

By the end of the course, at a minimum, you should be able to:

- Determine whether a linear system has a solution, and if so, how many.
- Solve linear systems using both determinants and the Gauss-Jordan method.
- Find eigenvalues and eigenvectors.
- Use the functional calculus.
- Determine whether an optimization problem has a solution.
- Characterize the solutions of optimization problems via the first order conditions.
- Solve unconstrained optimization problems using first and second order conditions.
- Solve constrained optimization problems using the Kuhn-Tucker Theorem.
- Exploit special features such as homogeneity or convexity when solving optimization problems.
- Solve linear difference and differential systems.
- Characterize the long-run behavior of difference and differential systems using eigenvalues.

Textbook

- Carl Simon and Lawrence Blume, *Mathematics for Economists*, W. W. Norton, New York, 1994.

Simon and Blume's book is the main text. I plan to cover Parts II-IV and VII of Simon and Blume, with some excerpts from Part VI. Time permitting, we will then turn our attention to Part V and dynamic models.

Lecture Notes

1. Chapters 6 and 7.

Selected Mathematical Economics Books

The first group focuses on mathematical economics. The following have been widely used and I am familiar enough with them to comment.

- Alpha Chaing and Kevin Wainwright, *Fundamentals of Mathematical Economics*
  Now in its 4th edition, this book is easier than Simon & Blume. I haven't seen this edition. I gather it adds material on probability and optimal control. Compared to S&B, it focuses more on how to use techniques rather than mathematical rigor.
- Avinash K. Dixit, *Optimization in Economic Theory*
  A nice short book on both static and dynamic optimization.
- Angel de la Fuente, *Mathematical Methods and Models for Economists*
  This book is more advanced than S&B and includes material on correspondences and fixed point theorems.
- Rangarajan K. Sundaram, *A First Course in Optimization Theory*
  Raghu's book focuses on optimization. It's at a higher level than Dixit.
- Akira Takayama, *Mathematical Economics*
  Akira's book is certainly mathematical, but the focus is on microeconomics, including general equilibrium and optimal growth. It's not really suitable for this course.

Selected General Mathematics Books

The book by Garrity covers a fair chunk of the math we cover, plus quite a bit we don't use. I found its preface outstanding. The other two are on how to solve mathematical problems, especially those involving proofs.

- Thomas A. Garrity (2021), *All the Math Your Missed*, 2nd ed., Cambridge University Press, Cambridge UK.
  This is a book aimed at beginning graduate math students. There are many relatively elementary topics that students may encounter in their undergraduate classes, that are nonetheless assumed known when they show up in grad school. Surprise, surprise, much of that material is needed in our class too. While I cover some of it, I don't time to do everything. Even if you don't read any of the rest, use Amazon's Look Inside feature to read the excellent preface and other materials prior to chapter one.
- George Polya (1957), *How to Solve It: A New Aspect of Mathematical Method*, 2nd ed., Princeton University Press, Princeton, NJ.
  If you're having difficulty understanding the logic of proofs, or how to find and construct them, try this classic.
- Daniel J. Velleman (2019), *How to Prove It: A Structured Approach*, 3rd ed., Cambridge University Press, Cambridge UK.

This is another take on the question of how to prove things.

Two Free Math Books

There are a couple of decent math books available for free.

- For Linear Algebra, there's Jim Hefferon's free book, Linear Algebra.
- For basic (point set) topology, try Sidney A. Morris's Topology without Tears, which now includes some supplementary videos and translations of earlier versions into 8 languages (Arabic, Chinese, Greek, Korean, Persian, Russian, Spanish, and Turkish).

Optimization Handout

You may find the following handout on basic optimization helpful, particularly in your micro course: Constrained Optimization Survival Guide.

Office Hours and Contact Info

If you have questions, you may ask immediately after class, or come to my office. Regular office hours are 12:45-1:45pm and 3:30-4:15pm on Tuesdays and Thursdays. I will be happy to make an appointment for another time if that is more convenient. My office is **DM-311A**, my phone number is **305-348-3287**, and my email is <boydj@fiu.edu> or <John.Boyd@fiu.edu>.

Exams and Homework

Grades will be based on two midterm exams (worth 25% each), a final exam (40%), and homework assignments (10%). In addition to being announced in class, homework assignments will be posted below.

Homework will be submitted in person or by emailing it to me. If you email it, it may be easiest to write it out and then photograph it with your phone. If so, please combine the pages into a **single pdf**. I will not be happy if I see 10 separate files for one assignment.

Homework is graded as follows: ✓+ (3 pts) means that it is mostly correct, no major errors. ✓ (2 points) indicates you've missed at least one problem. ✓- (1 point) means that at least two problems or equivalent are mostly incorrect. On difficult assignments three misses may be required for a ✓-.

A zero is also possible, and usually means it wasn't turned in.

Homework Assignments and Answers

Assignments will appear here. Answers will be posted sometime after the homework is collected.

1. TBA

Exams

There will be two midterm exams, each worth 25% of your grade, and a final, worth 40% of your grade.

- The first midterm is scheduled for **Thursday, September 22**.
- The second midterm is scheduled for **Thursday, October 27**.
- The final will be at the officially scheduled time, **5pm on Tuesday, December 6, 2022** in our regular classroom, DM-164.

Sample Exams

The material covered varies from year to year and some of the questions on previous exams may not be relevant for the material we cover this year. A few of the answers contain minor errors.

Old First Midterms	Old Second Midterms	Old Finals
2000	2000	2000
2001	2001	2001
2002	2002	2002
2003	2003	2003
2011 questions, answers	2011	2011
2012, with answers	2012, with answers	2012, with answers
2013, with answers	2013, with answers	2013, with answers
2014, with	2014, with	2014, with

answers	answers	answers
2015, with answers	2015, with answers	2015 questions, answers
2016 questions, answers	2016 questions, answers	2016 questions, answers
2017, with answers	2017, with answers	2017, with answers
2018 questions, answers	2018 questions, answers	2018 questions, answers
2019 questions, answers	2019 questions, answers	2019 questions, answers
2020 questions, answers	2020 questions, answers	2020 questions, answers
2021 questions, answers	2021 questions, answers	2021 questions, answers

Course Outline

Tentative and subject to change, especially if there are hurricanes. It is probably a bit over ambitious, but we'll see. There have been years when we've made it all the way to the end. Once the semester is underway, the lecture notes will show what was actually covered.

Aug. 23	6: Intro to Linear Algebra (and use in Economics)
Aug. 23, 25	7: Linear Systems
Aug. 30	8: Matrix Algebra
Sept. 1	9: Determinants & 26: Determinants
Sept. 6, 8	10: Euclidean Spaces
Sept. 13, 15	11: Linear Independence, Bases (see also Chapters 27 & 28)
Sept.	

20	Counting, 12: Limits and Open Sets
Sept. 22	**Exam #1 — through Chapter 11 + parts of 26, 27, and 28**
Sept. 27	12: Limits and Closed Sets
Sept. 29	12: Limits and Closed Sets
Oct. 4	13.4: Continuous Functions
Oct. 6	29.1-2, 5: Monotone Convergence, Completeness, Compact Sets, 30.1: Weierstrass Theorem
Oct. 11	14: Calculus of Several Variables I
Oct. 13	30: Calculus of Several Variables II Rolle's Theorem, Mean Value Theorem, Taylor Formulas 29.3: Connected Sets, Intermediate Value Theorem
Oct. 18	15: Implicit Functions and their Derivatives
Oct. 20	16: Quadratic Forms and Definite Matrices
Oct. 25	17: Unconstrained Optimization
Oct. 27	**Exam #2 — Chapters 12-17, 29 & 30**
Nov. 1	18: Constrained Optimization I: First-order Conditions (continued)
Nov. 3	19: Constrained Optimization II: Multipliers and Second-order Conditions
Nov. 8	20: Homogeneous and Homothetic Functions
Nov. 10	21: Concave and Quasiconcave Functions
Nov. 15, 17	21: Eigenvalues and Eigenvectors

Nov. 22	24: Ordinary Differential Equations: Scalar Equations
Nov. 24	***Thanksgiving Holiday (no class)***
Nov. 29	25: Ordinary Differential Equations: Systems of Equations
Dec. 1	Optimal Growth
Dec. 6	**Final Exam: At 5pm in our regular classroom, DM-164**