It’s Now or Never:

Deadlines and cooperation

Abstract: Opportunistic agents can cooperate over time under the threat of punishment. However, if there is no monitoring and outcomes are random, deviations cannot be detected with certainty. In such circumstances, deadlines are shown to induce cooperation. There is an optimal deadline, which is positively related to the reward from success and negatively related to the number of cooperating agents. The equilibrium under the optimal deadline is compared with that under the optimal two-phase strategy a la Green-Porter (1984).

Correspondence: Kaz Miyagiwa, Department of Economics, Emory University, Atlanta, GA 30322, U.S.A.; E-mail: kmiyagi@emory.edu; Telephone: 404-727-6363; Fax: 404-727-4639

Acknowledgments: I have benefited from comments by seminar participants at Emory and Osaka Universities. I alone am responsible for errors.
1. Introduction

Suppose you try to prove a theorem or look for a new research idea. Since two heads are better than one, your chance of success will improve if you have a co-author. However, co-authoring can give rise to the temptation to shirk. Such a temptation would intensify if co-authors cannot monitor each other’s effort.

How to induce cooperation among opportunistic agents has long been a subject of intensive research among social scientists, possibly dating back to Rousseau, who posed the prisoner’s dilemma problem in his parable involving stag hunters. Game theory has taught us that, if agents interact repeatedly over time, it is possible to maintain cooperation under the threat of punishment. However, with random outcomes, if actions cannot be monitored, it is impossible to detect cheating because failures from lack of efforts cannot be disentangled from those due to bad luck. If cheating goes undetected, the threat of punishment cannot be used to induce cooperation.

In this paper I show that in such circumstances setting a deadline can induce cooperation. I further show that there is an optimal deadline, which is negatively correlated to the number of agents involved and positively correlated to the value of success. Thus, although typically regarded as sheer nuisances, deadlines can help agents cooperate and increase their welfare in situations where cooperation would otherwise difficult to maintain.

Why deadlines can induce cooperation has an intuitive explanation. If an agent does not exert effort, probability of success falls for the team. Without punishment,

---

1 A contract-theoretic approach has recently been applied to study how a principal can induce cooperation among agents. The present paper concerns the desirability to cooperate outside principal-agent relationships.
however, an agent will face the same continuation payoff the following period that he does now. Now introduce a deadline and suppose that agents are in the final period of cooperation. Then, shirking would not only increase probability of failure but also result in a smaller continuation payoff the next period, because there will be no more cooperation. Thus, a deadline makes cheating less attractive.

There are other papers addressing this type of problems, collectively known as repeated moral hazard problems with imperfect public information. The pioneering work of Green and Porter (1984) focuses on collusion enforcement in oligopoly under stochastic demand, where firms can observe prices, ex post, but not one another’s outputs. They propose a two-phase solution in the class of trigger strategies, whereby firms start out in a cooperative phase, producing collusive outputs until the market price falls below a certain level. Then, firms switch to the punishment phase, producing competitive (Cournot) outputs for a fixed number of periods, after which they return to the collusive phase, and this cycle repeats itself over time. The optimal strategy for firms calls for minimization of the punishment phase subject to the constraint that no deviation occurs in the collusive phase. Below, I will devise a similar two-phase solution and compare it with the solution based on deadlines.

In terms of setup, the present model is related to the repeated partnership model of Abreu, Milgrom and Pearce (1991). However, the two differ in focus; their main interest lies in demonstrating how the folk theorem fails under imperfect public information when
each period gets shorter, whereas here the main objective is to show how a deadline can induce cooperation.\footnote{There is also a difference in setup. Here the game ends with success (except in the model of section). In contrast, in Abreu, Milgrom and Pearce (1991) a game never ends, whether there is success or failure.}

The remainder of the paper is organized in 7 sections. The next section describes the environment of the analysis in details. Section 3 examines the case in which there is no monitoring. Section 4, the main section, introduces deadlines and identifies the mechanism by which setting a deadline can induce cooperation. Section 5 recasts the Green-Porter model in the present context and compares its optimal strategy with the optimal deadline strategy.

Section 6 introduces asymmetry into the analysis and finds intensity reversals. An agent with higher probability of success has more of an incentive to cheat when a deadline is set not too distant but if the deadline is distant an agent with lower probability of success may be more likely to deviate. Section 7 considers situations in which success confers only transitory benefits, so that agents must return to search the following period. Here too setting a deadline can induce cooperation. The final section concludes.

2. Setup

2.A. Going it alone

Consider an agent engaged in “search” in an infinite time horizon, where time is discrete. All actions take place at $t = 1, 2, \ldots$, which are called ‘dates.’ At each date $t$, conditional on not having succeeded yet, an agent decides whether to make an effort or not. Making an effort, an agent incurs disutility $e$ but has success with (conditional)
probability \((1 - p) > 0\), where \(p\), probability of failure, is time-invariant. Success yields the benefit \(b\) per period in perpetuity and hence has the present value of \(\pi = b/(1 - \delta)\), where \(\delta \in (0, 1)\) is the discount factor. An agent exits with the reward \(\pi\) after having success.

Since the environment is stationary, the present value \(v\) of search is defined recursively by:

\[
v = -e + (1 - p)\delta\pi + p\delta v.
\]

Making an effort, an agent incurs disutility \(e\) but has success with probability \(1 - p\). If there is failure, an event with probability \(p\), then the following period an agent faces exactly the same environment – and hence the expected value \(v\) – that he does this period. Collecting terms I can express \(v\) in terms of the parameters:

\[
(1) \quad v = \frac{(1 - p)\delta b - e}{1 - \delta p}.
\]

If he chooses not to make an effort, an agent incurs no disutility but faces no chance of success. Normalize this utility to zero and assume \(v \geq 0\); i.e., search is worthwhile.\(^3\)

2.B. Cooperation with monitoring

Turn next to cooperation among \(m \geq 2\) identical agents. Forming a team, all agents independently and simultaneously decide whether to make an effort. Probability of failure per individual remains constant at \(p\). Therefore, if all agents make efforts, the team

\(^3\) This is just for convenience’s sake. If the inequality is reversed, an agent’s reservation utility is till zero, and the rest of the analysis is unaffected.
fails with joint probability $p^m$. Since $p^m < p$, each agent can lower the probability of failure by cooperating. Thus, cooperation functions as insurance.

Cooperation may change the benefit of success. Let $B$ denote the per-period benefit success brings each agent, and write its present value as $\Pi = B/(1 - \delta)$. In general $B$ can differ from $b$ (and hence $\Pi$ from $\pi$).

In the remainder of this subsection I assume perfect monitoring and consider cooperation under the following trigger strategy.

At $t = 1$, agree to cooperate and then make an effort. At any $t \geq 2$, make an effort as long as all other agents made efforts in all dates up to $t - 1$; otherwise go it alone.

Suppose that agents adopted the above strategy and the team reached date $t$ (without success). If all agents play the above strategy, the expected net value $V$ per agent can be written recursively as follows:

\begin{equation}
V = (1 - p^m)\delta \Pi + \delta p^m V - e.
\end{equation}

The interpretation is similar to the one given for the individual search case. Collecting terms I obtain $V$ in terms of the parameters.

\begin{equation}
V = \frac{(1 - p^m)\delta \Pi - e}{1 - \delta p^m}.
\end{equation}

Assume that cooperation is desirable; i.e., $V > v$.

Now examine an agent’s unilateral incentive to deviate from the equilibrium. A cheater saves the disutility $e$ but increases team probability of failure from $p^m$ to $p^{m-1}$. He also triggers the punishing phase, reducing the continuation payoff from $V$ to $v$. That is, a would get
\[ V_d = (1 - p^{m-1})\delta \Pi + \delta p^{m-1}v \]

There is no incentive to cheat if and only if \( V \geq V_d \). This condition can be expressed as

\[ (1 - p^m)\delta \Pi + \delta p^m V - e \geq (1 - p^{m-1})\delta \Pi + \delta p^{m-1}v. \]

Rearrangement yields:

\[ \text{(4)} \quad p^{m-1}(1 - p)\delta (\Pi - V) + p^{m-1}\delta (V - v) \geq e. \]

**Assumption 1:** When efforts are observable, cooperation is stable; i.e., (4) holds.

3. **Cooperation without monitoring**

Suppose now that agents cannot monitor one another’s behavior. Without monitoring it is impossible to identify and punish a cheater for lack of effort when outcomes are stochastic. In this subsection I derive the condition in which there would be no cooperation if all agents simply exerted effort at each date without punishment.

If cooperation occurs, agents face the same equilibrium payoff \( V \) defined in (3). However, since cheating goes unpunished, the continuation payoff remains also at \( V \). Thus, a cheater is expected to receive

\[ W_d = (1 - p^{m-1})\delta \Pi + p^{m-1}\delta V. \]

A calculation shows that

\[ \text{(5)} \quad V - W_d = p^{m-1}(1 - p)\delta (\Pi - V) - e. \]

Thus, cooperation would not be possible without punishment if \( V < W_d \).
Assumption 2: $V < W_d$, i.e., $p^{m-1}(1-p)\delta(\Pi - V) < e$.

I focus on the case in which assumptions 1 and 2 hold simultaneously. There is a range of $e$ that satisfies both assumptions (proof is trivial since $V > v$).

Lemma 1: There is $e$ satisfying

$$p^{m-1}(1-p)\delta(\Pi - V) + p^{m-1}\delta(V - v) \geq e > p^{m-1}(1-p)\delta(\Pi - V)$$

so that both Assumptions 1 and 2 are satisfied.

4. Deadlines

Cooperation breaks down if a cheater goes unpunished. The purpose of this section is to show that in such cases a deadline may induce cooperation for a limited time. I begin by defining strategy $C_n$; $n = 1, 2, \ldots$ as follows:

At dates $t$ (where $1 \leq t \leq n$) make an effort. At $t > n$, go it alone.

Define by $U(n)$ the equilibrium payoff from strategy $C_n$ in date 1’s values. Likewise let $U_d(n)$ denote the payoff obtainable if an agent deviates from $C_n$ at date 1.

Now, consider strategy $C_1$; agents cooperate just for the first period. Hence, the expected equilibrium net benefit is

(7) \[ U(1) = (1 - p^m)\delta\Pi + p^m\delta v - e. \]

A cheater will get

$$U_d(1) = (1 - p^{m-1})\delta\Pi + p^{m-1}\delta v.$$
Strategy $C_1$ is an equilibrium strategy if the following holds under assumptions 1 and 2:

$$U(1) - U_d(1) = p^{m-1}(1 - p)\delta(\Pi - v) - e \geq 0.$$  

Since $V > v$, clearly there is $e$ satisfying:

$$p^{m-1}(1 - p)\delta(\Pi - v) \geq e \geq p^{m-1}(1 - p)\delta(\Pi - V).$$

This $e$ also satisfies the inequalities in lemma 1 because the set of $e$ satisfying (9) is a proper subset of the set of $e$ satisfying (8). This proves the next proposition

**Proposition 1**: For $e$ satisfying (9), all agents cooperate and exert efforts period 1 without monitoring under strategy $C_1$.

Thus, the deadline at $n = 1$ can induce cooperation without monitoring. This result has the following intuitive explanation. With or without a deadline, cheating changing probability of failure from $p^m$ to $p^{m-1}$. However, failure leads to the continuation payoff $V$ without a deadline and $v < V$ with a deadline. Thus, a deadline makes cheating less attractive by reducing the continuation payoff from failure.

Supposing that condition (9) holds, I try to extend the deadline to $n = 2$. If all agents adopt $C_2$, at date 1, the expected net benefit is given by

$$U(2) = (1 - p^m)\delta\Pi + p^m\delta U(1) - e,$$

whereas cheating yields

$$U_d(2) = (1 - p^{m-1})\delta\Pi + p^{m-1}\delta U(1).$$

There is no incentive to cheat if
If this inequality does not hold, then cooperation can be maintained only during the first period. If the inequality holds, cooperation can be maintained for at least the first two periods. In that case, the deadline may be extended further. However, this process cannot continue forever as shown in the next proposition.

**Proposition 2:** If \( U(1) - U_d(1) \geq 0 \), there exists the unique integer \( n^* \geq 1 \) such that

\[
U(n^* + 1) - U_d(n^* + 1) < 0 \leq U(n^*) - U_d(n^*). 
\]

To prove the proposition, assume that cooperation can be maintained for the first \( n - 1 \) periods. Then the expected equilibrium payoff under strategy \( C_n \) (cooperation over the first \( n \) periods) is written as:

\[
U(n) = (1 - p^m)\delta \Pi + p^m \delta U(n - 1) - e. 
\]

Then:

\[
U(n) - U(n - 1) = p^m \delta (U(n - 1) - U(n - 2)) = \cdots = (p^m \delta)^{n-1} (U(1) - v) > 0. 
\]

This shows two things. First, \( U(n) \) increases monotonically, implying that, the longer agents can cooperate, the more valuable cooperation is. Second, since \( (p^m \delta)^{n-1} \) decreases in \( n \), (12) also implies that

\[
U(n) - U(n - 1) < U(n - 1) - U(n - 2); 
\]

that is, there are diminishing returns.
On the other hand, cheating in period 1 yields:

\[(13) \quad U_d(n) = p^{m-1}(1-q)\delta\Pi + p^{m-1}\delta U(n - 1)\]

There is no incentive to cheat in period 1 if

\[(14) \quad U(n) - U_d(n) = p^{m-1}(1-p)\delta(\Pi - U(n - 1)) - e \geq 0.\]

Since \(U(n - 1)\) is increasing in \(n \geq 2\), the middle expression in (14) decreases monotonically. Since \(U(1) - U_d(1) \geq 0\), if I show that there is \(n \geq 2\) such that \(U(n) - U_d(n) < 0\), the proof is complete.

To do so, first substitute from (2) into (11) to obtain

\[
U(n) = V - p^m\delta(V - U(n - 1)).
\]

Similarly, substituting from (5) into (13) yields

\[
U_d(n) = W_d - p^{m-1}\delta(V - U(n - 1))
\]

Hence,

\[(15) \quad U(n) - U_d(n) = V - W_d + \delta p^{m-1}(1-p)(V - U(n - 1)).\]

Now, \(U(n - 1) \to V\) as \(n \to \infty\); i.e., a deadline set at infinity is no deadline at all. Thus,

\[
U(n) - U_d(n) \to (V - W_d) < 0
\]

where the inequality is by assumption 2. This completes the proof of the proposition.

Proposition 2 can be understood in terms of continuation payoffs. In the last period of cooperation a cheater faces the continuation payoff \(v\) from going it alone if failure occurs. In the penultimate period, the continuation payoff instead is \(U(1) > v\).
Thus, by the argument following proposition 1 there is more of an incentive to cheat in the penultimate period. This incentive to cheat augments in the antepenultimate period because the continuation payoff in the first period is $U(2) > U(1)$, and so forth. At $n^* + 1$ the incentive is so high that an agent indeed prefers to cheat, thereby limiting the length of cooperation to $n^*$. 

Thus, $n^*$ is the maximum number of periods of cooperation without monitoring. Given that the benefit of cooperation is monotone increasing, $n^*$ also signifies the optimal length of cooperation. In other words, $n^*$ is the optimal deadline.

I next turn to the relationship between the benefit from success and the optimal deadline. The finding is stated in

**Proposition 3.** Assume there is no monitoring. Then, the greater the benefit of cooperation, the longer it is possible to maintain cooperation.

To prove this proposition, differentiate both sides of (13) with respect to $\Pi$, and note that

$$\text{sgn } \{d(U(n) - U_d(n))/d\Pi\} = \text{sgn } \{d(\Pi - U(n - 1))/d\Pi\} > 0,$$

The inequality holds since $U(n - 1)$ contains $\Pi$ only with positive probability. Thus, an increase in benefit intensifies the incentive to cooperate. Thus, if $U(n^* + 1) - U_d(n^* + 1) > 0$ in (10), then cooperation can be maintained at least for the first $n^* + 1$ periods instead of the first $n^*$ periods. This establishes proposition 3.
Turning next to the effect of the number of cooperating agents, assume momentarily that the benefit $\Pi$ from success is independent of the membership size $m$. Then I can show the following.

**Proposition 4**: The greater the number of cooperating agents $m$, the shorter the optimal length of cooperation $n^*$ is.

I prove this by showing that an increase in $m$ increases the incentive to deviate, given $n$. The first things to show however is that the expected value of success increases with $m$. To do so, let $U(n; m)$ be the value $U(n)$ with $m$ cooperative agents. Suppose $U(n; m)$ is positive. Then I can show that $U(n; m) < U(n; m + 1)$ by induction on $n$. It is easy to check using (7) that this holds for $n = 1$. Suppose that $U(n; m) < U(n; m + 1)$ for $n \geq 1$. Then by (11)

\[
U(n + 1; m + 1) - U(n + 1; m) = (p^m - p^{m+1})\delta \Pi + p^{m+1}\delta U(n; m + 1) - p^m\delta U(n; m)
\]

\[
= \delta p^m ((\Pi - U(n; m)) - p(\Pi - U(n; m + 1))) > 0.
\]

While the benefit of cooperation increases in $m$, the incentive to deviate increases even faster. To see it, observe that by (12)

\[
[U(n; m + 1) - U_d(n; m + 1)] - [U(n; m) - U_d(n; m)]
\]

\[
= p^m(1-p)\delta (\Pi - U(n - 1; m + 1)) - p^{m-1}(1-p)\delta (\Pi - U(n - 1; m))
\]

\[
= \delta (1-p)p^m \{p[\Pi - U(n - 1; m + 1)] - [\Pi - U(n - 1; m)]\} < 0.
\]

The inequality follows because $U(n - 1; m + 1) > U(n - 1; m)$. 
The proposition says that it is more difficult to maintain cooperation as the number of cooperative agents grows. The proposition is based on the assumption that the benefit $\Pi$ to each agent stays constant as the number of cooperative agents increases. In some case, however, $\Pi$ may increase due to the synergy or complementarity among agents. In other case $\Pi$ may fall, especially if success yields a fixed total sum to be divided among agents. In such cases, depending the rate at which $\Pi$ changes, the result may not hold locally, although it must as $n$ gets large enough.  


As mentioned in the introductory section, Green and Porter (1984) pioneered a two-phase strategy for inducing cooperation without monitoring. Although their setting differs from that of this paper, their methodology can be applied to the present case. Translated into our context, the two-phase strategy a la Green-Porter (henceforth, the GP strategy) has agents exerting effort in the first period but switching, when there is no success, to the punishment phase of a given length, during which agents go it alone. At the end of the punishment phase agents return to cooperation another period, and so forth. Thus, the equilibrium payoff $V^c$ is given by

$$V^c = -e + (1 - p^m)^n \delta \Pi + \delta p^m V^p$$

4 With $n$ approaching infinity, cheating by one agent will have no effect on success probability but a cheater saves the disutility $e$ and hence deviates.
where \( V^P \) is the payoff from the punishment phase to be defined below. Suppose that the punishment phase lasts \( B - 1 \) periods. Since agents cooperate again the following period, \( V^P \), the payoff from the punishment phase, is expressed, after some manipulation, as

\[
V^P = (1 - (\delta p)^B) v + (\delta p)^B V^c.
\]

(16) and (17) can be solved for the values \( V^c \) and \( V^P \), and can be expressed in terms of \( v \) and \( V \):

\[
V^c = \frac{[1 - (\delta p)^B] \delta p^m v + (1 - \delta p^m) V}{[1 - (\delta p)^B] \delta p}
\]

(18)

\[
V^p = \frac{[1 - (\delta p)^B] v + (\delta p)^B (1 - \delta p^m) V}{[1 - (\delta p)^B] \delta p}
\]

(19)

There would not be deviations if

\[
V^c - V^c_d = -e + p^{m-1} (1 - p) \delta (\Pi - V^P) \geq 0,
\]

(20)

where \( V^c_d \) is the payoff to a cheater:

\[
V^c_d = (1 - p^{m-1}) \delta \Pi + p^{m-1} V^p.
\]

Since all these values depend on \( B \), write them as \( V^c(B) \), \( V^P(B) \) and \( V^c_d(B) \). It is easy to check that \( V^P(B) \) is monotone decreasing with \( V^P(0) = V \) and \( V^P(\infty) = v \).

The optimal GP strategy maximizes the equilibrium payoff \( V^c(B) \) with respect to \( B \) (when to end the punishment phase) subject to the no-deviations constraint (18) and the integer constraint. That is

\[
\text{Max}_{\{B \in \mathbb{N} \cup \{0\}\}} V^c \text{ subject to (20).}
\]
where $\mathbb{N}$ is the set of natural numbers. Let $B^*$ be the optimizer of the above problem, and denote the corresponding values by $V^c(B^*)$, $V^p(B^*)$ and $V^c_d(B^*)$.

Now I compare the GP strategy with the deadline strategy. Consider the following companion problem:

\[(22) \quad \max_{B \in \mathbb{R}^+} V^c(B) \text{ subject to } (20).\]

(22) differs from (21) only in the choice set. Since $V^c(B)$ and $V^p(B)$ are continuous, (22) is a standard constrained optimization problem. Let $B_r$ be an optimizer of the companion problem (22). $B_r$ is unique. Since it is real, it satisfies the constraint (20) with equality:

\[(23) \quad - e + p^{m-1}(1-p)\delta(\Pi - V^p(B_r)) = 0.\]

**Proposition 5**: If $B_r$ is an integer so that it equals $B^*$, and $V^p(B^*) \geq U(B^*)$, GP payoff-dominates the optimal deadline.

**Proof**: By (23)

\[
0 = - e + p^{m-1}(1-p)\delta(\Pi - V^p(B^*)) \\
\leq - e + p^{m-1}(1-p)\delta(\Pi - U(B^*)) = (U(B^* + 1) - U_d(B^* + 1)) \\
\]

and hence the deadline at $B^* + 1 = B_r + 1$ is incentive-compatible.

\[
U(B^* + 1) = - e + (1-p^m)\delta\Pi + p^mU(B^*) \\
\leq - e + (1-p^m)\delta\Pi + p^mV^p(B^*) = V^c(B^*).
\]

Finally,
\begin{align*}
U(B^* + 1) - V^p(B^*)
&= -e + p^{m-1}(1-p)\delta(\Pi - U(B^*)) - V^p(B^*) \\
&= -e + p^{m-1}(1-p)\delta(\Pi - V^p(B^*)) - p^{m-1}(1-p)\delta U(B^*) \\
&\quad - [1 - V^p p^{m-1}(1-p)\delta] U(B^*) \\
&= - [1 - V^p p^{m-1}(1-p)\delta] U(B^*) < 0
\end{align*}

Then the deadline at \( B^* + 2 \) is not incentive-compatible. \( \square \)

Suppose that \( B_r \) is not a natural number. Then the solution to the GP problem is the smallest integer that is not greater than \( B_r \), which we write as \([B_r] = B^*\). We have that

**Proposition 6:** Suppose that \( B_r \) is not a natural number. If \( V^p(B_r) \geq U(B^*) > V^p(B^*) \), the deadline strategy yields a greater payoff than the GP strategy.

**Proof:**

\begin{align*}
U(B^* + 1) - U_d(B^* + 1) &= - e + p^{m-1}(1-p)\delta(\Pi - U(B^*)) \\
&\geq - e + p^{m-1}(1-p)\delta(\Pi - V^p(B_r)) = 0,
\end{align*}

where the equality is by (23). Thus, \( B^* + 1 \) is an incentive-compatible deadline date. Furthermore, since

\begin{align*}
U(B^* + 1) &= - e + \delta(1-p^m)\Pi + \delta p^m U(B^*)
\end{align*}
\[ > - e + \delta(1-p)\Pi + \delta p V^p(B^*) = V^c(B^*). \]

the deadline at \( B^* + 1 \) yields a greater payoff than the optimal GP strategy. □

6. Asymmetric agents

5.A. Setup

This section extends the analysis to asymmetric agents. I assume agents differ in regard to probability of success. To simplify the analysis I consider the possibility of cooperation between two agents, whom I call Senior (s) and Junior (j). Senior has a better chance of success, or a lower probability of failure; i.e., \( p_s < p_j \). Agents are identical in all other respects agents. In particular, they have the same discount factor, the same disutility from effort and the same benefit \( b \) from individual search and \( B \) from cooperative search.

The analysis of this section follows closely that from the preceding section, so I below omit details unless confusions arise. A procedure analogous to the one leading to equation (1) yields the following payoff from individual search for agent \( i = s, j \):

\[
(26) \quad v_i = \frac{[(1 - p_i)\delta b - e]}{(1 - \delta p_i)}
\]

Straightforward differentiation shows that \( dv_i/dp_i < 0 \) and hence

\[
(27) \quad v_j < v_s.
\]

That is, Junior faces a lower payoff from individual search. Assume \( v_j \geq 0 \) so both agents find individual search worthwhile.

5.B. Cooperation
I now turn to the possibility cooperation without monitoring. Agents collectively fail with probability $p_s p_j$. Therefore, with some abuse of notation I can write the equilibrium payoff as

$$V = (1 - p_j p_s) \delta \Pi + \delta p_j p_s V - e.$$ 

Collecting term gives

$$V = [(1 - p_j p_s) \delta \Pi - e] / (1 - \delta p_j p_s).$$

If Junior cheats, he will free-ride on Senior’s success so his payoff will be

$$V_{jd} = (1 - p_j) \delta \Pi + \delta p_s V,$$

where the continuation payoff $V$ implies that cheating occurs with impunity. Junior has no incentive to cheat if

$$V - V_{jd} = p_s (1 - p_j) \delta (\Pi - V) - e \geq 0.$$  

Similarly, Senior has no intension to cheat if

$$V - V_{sd} = p_j (1 - p_s) \delta (\Pi - V) - e \geq 0.$$  

Since

$$p_s (1 - p_j) - p_j (1 - p_s) = p_s - p_j < 0,$$

a comparison between (29) and (30) shows that

$$V - V_{jd} < V - V_{sd}.$$  

Thus, **Junior has a greater incentive to cheat than Senior.** Intuitively speaking, Junior’s cheating has less impact on the team’s success; that is, his cheating does not decrease the expected payoff as much as cheating by Senior, but each agent can save the same disutility from cheating. Thus, cheating is more attractive to Junior.
For the remainder of this section I focus on the case in which both agents have the incentive to cheat. By the above discussion if Senior has the incentive to cheat, so does Junior. Thus, I posit that $V < V_{sd}$. Substituting from (18) into (30) and reversing the inequality yields, after some manipulation, this equivalent condition

$$\tag{31} (1 - \delta_p) e > p_j (1 - p_s)(1 - \delta) \delta \Pi.$$  

5.C. Deadlines

Now consider the effect of deadlines. As before, begin with the possibility of cooperating for just one period. Then, the payoff to agent $i$ is

$$U_i(1) = (1 - p_j p_s) \delta \Pi + \delta p_j p_s v_i - e.$$  

The payoff difference is

$$\tag{32} U_s(1) - U_j(1) = \delta p_j p_s (v_s - v_j) > 0$$  

where the inequality follows from (27): individual search yields a lower return to Junior. Thus, cooperation with a deadline set at $n = 1$ yields a greater payoff to Senior than Junior.

Consider next the payoff from cheating. Junior gains by cheating

$$U_{jd}(1) = (1 - p_s) \delta \Pi + \delta p_s v_s$$

so he has no intention to cheat if

$$\tag{33} U_j(1) - U_{jd}(1) = p_s (1 - p_j) \delta (\Pi - v_j) - e \geq 0.$$  

---

5 Implicitly, I also need $\Pi < \pi$. If this latter holds in reverse, then Senior would be willing to let Junior cheat and still share the fruit of success. However, it is difficult to imagine how the benefit of success increases from $\pi$ to $\Pi$ without the participation of Junior.
Similarly, Senior does not cheat if

\[ U_s(1) - U_{sd}(1) = p_j(1 - p_s)\delta(\Pi - v_s) - e \geq 0. \]  

By (27) \( v_j < v_s \), which implies that \( \Pi - v_j > \Pi - v_s \). However, \( p_s(1 - p_j) < p_j(1 - p_s) = p_s - p_j < 0 \). Therefore it is in general ambiguous which agent has a greater incentive to cooperate.

Substituting from (26) into (33) and (34), after some manipulation, shows that if

\[ \delta \Pi(1 - p_j)(1 - p_s)[1 - (p_j + p_s)\delta] > [1 - (p_j + p_s - p_j p_s)\delta]e, \]

then Senior has a greater incentive to cheat than Junior. \( e \) and \( \Pi \) must also satisfy (31). It can be shown that there is a range of \( e \) satisfying both (31) and (35); i.e.,

\[ \delta \Pi(1 - p_j)(1 - p_s)[1 - (p_j + p_s)\delta]/[1 - (p_j + p_s - p_j p_s)\delta] > e > \delta \Pi(p_j(1 - p_s)(1 - \delta))/(1 - \delta p_j).^6 \]

The next proposition summarizes the results obtained so far.

**Proposition 7**: Suppose there is no monitoring.

(A) Without deadlines, Junior has a greater incentive to cheat than Senior.

(B) With a deadline set at \( n = 1 \), if the condition (36) is satisfied, Senior has a greater incentive to cheat than Junior.

The reversal in incentives to cheat occurs because with a deadline in effect Junior has a lower continuation payoff \( v_j < v_s \), and hence faces a more severe now-or-never

---

^6 The sufficient condition for (25) to hold is \( 1 - p_j^2 \delta > 0 \), which holds for all relevant values of \( p_j \) and \( \delta \).
dilemma than Senior. Therefore, if $\Pi$ is large enough to satisfy the condition in (34) Junior has a greater incentive to cooperate than Senior. In contrast, if $e$ exceeds the left-hand side bound of the condition (36), Junior has a greater incentive to cheat than Senior.

Suppose that both agents cooperate during the first period; that is, $U_i(1) - U_{id}(1) > 0$ for both $i = j$. Then, I consider extending the deadline, as before. Suppose that agents can cooperate for the first $n - 1$ periods. If agents can extend cooperation by one more period; that is, the first $n$ periods, their payoff is

$$U_i(n) = (1 - p_j p_s) \delta \Pi + \delta p_j p_s U_i(n - 1) - e.$$ 

It is easy to show that $U_i(n)$ is monotone increasing. Further, the payoff difference between agents is given by

$$U_s(n) - U_j(n) = \delta p_j p_s (U_s(n - 1) - U_j(n - 1)).$$

This and (32) imply that Senior always has a greater payoff than Junior from cooperation now matter how long a deadline they face.

Junior will not cheat if

$$U_j(n) - U_{jd}(n) = p_s (1 - p_j) \delta (\Pi - U_j(n-1)) - e \geq 0$$

while Senior will not if

$$U_s(n) - U_{sd}(n) = p_j (1 - p_s) \delta (\Pi - U_s(n-1)) - e \geq 0.$$ 

Now letting $n$ approach infinity, the left-hand side of (37) and (38) approach

$$p_s (1 - p_j) d(\Pi - V) - e = V - V_{jd} < 0$$

and

$$p_j (1 - p_s) d(\Pi - V) - e = V - V_{sd} < 0.$$
respectively, where the inequalities follow from assumption 3. Thus, there is $n_i^*$ satisfying that

$$U_i(n) - U_{id}(n) \geq 0 > U_i(n + 1) - U_{id}(n + 1).$$

Let $n^* = \min \{n_s^*, n_j\}$. Then agents can cooperate to $n^*$ periods without monitoring.

The next proposition summarizes the discussion of this section.

**Proposition 8.** Assume no monitoring. Then

(A) Without a deadline, if Junior has an incentive to cheat, there is no cooperation.

(B) Suppose that agents can cooperate if there is a deadline set at $n = 1$. Then, there is a deadline set at $n^* \geq 1$ that induce cooperation without monitoring from period 1 to period $n \leq n^*$. $n^*$ is the optimal length of cooperation for both agents.

6. Cooperation in hunter society

6. A. Setup

In the preceding analysis “search” ended whenever an agent had success. In this section I consider the case in which success yields only transitory benefits as in Rousseau’s hunting game. To be specific, I assume that the benefits last only one period so that agents return to search the following period. Thus, when an agent goes it alone, the expected net benefit can be expressed as

$$v = -e + (1 - p)(\delta b + \delta^2 v) + p\delta v.$$

An agent expending effort $e$ in period $t$ succeeds and enjoys the benefit $b$ with probability $(1 - p)$ in period $t + 1$. However, he returns to search the following period $(t + 2)$, when
he faces the continuation payoff v. If his search fails, he also faces the continuation 

payoff v. Collecting terms, I obtain

\[ v = \frac{(1 - p)\delta b - e}{1 - \delta p - (1 - p)\delta^2}. \]

I again assume that search is worthwhile, or \( v \geq 0 \).

6. B. Cooperation

Now, turning to cooperation, I retain the assumption that there are two agents to 

keep the notation simple. Success yields only the transitory benefit B so the following 

relation holds for the expected payoff \( V \) per agent:

\[ V = -e + (1 - p^2)(\delta B + \delta^2 V) + p^2\delta V. \]

Collecting terms, I find that

\[ V = \frac{(1 - p^3)\delta B - e}{1 - \delta p^2 - (1 - p^3)\delta^2}. \]

It is assumed that \( V > v \).

Suppose that agents can monitor each other’s effort. Then, a deviation triggers the 

punishing phase of the equilibrium strategy the following period. Therefore, and cheating 

yields:

\[ V_d = (1 - p)(\delta B + \delta^2 v) + p\delta v. \]

As before, assume that there is no incentive to cheat with monitoring.

\[ V - V_d = -e + p(1 - p)\delta B + \delta V[(1 - p^2)\delta + p^2] - \delta v[(1 - p)\delta + p] \geq 0. \]
Turn next to the case where there is no monitoring. A deviation, which goes undetected and with impunity, then yields

\[ W_d = (1 - p)(\delta B + \delta^2 V) + p\delta V. \]

Cooperation is impossible to maintain if

\begin{equation}
V - W_d = -e + p(1 - p)\delta B - p(1 - p)(1 - \delta)\delta V < 0.
\end{equation}

It is easy to check that there is a range of \( e \) satisfying conditions (40) and (41) such that cooperation is maintained with monitoring but not without monitoring.

6. C. Deadlines

I now examine the role of deadlines in inducing cooperation without monitoring under the conditions of (40) and (41). With slight abuse of notation, I retain \( U(n) \) for the equilibrium payoff with the deadline \( n \), and likewise \( U_d(n) \) for the corresponding payoff from cheating in period 1. Then, for \( n = 1 \)

\[ U(1) = -e + (1 - p^2)(\delta B + \delta^2 v) + p^2 \delta v \]

and

\[ U_d(1) = (1 - p)(\delta B + \delta^2 v) + p\delta v. \]

Hence,

\begin{equation}
U(1) - U_d(1) = -e + p(1 - p)\delta[B - (1 - \delta)v].
\end{equation}

Notice that

\begin{align*}
(V - W_d) - (U(1) - U_d(1)) &= p(1 - p)\delta B + p\delta V[(p - 1)(1 - \delta)] - p(1 - p)\delta[B - (1 - \delta)v]
\end{align*}
\[ = p\delta V[(p - 1)(1 - \delta)] + p(1 - p)\delta(1 - \delta)v \]
\[ = p\delta(1 - p)(1 - \delta)(v - V) < 0. \]

Thus, there is a range of \( e \) such that the right-hand side of (42) is positive under assumption (41). For such \( e \), agents can cooperate with a deadline of \( n = 1 \) without monitoring.

If \( U(1) - U_d(1) > 0 \), I can try extension of the deadline. I show, in the appendix, that the incentive to cooperate diminishes monotonically after \( n \geq 4 \). Furthermore \( U(n) \) is monotone increasing. The main result, stated below, therefore closely resembles proposition 5 (the proof is in the appendix).

**Proposition 9:** Suppose that \( U(1) - U_d(1) > 0 \). Then there is the optimal deadline \( n^* \geq 1 \), which yields the maximum welfare \( U(n^*) \) per agent.

7. **Concluding remarks**

I study the repeated-game setting in which cooperation is impossible to maintain without monitoring. In such circumstances I show that setting a deadline can induce agents to cooperate. Furthermore, I show that there is the optimal deadline, which defines the maximum length of cooperation that can be maintained and also yields the greatest payoff to each agent.

The analysis here may illuminate the usefulness of deadlines for successful co-authorships and other real world entities such as pharmaceutical firms jointly trying to discover a cure for a new disease or more generally firms forming a research joint
venture. In reality, governments may help research firms set deadlines. For example, the Advanced Technology Program, a program within the U.S. Department of Commerce, requires an applicant to give the duration of the RJV on its application form.

In this paper I maintained several special assumptions to obtain clean results such as binary effort decisions and known and time-invariant probability of success. A natural extension is to make effort choice continuous. Another possible extension is to make probability of success dependent on the past efforts, i.e., learning from failures. Also, asymmetries can be introduced with regard to other aspects of the model besides probabilities of success, for example, the disutility of efforts, which can represent an agent’s opportunity cost. I leave these extensions for future research but since the mechanism by which deadlines induce cooperation is so intuitive that the main result of this paper remains valid in such extensions.
Appendix A: I construct an example for proposition 6. Choose parameter values in the model such that $B_0 = 1 + \epsilon$, where $\epsilon$ is an arbitrarily small positive number satisfying the condition

(1a) $\epsilon \leq \delta p^2 (1 - \delta p) v / [(1 - \delta p^2) V^p(1)]$

where $V^p(1) = dV^p(1)/dB < 0$ since $V^p(B)$ is monotone decreasing. Hence the right-hand side of (1a) is positive. Further, $V^p(1) > V^p(B_0)$ so the incentive compatibility constraint (20) is violated at $B = 1$. It follows that $B^* = 2$ is the solution to the GP problem (21).

Now I demonstrate that there is a integer $n$ such that

(2a) $V^p(B_0) > U(n) > V^p(2)$.

Namely, $n = 1$. Since $V^p(B)$ is decreasing and convex,

$V^p(B_0) > V^p(1) + \epsilon V^p(1)$

Therefore,

$V^p(B_0) - U(1) > V^p(1) - U(1) + \epsilon V^p(1)$.

I have

$V^p(1) = \{(1 - \delta p) v + \delta p (1 - \delta p^2) v\}/(1 - \delta p^2)$

and

$U(1) = (1 - \delta p^2) V + \delta p^2 v.$

Straightforward calculations show that

$V^p(1) - U(1)$

$= \{(\delta p - 1 + \delta p^2) (1 - \delta p^2) V + (1 - \delta p - \delta p^2 + \delta^2 p^4) v\}/(1 - \delta p^2)$.
Replacing \( V \) with \( v \) and collecting terms, I obtain

\[
V^P(1) - U(1) > \delta p^2(1 - \delta p)v/(1 - \delta p^2).
\]

Therefore,

\[
V^P(1) - U(1) + \epsilon V^P(1) > \delta p^2(1 - \delta p)v/(1 - \delta p^2) + \epsilon V^P(1) \geq 0
\]

by (1a). This proves that \( V^P(B_o) - U(1) > 0 \).

Turning to the other inequality, using definitions I obtain

\[
V^P(2) = (1 - \delta p)(1 + \delta p)v + (\delta p)^2 V^c(2)
\]

and

\[
V^c(2) = (1 - \delta p^2)V + \delta p^2 V^P(2).
\]

Solving these equations yields

\[
V^P(2) = \{[1 - (\delta p)^2]v + (\delta p)^2(1 - (\delta p^2)V)/(1 - \delta^3 p^5).
\]

Then straightforward computation, after manipulation, shows that

\[
V^P(2) - U(1) \geq \nu \delta p^2(3 - \delta^2 p^2 - \delta p^2 - \delta^3 p^5)/(1 - \delta^3 p^5) > 0.
\]

This proves (2a) for \( n = 1 \). Then the remainder of proposition 7 goes through showing that \( U(2) > V^P(B^*) \).

The proof can be visualized in figure 1, where \( V^P(B) \) is a continuous function. \( B_o = 1 + \epsilon \) is the optimum for the companion problem so at \( B = 1 \) the no-deviations constraint is violated. Furthermore, due to the integer constraint \( B^* = 2 \) is the optimal solution to the GP problem. Then, since \( \epsilon \) small enough to satisfy (25) the conditions in (61) holds as shown in the appendix, then \( U(2) = U(B^*) > V^c(B^*) \).
Appendix B: Proof of proposition 9

First define $\Delta U(n) = U(n) - U(n - 1)$, with $\Delta U(1) = U(1) - v > 0$. I prove two lemmas before presenting the proof of proposition 9.

Lemma B1: $U(n)$ is monotone increasing.

Proof. Given that $\Delta U(1) > 0$, calculation shows that

(1b) $\Delta U(2) = p^2 \delta \Delta U(1) > 0$

Now assume that $\Delta U(n - 2)$ and $\Delta U(n - 1)$ are both positive. Then

(2b) $\Delta U(n) = (1 - p^2) \delta^2 \Delta U(n - 2) + p^2 \delta \Delta U(n - 1)$

which is positive under the assumption. Thus, $U(n + 1) > U(n)$, for all $n \geq 1$.

Lemma B2. $\Delta U(n) > \Delta U(n - 1)$ for $n \geq 3$.

Proof. (1b) implies

(3b) $\Delta U(2) - \Delta U(1) = (p^2 \delta - 1) \Delta U(1) < 0$.

So, “marginal” utility is diminishing between $n = 1$ and $n = 2$. However, by (2b)

$\Delta U(3) = (1 - p^2) \delta^2 \Delta U(1) + p^2 \delta \Delta U(2) > 0$

and hence

(4b) $\Delta U(3) - \Delta U(2) = \delta^2 [1 - p^2 + p^4] \Delta U(1) > 0$.

Marginal utility is increasing between $n = 2$ and $n = 3$. For $n \geq 4$ by (2b)

(5b) $\Delta U(n) - \Delta U(n - 1) = (1 - p^2) \delta^2 (\Delta U(n - 2) - \Delta U(n - 3))$

$+ p^2 \delta (\Delta U(n - 1) - \Delta U(n - 2))$. 
This is a homogeneous second-degree linear difference equation with two initial values given in (3b) and (4b). Its characteristic equation has two real roots, each of which is less than one in absolute value. Thus, \(\Delta U(n) - \Delta U(n - 1)\) converges monotonically to zero. Since the second initial value \(\Delta U(3) - \Delta U(2) > 0\) by (4b), it follows that \(\Delta U(n) - \Delta U(n - 1) > 0\) for \(n \geq 3\).

**Proof of Proposition 9**

Suppose that \(U(1) - U_d(1) > 0\). Calculation shows that

\[
[U(2) - U_d(2)] - [U(1) - U_d(1)] \\
= - p(1 - p)\delta(U(1) - \delta v - (1 - \delta)v) = - p(1 - p)\delta\Delta U(1) < 0.
\]

If \(U(2) - U_d(2) > 0\), agents can cooperate at least for the first two periods. For \(n \geq 3\) since

\[
U(n) - U_d(n) = - e + p(1 - p)\delta(B + \delta U(n - 2) - U(n-1)),
\]

(6b) \[
(U(n) - U_d(n)) - (U(n - 1) - U_d(n - 1)) \\
= p(1 - p)\delta\{\delta\Delta U(n - 2) - \Delta U(n - 1)\}.
\]

Evaluating (6b) at \(n = 2\) and \(n = 3\), I obtain

\[
U(3) - U_d(3) - (U(2) - U_d(2)) = p(1 - p)\delta\{\delta\Delta U(1) - \Delta U(2)\}.
\]

The sign is indeterminate due to (3b). That is, it is possible that the incentive to cooperate can be even stronger at \(n = 3\) than at \(n = 2\). For \(n \geq 4\), \(\Delta U(n - 1) > \Delta U(n - 2)\) by lemma B2, which implies

\[
\Delta U(n - 1) > \delta\Delta U(n - 2),
\]
so that the right-hand side of (6b) is negative. This implies that the incentive to cooperate diminishes monotonically after \( n = 3 \). Thus proof is complete if I show that this incentive becomes negative at some \( n \geq 3 \). To do so, I use the definition of \( V \) to rewrite

\[
U(n) = V - (1 - p^2) \delta^2 V - p^2 \delta V + (1 - p^2) \delta^2 U(n - 2) + p^2 \delta U(n - 1).
\]

Similarly, by the definition of \( W_d \), I can rewrite \( U_d(n) \) as

\[
U_d(n) = W_d - (1 - p) \delta^2 V - p \delta V + (1 - p) \delta^2 U(n - 2) + p \delta U(n - 1).
\]

Taking the difference and collecting terms, I obtain

\[
(7b) \quad U(n) - U_d(n) = (V - W_d) + p(1 - p) \delta \{(1 - \delta) V - [U(n - 1) - \delta U(n - 2)]\}.
\]

Now, let \( n \) approach infinity and note that \( U(n) \) approaches \( V \). Then, the right-hand side of (7b) approaches \( V - W_d < 0 \).
References


Figure 1: $V^p(B)$