This paper proposes a theory of free movement of goods and labor between two large economies with imperfect labor contracts. Each country is incompletely specialized in producing two final goods that differ in their complexity of production. The most complex good is produced by workers and managers who pair up with each other according to an efficient matching process, where the most talented manager matches with the most talented worker. The least complex good is produced by firms that consist of one individual. The most talented individual is defined as the one with the highest level of optimal job training. The heart of our analysis lies in the determinants of talent development. We show that in a world economy with two otherwise similar countries that have different institutional quality, or/and a different system of early education, a country that has the best quality of institution, combined with the best early educational system, will be the host country of immigrants. Under free trade and labor, the best institutions and the best early educational system can serve as complementary sources of comparative advantage in the most complex industries. Consequently, the host country of immigrants will export the most complex goods produced by the most talented individuals. The economic progress of a source country will be shown to be related to its ability to improve its quality of institutions and its early educational system. It also is shown that individuals’ decisions to emigrate are related to the fixed costs of migration, such as language barriers. Finally, emigration affects the income of both countries via an indirect effect on individuals’ incentives to invest in their job training and a direct effect on prices of goods.

**JEL Classification:** B52, I21, F10, F16, F22, J24.

**Keywords:** Comparative advantage, Occupational Choice, Education, Institutions, Immigration, Moral hazard, Organization of production.
1. Introduction

In the modern world, a number of recent political developments have intensified the free movement of goods and labor. According to Hatton and Williamson (2005), average industrial tariffs rates around the world have fallen over the last half century from about 40 percent to 3 percent. Over the last thirty years the ratio of exports of goods and services to GDP has doubled. The proportion of the world’s population that are immigrants also has increased. The United Nations estimates that international migrants constituted 3 percent of the world population in 2005.¹ The tendency toward the world liberalization of goods and labor might affect the development of talent in each country.

We could think of talent as something that an individual develops. Individuals differ in their level of training. Along this line, the most talented individual might be defined as the one with the highest level of job training. In a world where national institutions mitigate moral hazard, contracts consist of wages that are derived from a matching process, where the most talented workers pair up with the most talented managers during the team production process. In this environment, some individuals might choose to incrementally increase their level of training and therefore be more productive in their jobs than others. But, who are these individuals? One might answer the above question by assuming that an individual who accumulates higher human capital, such as, a technical, undergraduate, master, PhD, or post doctorate degree, will choose to obtain higher level of training in her prospective job because it becomes relatively easier for her to do so, as compared to another individual who obtains a degree of a lower level but will work in the same prospective job as the former. But, what factors can impact an individual’s decision to go to college or to purse a higher degree? One can argue that the economic environment; her childhood environment such as the intellectual and financial support from her parents, relatives and friends; the culture in which she grew up; the system of primary, secondary and high school education all could be considered as factors that can push or pull an individual from pursuing a higher level of education, and therefore, from accumulating higher levels of human capital. In this paper, the above factors with the exception of the economic environment factor, determine what we call the early educational level of an individual. Consequently, individuals with higher levels of early education will tend to pursue a higher degree because is relatively easier for them to do so.

In a perfect labor contract’ world with competitive firms that operate in complete markets, there exists perfect information about the productivity of a worker. Therefore, the most productive firms will tend to employ the most productive workers who, according to the above information, are the most skilled individuals. Moreover, the most skilled individuals will seek employment in the most productive firms, or run their own firms in order to optimize their returns to education.

However, in the real world not all sectors operate in a perfect labor contract environment. This is related to the fact that in some sectors it might be very difficult for a manager of a firm to observe the productive efforts

¹“In 2005, the number of international migrants in the world reached almost 191 million, up from 155 million in 1990. The number of international migrants increased by 10 million from 1990 to 1995, going from 155 to 165 million. The estimated increase was close to 12 million from 1995 to 2000 and above 14 million from 200 to 2005.” For more details see United National: The International Migration Report (2006, p. 1).
of her employees that engage in a team project during the production process. Consequently, the manager has perfect information about the skill level of her employers but imperfect information about their productive efforts during the production process. Thus, a manager will tend to offer an optimal contract where workers’ wages depend on the workers’ skill levels and on the quality of national institutions because these are the only things that she can observe perfectly. In other words, it is difficult to base the contract directly on the firm’s output and workers’ effort levels because it may be difficult for a court to measure the above due to the assumption of imperfect information on the worker’s productive effort levels. However, the firm’s manager is able to measure the level of a worker’s performance. The latter is partially related to her efforts. In the same way, the worker may be able to measure the performance of the firm that is related to a degree to its output levels. As a result, because of the imperfect monitoring, the degree of labor contract perfectibility could be proxied by the degree of the quality of the institutions. With imperfect contracts, the higher the quality of institutions, the higher the effort levels exerted by a worker in the team production process. This is because the higher the quality levels of institutions in a country, the higher the verifiability of distortion levels of a worker and therefore the higher the levels of effort exerted by the same worker. Better national institutions provide higher quality of the performance and verifiability measures. The quality of national institutions can be related to the quality of the national judicial system.\(^2\) The better a country’s legal establishments, the more precisely courts can assign credit for each individual contribution to the team production process of a firm operating in this country. In such economic environment, where some sectors operate under perfect labor contract, while some other sectors operate under imperfect labor contract, one can raise the following question. In what sectors will the most skilled individuals of a country seek employment?

We take the stand developed in the recent literature on institutions and international trade and assume that the answer to this question is related to the quality of national institutions. In other words, institutions’ quality affects the productivity only in those sectors where firms are unable to measure precisely the productive efforts of each individual involved in their production process. Consequently, the quality of institutions in a country will affect an individual’s decision about the industry in which she will seek employment, and the early educational system will affect an individual’s decision about her skills’ level. All things considered, the development of talent will depend on the quality of a country’s early educational system and its institutions. Since countries differ in their institutions and early educational system, they will differ in their skills’ distribution of their labor force. Put differently, countries differ in their talent development because they obtain different quality level of their institutions and early educational systems.

One of the objectives of this paper is to provide a theoretical understanding of the relationship between a country’s early educational system and its institutions on the one hand and the development of talent on the other. By focusing on the economic function of early educational systems and institutions, our theory offers an

\(^2\) According to Vogel (2007) the quality of national institutions can be related not only to the national judicial system but also to the national accounting system. He argues that the more effective national accounting system, the better the reports of data on both the productivity of a firm and the contribution of each individual involved in its production process.
explanation on the distribution of skills in the labor force of a country. Another objective of this paper is to analyze the consequences of the endogenous talent development: first in a free trade world, such as NAFTA, and second in a common labor market world, such as European Union. To this end, we develop a theory that links the development of talent, as a consequence of the early educational system and institutions with its impact on the organization of production.

This paper develops a framework with imperfect labor markets in a world with free movement of goods and labor, which consists of two large economies. In each country there exist a large number of firms grouped into two sectors, an agriculture sector, where only firms that consist of one individual each operate, and an industrial sector, where only firms that are involved in team production operate. There are two final goods produced (one good in each sector) using one factor of production, labor, that is heterogeneous in terms of skills. The heart of our study lies in the determinants of the distribution of skill in the labor force of each country. The latter is determined endogenously by individuals of each country. In a world with imperfect labor contracts, the productive efforts of the workers involved in the production of the industrial good cannot be measured perfectly. Thus, each individual involved in the production process of the industrial good has perfect information about her own level of productive efforts, but imperfect information about the levels of productive efforts of others. Individuals choose their level of skill subject to their early level of education and the quality of institutions that exist in their country. Individuals with high levels of skill choose high levels of job training in order to maximize their utilities. These types of individuals seek employment in the industrial sector. This is related to the fact that it is easier for high skilled workers to exert high effort levels in the team production process since they have accumulated higher level of human capital. Consequently, is more effective for them to shirk less in the team production process. An efficient matching process takes place, within each country, where the most talented individuals pair up with the most talented managers. The most talented individuals enter in the industrial sector either working as workers in team production firms, or running their own firms as managers. The least talented individuals enter in the agricultural sector, where they work as self-employed, operating their own individualistic firms. We examine the implications of international trade when both countries move from autarky to a free trade world. Then, we go a step further and examine the pattern of emigration when both countries open up their labor markets between themselves in an already free trade world.

We show that in a world with two open economies under free trade and incomplete specialization, the country with relatively higher levels of early education or quality of institutions, ceteris paribus, will have relatively more talented individuals, and therefore, will export the industrial good and will import the agricultural good.

In a world with free movement of goods and labor, we show that only the most talented individuals have an incentive to emigrate towards the country with the best institutions and early educational systems. This is related to the fact that if they are able to afford the fixed costs of emigration, such as language barriers, they capture higher income to their level of skill in the host country of immigrants, as compared to their income in their country of origin.
We consider certain scenarios, where the government of an origin country of immigrants could reinforce the incentives of its citizens to accumulate skills through its ability to improve the quality of its institutions and early educational system. In particular, we describe a scenario where the government of the origin country can promote the development of more talented individuals in the world and simultaneously increase the income of most of its citizens by simply improving the quality of its early educational system. The latter will increase the intensity of talent development in the world because of emigration towards the country with the best institutions. This in turn, causes a raise in the relative price of the agriculture good, therefore increasing the income of all individuals who work in the agriculture sector. Since the host country is exporting the agriculture good, most of its labor force will enjoy higher income than before as a result of the emigration of its most talented individuals to the host country of immigrants. However, in such a scenario the government of the origin country of immigrants will fail to achieve its goal if we relax the assumption of large economies. Thus, in the case of small open economies, since the price of each good will not be affected by trade or immigration, the improvement of the early educational system in the origin country of immigrants will raise only the volume of its emigrants for sufficient low immigration costs and it will not affect the income of the individuals who work in the agriculture sector. Consequently, the only way for the government of the origin country of immigrants to ameliorate the income of its citizens is to encourage the development of its institutions. Therefore, emigration influences the individuals’ income via an indirect effect on their incentives to invest in their level of skills and a direct effect on the goods’ prices only under the assumption of large economies.

Our paper is original in four key dimensions. It provides two separate contributions to the burgeoning literature of international trade through the involvement of institutions and endogenously determined human capital accumulation. Also, it makes two equally distinct contributions to the recent literature on immigration and economic development through the involvement of the quality of a country’s early educational system and its institutions.

First, our paper contributes to the recent and burgeoning literature on institutions and international trade. The paper argues that the quality of institutions acts as an independent source of comparative advantage in a country, and therefore, determines the pattern of trade. This is related to the fact that institutions affect more the productivity in certain sectors of the economy than in others. This is consistent with Acemoglu, Johnson and Robinson (2001, 2002), Acemoglu, Antras, Helpman (2006), Costinot (2009), Cunat and Melitz (2006), Grossman (2004), Levchenko (2007), Matsuyama (2005), Nunn (2007), and Vogel (2007). In this context, I follow the steps of Vogel (2007) by developing a simple theoretical game in which each individual chooses her skill level, sector of employment, training level, matching co-worker, and level of efforts and distortions. However, my model differs from Vogel (2007) in terms of the endogeneity of individuals’ skill level. In my framework an individual chooses her skill level, subject to the quality of institutions and her level of early education, prior to her choice of the sector where she will seek employment. In Vogel’s model an individual’s skill level is considered exogenous. Consequently, the endogeneity of an individual’s skill level that depends on the interaction of institutions and the early educational system makes this paper unique in the literature.
Second, it contributes to the latest and increasing literature on international trade and allocation of talent. Human capital accumulation and institutions act as complementary sources in the determination of organization of production. Thus, a country with better institutions and a labor force that consists of more talented individuals has a comparative advantage in the production of the more complex goods. This idea is similar to Costinot (2009), Grossman and Maggi (2000), Ohnsorge and Trefler (2007), Lucas (1978), Rosen (1981), and Murphy et al. (1991). Our paper differs considerably from the above papers, mainly in the definition of talent. In our paper talent is defined as something that an individual develops through the interaction of her early level of education and the quality of her country’s institutions. Thus, a distinct contribution that this paper offers to the literature is the ability of our model to make the early educational system of a country the sole determinant of the pattern of trade.

Third, the paper contributes to the growing literature on economics of immigration. It argues that institutions and early educational systems can determine the pattern of labor migration. It also shows that only the most talented individuals have an incentive to emigrate to the country with the best quality of institutions and early educational system. In other words, only skilled individuals have incentive to emigrate and therefore afford the emigration costs due to the existence of high differences between their earning in their country of origin and their destination country. The latter is consistent with Abowd and Freeman (1991), Blanchard and Katz (1992), Borjas (1987, 1992, and 1993), Freeman (1993), Jensen (1988), and Lucas (1988). Our paper differs notably in terms of the mechanism through which the incentives of individuals to emigrate are determined. In particular, our paper is the first paper to shed light on two separate channels, the institutions and the early educational system, that can determine the pattern of emigration between two otherwise similar countries.

Fourth, it contributes to the recent literature on human capital accumulation, immigration, and economic development. Our paper shows that the volume of talented individuals increases when countries that differ in the qualities of their early educational systems and institutions move to a world with free labor mobility. This is somewhat related to the literature on immigration and beneficial and non-beneficial brain drain as developed in Bhagwati et al. (1974), Beine et al. (2001, 2008), Di Maria et al. (2009), Galor et al. (1997), Miyagiwa (1991), Mountford (1997), Stark et al. (1997, 1998), and Vidal (1998). The papers that study the theory of brain drain argue that the emigration of skilled workers hurts the origin country of immigrants and promotes the host country of immigrants because the existence of immigration increases the volume of skilled workers in the host country and decreases their volume in the origin country of immigrants. On the other hand, the papers that develop the theory of beneficial brain drain argue that the increase in the possibility of emigration increases the volume of skilled workers in the origin country of immigrants because it increases the stock of human capital there. Our paper is different from the above papers on the literature of brain drain since it

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3 The model developed in this paper assumes that the wages of unskilled workers (the less talented individuals that seek employment in the agriculture sector) are the same in all countries (developed or developing countries). Thus, there are no economic incentives for the unskilled workers of a developing country to immigrate to the developed country.
provides another mechanism through which the incentives of individuals on the development of their talent increase with the existence of a common labor market. Moreover, it provides a scenario where the government of the origin country of immigrants can increase the human capital accumulation in its country simply by improving the early educational system or/and its quality of institutions in a common labor market world. This also is very different with the literature of beneficial brain drain in the sense that the existence of free movement of labor could improve the capital accumulation in the host country of immigrants, not simply the possibility of emigration.

The rest of the paper is organized into six sections. We present the model in Section 2, where we analyze an individual’s decision on her skill accumulation, and therefore, on her choice of the sector where she will seek employment. Here, we describe a five-stage theoretical game in a two sector economy that produces two goods, where labor is considered heterogeneous and is the only input. In Section 3, we analyze an individual’s decision on her level of skills’ accumulation in a world of perfect labor contracts and competitive firms that operate in complete markets. In Section 4, we solve the five-stage theoretical game for a symmetric subgame perfect equilibrium in a closed economy with imperfect labor contracts. We divide this section into two main subsections that analyze the same game, but under different contract design structures. Section 5 investigates the pattern of trade when two large economies enter into a free trade agreement. Section 6 explores the pattern of emigration and its effects on individuals’ income, when both countries that already enjoy free trade of goods enter into a free common labor market. Section 7 presents conclusions. All the proofs of propositions and corollaries are relegated to the appendixes.

2. The Model

The economy has two sectors (X and Y). In the y sector, individuals work alone. They own their own firm where they produce a final good. Therefore each individual in the y sector will provide only productive efforts in order to maximize her profits. For convenience, let me call this sector “the agriculture sector.”

In the x sector, production of the final good is determined as a result of a team work of a worker and a manager. Managers are matched with workers in order to produce the final good. In this case, managers (or firm owners) have no incentive to shirk since their final objective is to maximize the firm’s profit. Thus, they will only provide productive efforts in order to increase the firm’s productivity. On the other hand, workers might have an incentive to shirk, since they care about their wage and not the firm’s profits. Therefore, if a manager is able to measure perfectly a worker’s efforts, then the worker will not have any incentive to shirk. Thus, she will provide only productive efforts. However, if the manager is unable to identify perfectly the worker’s efforts, then the worker might have an incentive to shirk. In this case, the worker might choose to put forth some productive as well as some unproductive efforts. The amount of productive and unproductive effort

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4 The term talent’ development, introduced in this paper, essentially is the same as the term human capital accumulation, which is used in the literature on the brain drain.
will depend on the wage that the worker receives, but also on the degree of imperfectability of the labor contracts. We assume that the later is related to the country’s institutions. The better the institutions in a country, the more perfect the labor contract market, the lower the amount of unproductive efforts provided by workers. The amount of unproductive efforts (as shown later in the paper) also will be related to the level of a worker’s ability. This will be related to the level of skill that an individual has, which in turn is related to the amount of training that an individual has obtained during her lifetime. The more talented, and therefore trained, a worker is, the less the amount of unproductive efforts that she can provide for the firm. The latter intuition is related to the relatively easiness of the more talented and trained individual to put forth productive efforts. We refer to the \( x \) sector as the “industry sector”. Also, let me refer to the productive efforts as simply “efforts” and the unproductive efforts as “distortions.”

![Fig. 1. The Five-Stage Game](image)

We describe an individual’s decision on whether to work in the agriculture sector or in the industrial sector with a five-stage game. The timing of such a decision is illustrated in fig. 1. After an individual chooses her level of early education, she determines the sector in which she will look for employment. After determining the sector of employment, she decides on how much education (or sector specific training) to achieve. After the implementation of her training, she must select the production team (if any, dependent on the sector of employment choice) that is matched with her training. At the last stage, after she gets the job, she chooses how much effort and how much distortion to supply in her work. The above five-stage game is solved for a symmetric, subgame-perfect equilibrium. Using this method, we first find the equilibrium effort and distortion levels. Second, we determine the equilibrium level of training. Third, we establish the equilibrium matching of the production teams subject to their skill levels. Fourth, we find the equilibrium level of skill that makes an individual indifferent to the choice of the sector. Finally, we determine the equilibrium level of early education.

The utility of an agent, which consumes \( C_i \) units of the final good \( i \), with ability (skill level) \( q \), and with observed and verified sector-specific training \( t \), which supplies an amount of effort equal to \( a \) and an amount of distortion equal to \( d \), is given by:

\[
U = u(C_x, C_y) - \frac{1}{2q} (a^2 + d^2) - \frac{1}{2q} t^2
\]  

(1)

All individuals have identical and homothetic preferences represented by the subutility function \( u \). Let \( u(C_x, C_y) = C_y^\beta C_x^{1-\beta} \) and the income of an individual be \( I = x + py \), where the price of good \( x \) is considered
as numeraire and therefore the relative price of \( y \) is denoted by \( p \). Each individual maximizes her utility function subject to her income level. Hence, the indirect utility function is:

\[
V(a,d) = RL - \frac{1}{2t} (a^2 + d^2) - \frac{1}{2q} t^2
\]

where \( R \equiv \beta^\theta (1 - \beta)^{1-\beta} p^{-\beta} \). There are many competitive firms in each sector. Each firm in the agriculture sector is characterized by the same individual production process, exerting \((a)\) effort. On the other hand, each firm in the industrial sector is characterized by the same production process, where the manager exerts \( a_m \) effort and \( d_m \) distortion, while the worker exerts \( a_w \) effort and \( d_w \) distortion. The production functions in both sectors exhibit constant returns to scale in regards to effort. More specifically, the production functions in both sectors are:

\[
y(a) = a \quad \quad \quad x(a_w, a_m) = 2\sqrt{a_w a_m}
\]

Thus, in the agriculture sector, an individual who provides one unit of effort \((a)\) gets in return \((a)\) units of final good \( y \). In the industry sector, the final good is produced as a result of matched efforts and distortions of a manager and a worker. In this sector, the firm’s output depends on the effort levels of both manager and worker matched together in order to produce one unit of final good \( x \). Here, we follow the assumption of complementarities in the production (as indicated later by the supermodularity condition of the indirect utility function), meaning that it is more efficient for a manager to match with a worker with the same level of training in order to produce the final good \( x \). Therefore a firm, which consists of a manager who exerts one unit of effort \((a_m)\) and one unit of distortion \((d_m)\) matched with a worker who exerts one unit of effort \((a_w)\) and one unit of distortion \((d_w)\), will produce \(2\sqrt{a_w a_m}\) units of \( x \).

In the fifth stage, each individual inelastically provides one unit of labor. In the agriculture sector, each individual is a firm owner. Thus, she provides the amount of effort that maximizes firm’s profit. In the industrial sector, we have to introduce the contract imperfection market due to imperfect monitoring. Thus, the managers must pay the workers their income earned from their engagement in the production process. The managers will exert the amount of effort that maximizes the firm’s profit while the workers will exert the amount of effort that maximizes their income defined in the contract as follows:

\[
\begin{align*}
K(\theta, q_m, a_w, d_w) &\equiv a_w + \beta \left( e^{1-\theta} - 1 \right) d_w, \quad \text{for } \beta \in [1, q_m) \\
K(\theta, a_w, d_w) &\equiv a_w + \left( e^{1-\theta} - 1 \right) d_w, \quad \text{for } \beta = q_m
\end{align*}
\]

where \( q_m \) denotes the skill level obtained by the manager, \( a_w \) and \( d_w \) respectively denote the amount of unobservable effort and distortion of the worker paired with the manager of a firm, and finally \( \beta \) denotes the degree of dependence of the manager’s skill level on the design of the contract. Therefore, since \( \theta \) denotes the institutional quality of a country, the higher the institutional quality, the better the manager’s monitoring of the worker’s distortion levels and therefore, the higher the worker’s effort. Hence, \( \theta \in [0,1] \). In other words, the ability of the manager to observe the amount of distortions that the worker exerts in the production function is
proxied by the institutional quality of a country. It should be obvious that the closer to one the parameter $\beta$ gets, the more important is the manager’s skill level on the contract design system. Finally, in this model, when the labor contracts are perfectly observed, the country’s institutions also must be perfect. This coincides with $\theta = 1$, which means that in a world with perfect labor contracts we have: $K \equiv a_w$. Therefore, the income of the workers will be translated perfectly from their effort levels in the production process.

In the fourth stage, in the agriculture sector, each individual provides the amount of job training that maximizes the firm’s profit. In the industrial sector, a worker decides on her optimal level of job training that she is going to obtain, subject to her wage, which is assumed to be her entire income. The wage is determined in the next stage from the efficient matching system between a worker and a manager. On the other hand, a manager chooses her optimal level of job training that she is going to obtain, in order to maximize the firm’s profit, which represents her only income. This is shown to be related to the matching system as described in the next stage.

In the third stage (in the industrial sector), production teams are paired up between a manager and a worker following the assumption of the complementary in production. The managers offer the contract to the workers as defined above. The managers can observe perfectly and verify the worker’s skill level but, in the imperfect labor contract case, are unable to perfectly observe the amount of training and effort that this worker will put into the production process. Also, workers accept the contract after observing the manager’s skill level. In the equilibrium with imperfect contracts, workers and managers sort themselves subject to their skill levels. Therefore, in equilibrium contracts also are defined from this type of costless matching between workers and managers conditional on their skill levels.

In the third stage, we find the symmetric equilibrium training after the contracts are determined in the previous stage. The more skilled managers will be matched with the more skilled workers. Moreover, the more skilled individuals obtain more training since it is relatively easier for them to do so; therefore, they exert more productive efforts.

In the second stage, we determine the distribution of the labor force in a country subject to individuals’ skill level. Put differently, here we construct the labor force of a country, subject to individuals’ decisions to join one sector in a two sector economy. The individuals can join the agriculture sector, where they create firms operated by one individual, or they can join the industrial sector by becoming either a manager or a worker. The individuals cannot be employed in both sectors at the same time. Thus, the labor force consists of a continuum of individuals indexed by their skill levels denoted by $q$. The unskilled individuals will become farmers and obtain less training, while the skilled individuals will create firms (that produce in teams of two), by being a manager or a worker of a firm. Thus, each firm in the industrial sector consists of two individuals who are self-paired with respect to their training levels.

Finally, in the first stage, we endogenize the skill level of individuals. We assume that the latter depends on the early education level, or the early childhood, or the economic environment, or the general culture of individuals in a country, or on all of the above denoted by $\gamma$. Also, the skill level of individuals depends on the
institutional quality of their country even before an individual determines the industry, where she will seek employment. In this stage, we find the equilibrium level of $\gamma$, which is defined as $\gamma \in [\gamma_{min}, \gamma_{max}]$. The higher the early education obtained by an individual, the greater her skill level and therefore the easier it is for her to obtain higher levels of training. Thus, a country that has a better system of early education, ceteris paribus, will have a labor force that consists of more skilled individuals. They might work in the industry sector, but this depends on the imperfectability of the labor contract market. Therefore, this country has a comparative advantage in the industry sector. In the same way, the country with better institutions, ceteris paribus, will consist of more individuals who will choose to obtain more training and self-select in the matching process of production in the industry sector. Hence, the country with better institutions has the comparative advantage in the industry sector.

3. Perfect Labor Contracts

In this section, we describe the equilibrium that must come out in a world of perfect labor contracts and competitive firms that operate in complete markets. This is a typical Walrasian equilibrium in a perfect information world that consists of one country with two sectors. The workers and managers effort and distortion levels are observable perfectly and verifiable in each industry. This corresponds to the case where $\theta = 1$, which implies perfect quality of institutions. Since all the individuals have perfect information of each other skill levels, the distortion levels of each individual will be equal to zero independent of the industry, or the position level that an individual has. Therefore, the contract that a manager assigns and a worker accepts is related only to the effort levels that the latter will provide during the production process.

Let us first look at the $y$ sector, where by using backward induction, we can determine the equilibrium level of $\gamma$. An individual operating in the agriculture sector endowed with $t$ units of training and with skill level $q$ has homothetic preferences generated by the indirect utility: $V_\gamma(a, t, q) = Rpa - \frac{1}{2t}(a^2 + d^2) - \frac{1}{2q}t^2$. So, in the fifth stage we find the optimal level of efforts exerted by individuals working in the agriculture sector subject to their training and skill levels (which in turn also depend on the early education levels as will be shown in the stage one). Each individual’s distortion levels, in this sector, are always equal to zero $d=0$, since each individual is operating her own firm. Her optimal effort levels are $a = Rpt$. This indicates that the optimal levels of effort in this sector are a monotonically increasing function of the levels of training. Thus, the indirect utility function with optimal efforts in the agriculture sector is given by:

$$V_\gamma(t, q) = \frac{1}{2}(Rp)^2t - \frac{1}{2q}t^2 \tag{5}$$

In the fourth stage, we determine the optimal level of efforts exerted form individuals working in the agriculture sector subject to their skill levels. This is achieved by maximizing the training level in the indirect utility of equation 5. Hence, optimal training levels of an individual working in the agriculture sector are $t = \frac{1}{2}(Rp)^2q$. The optimal levels of training of an individual working in the $y$ sector are a linearly increasing
function of her skill levels. Thus, the indirect utility function with optimal training of an individual working in the agriculture sector is given by:

$$V_q(q) = \frac{1}{\theta} (Rp)^{4}q$$

(6)

We skip the third stage in the agriculture sector since there is no team production in this sector. In order to analyze the second and the first stage of the $γ$ sector, we first should find the optimal levels of distortion, effort, training, and skill levels in the industrial sector.

Let’s assume that a manager endowed with $t_m$ units of training pairs up with a worker endowed with $t_w$ units of training in order to form a firm in the industrial sector. They will produce an efficient outcome if they choose their effort and distortion levels in order to maximize the sum of their indirect utilities: $V_x(a_i, d_i, t_i, q_i) = Rl_x \left( a_i^2 + d_i^2 \right) - \frac{1}{2t_m} (a_m^2 + d_m^2) - \frac{1}{2t_w} (a_w^2 + d_w^2) - \frac{t_m^2}{2q_m} - \frac{t_w^2}{2q_w}$. We start again in the fifth stage, where we find the optimal level of efforts exerted by individuals working in the industrial sector subject to their training and skill levels. The indirect utility of a worker is $V_w(a_w, d_w, t_w, q_w) = Rl_w - \frac{1}{2t_w} (a_w^2 + d_w^2) - \frac{t_w^2}{2q_w}$. The indirect utility of a manager is $V_m(a_m, d_m, t_m, q_m) = Rl_m - \frac{1}{2t_m} (a_m^2 + d_m^2) - \frac{t_m^2}{2q_m}$.

The income of a worker is $I_w = wK$, where $w$ represents the wage of a worker, which is derived from the matching process, and $K$ represents the contract. In this section, we are describing the case of a world with perfect labor contracts ($\theta = 1$). Therefore, both equations 4A and 4B now can be equal to a simpler contract design $K = a_w$ since $\theta = 1$. Hence, the income of a worker is $I_w = wa_w$. Substituting this in the indirect utility of a worker, we can find her optimal effort and distortion levels, which are $a_w = Rwt_w$ and $d_w = 0$. The optimal distortion levels for the worker are equal to zero since her efforts are perfectly observed and verified from the manager. The optimal effort levels of a worker are strictly increasing in her wage and in her training levels. Consequently, the indirect utility of a worker with optimal effort and distortion levels is $V_w(w, t_w, q_w) = \frac{1}{2} (Rw)^2 t_w - \frac{t_w^2}{2q_w}$.

The income of a manager is equal to the profits ($\pi$) of the firm that she is operating with one worker. The profit function is represented by the simple equation $\pi = x - w a_w = 2\sqrt{a_w a_m} - wa_w$. Substituting this in the indirect utility of a manager, we can find her optimal effort and distortion levels, which are $a_m = Rwt_m t_w^{\frac{1}{2}}$ and $d_m = 0$. It should be obvious that the manager will exert zero distortion levels since she wants to maximize her profits. The manager’s optimal efforts levels are strictly increasing in her training levels, worker’s training levels and wage. Consequently, the indirect utility of a manager with optimal effort and distortion levels is $V_m(w, t_i, q_m) = R^2 w^2 t_m^{\frac{1}{3}} t_w^{\frac{2}{3}} - R^2 w^2 t_w - \frac{t_m^2}{2q_m}$.

In the fourth stage, we find the optimal level of training obtained from individuals working in the industrial sector subject to their skill levels. Hence, using the indirect utility of a worker with optimal training,
we can establish the optimal training level of a worker that is \( t_w = \frac{1}{2} (Rw)^2 q_w \). The optimal level of training for a worker is strictly increasing in her wage and her skill level. In a similar way, using the indirect utility of a manager with optimal training, we can find that the optimal training level of a manager is \( t_m = \frac{1}{2} R^2 w^2 a^2_m q_w^2 \).

The optimal level of training for a manager is increasing in the wage, her skill level and workers skill level.

In the third stage, the optimal production team is chosen by the managers and the workers. Here, the workers and the managers attempt to match together to form a production team. The manager supplies the optimal efficient wage to the worker and then the worker decides on accepting the efficient wage subject to her and her manager’s skill level. A manager maximizes her profits subject to her optimal levels of effort and training, worker’s efficient wage, and optimal levels of effort and training. More specifically, the manager is maximizing her profits on her post skill indirect utility \( \Lambda_m = R\pi - \frac{a_m^2}{2t_m} - \frac{t_m^2}{2q_m} \) by designing an efficient wage that corresponds to her optimal levels of skills. On the other hand, a worker attempts to maximize her income levels by accepting an efficient wage that corresponds to her optimal level of skill. Thus, the worker is optimizing her income on her optimal post skill indirect utility \( \Lambda_w = Ra_w w - \frac{a_w^2}{2t_w} - \frac{t_w^2}{2q_w} \). Let’s denote with \( \Lambda \) the total post skilled level utility derived from matching a manager with skill level \( q_m \) with a worker with skill level \( q_w \). This is described in the following equation:

\[
\Lambda(a, d, t) = \frac{7}{6} R\sqrt{a_w a_m} - \frac{1}{2t_m} (a^2_m + d^2_m) - \frac{1}{2t_w} (a^2_w + d^2_w). 
\]

Substituting the optimal efforts, distortions, and training levels in the above equation can be written as:

\[
\Lambda(q, w) = \frac{5}{24} R^4 w^2 q_w^2 q_m^2 - \frac{(Rw)^4}{8} q_w. 
\]

Using this equation we can determine the equilibrium, efficient wage, which is \( w = \left( \frac{q_m}{q_w} \right)^\theta \) for each unit of effort exerted from the manager and the worker. Therefore, the aggregate post utility level with efficient wage can be written as:

\[
\Lambda(q_i) = \frac{1}{12} R^4 \sqrt{q_m q_w} 
\]

Therefore, each member of the team is maximizing (7) by choosing the right partner, where the worker’s and manager’s levels of effort and training are known perfectly and verified in this labor market. Hence, it is optimal for each manager to match with a worker who obtains the same skill level \( q_m = q_w \). This result is efficient because it satisfies the assumption of complementary in the production process. This assumption is related to the property of the supermodularity of the post training indirect utility described in (7).

Mathematically, the property of supermodularity is satisfied since \( \frac{\partial^2 \Lambda}{\partial q_m \partial q_w} > 0 \forall q_m, q_w > 0 \).

In the third stage, managers and workers choose the optimal amount of training. We demonstrated in the above paragraph that a manager will be paired with a worker who obtains the same skill level. Thus, the indirect utility of an individual with optimal levels of effort and training and efficient wage is given by:
Now, we are ready to proceed with the solution of the second stage, where we can find the optimal choice of individuals’ decisions on the selection of the industry where they will seek employment. We assumed that in a country there are two sectors. Therefore in a closed economy, a country is producing two types of final goods (the industrial good and the agriculture good). Comparing (6) to (8) it is easy to conclude that the indirect utility of an individual working in each sector is equal to zero where the skill level of individuals is equal to zero. Also, the indirect utility of (6) and (8) are a linear function of the skill levels with which an individual is endowed. Consequently, the indirect utility of an individual working in the agriculture sector must be equal to the indirect utility of an individual working in the industrial sector in a closed economy with two sectors. This indicates that \( V_x(q) = V_x(q) = V(q) \) or \( V(q) = \frac{1}{8} (Rp)^4 q = \frac{1}{8} R^4 q \Rightarrow p = 1 \). Thus, in autarky, the relative price of the agriculture good is equal to its return on technology, which is equal to unity by the design of the agriculture production function. So, each worker is indifferent as to what sector in which she will seek employment. Moreover, in the industrial sector the owner of the firm is not necessarily the manager of that firm. The worker of the firm also may be the owner of the firm; or the firm can be owned by some other third party who pays an efficient wage to the manager and the worker of the firm. This result is related closely to the assumption of perfectibility of the labor market. Clearly, when the product market clears, the interaction of preferences and technologies establish the distribution of skill levels to each sector. Under these circumstances the engagement of a country to international trade follows the comparative advantage theory according to a simple Ricardian model for two countries that exhibit the same technologies. Therefore, the autarky relative price of both countries that share the same production technologies in the agriculture sector is identical and is not determined by the allocation of the skill levels within each country. As a result these countries will not engage in international trade, but will produce the same amount of output in each sector as they did in autarky. The perfect labor contract model gives the same results as the Grossman’s (2004) model and Vogel’s (2007) perfect institution model.

What if we endogenize the skill level of individuals? We assume that the skill level depends on the early education levels \((\gamma)\) and on the degree of perfectibility of institutions \((\theta)\) of a country. Thus, in this stage, we find the equilibrium level of \(\gamma\), which is defined as \(\gamma \in [\gamma_{min}, \gamma_{max}] \forall \gamma > 0\). When \(p = 1\), the indirect utility of an individual working in the industrial sector with optimal levels of training is \( V_x(q) = \frac{1}{8} R^4 q_x \). After endogenizing the skill levels as described above, the indirect utility of an individual working in the industrial sector, with optimal training levels, can now be written as:

\[
V_x(q, \gamma) = \frac{1}{8} R^4 q_x - \frac{1}{3\gamma} q_x^3
\]  

(9)

Individuals of a country before making the decision on the selection of the sector in which they will seek employment, choose the amount of skill level they want to achieve, subject to their early education levels. Hence, all individuals working in the industrial sector maximize \( V_x(q, \gamma) \) over their choice of \( q_x \). The optimal
level of skill as a function of early education is \( q_x = \frac{1}{2\sqrt{2}} R^2 \sqrt{y} \). This shows that the optimal skill level of all individual in a country is a concave function of their early education levels. Consequently, the higher the early education obtained by an individual, to a certain degree, the greater her skill level and in turn the easier it is for her to obtain a higher level of training. Thus, a country that has higher levels of early education, before reaching the maximum level of early education, will have a labor force allocated with more skilled individuals.

Substituting the optimal skills in the indirect utility of equation 9, we obtain the indirect utility of an individual, working in the industrial level, with optimal skills. This can be written as:

\[
V_x(y) = \frac{1}{3 \cdot 2^{7/2}} R^6 \sqrt{y}
\]

(10)

The indirect utility of an individual working in the agriculture sector, with optimal training levels, now can be written as:

\[
V_y(q, y) = \frac{1}{8} (Rp)^4 q_y - \frac{1}{3y} q_y^3
\]

(11)

In a similar way to the industrial sector, individuals in the agriculture sector maximize equation 10 over their choice of skill level. Thus, the optimal level of skills as a function of early education is \( q_y = \frac{1}{2\sqrt{2}} (Rp)^2 \sqrt{y} \). Therefore, analogous with the industrial sector case, in the agriculture sector, the higher the early education (before reaching its maximum level) obtained by an individual, the greater her skill level. The higher the relative price of the agriculture good, the higher the optimal level of skills obtained by an individual working in the \( y \) sector. Substituting the optimal skills of an individual working in the \( x \) sector into the \( V_y(q, y) \) of equation 11, we can write the indirect utility of an individual, working in the agriculture sector, with optimal skills as:

\[
V_y(y) = \frac{1}{3 \cdot 2^{7/2}} (Rp)^6 \sqrt{y}
\]

(12)

Thus, the indirect utility with optimal skill levels of an individual working in the agriculture sector is concave in her skill levels: \( \frac{\partial^2 V_y(y)}{\partial y^2} > 0 \), \( \frac{\partial^2 V_y(y)}{\partial y^2} < 0 \). In other words, the higher the levels of skills an individual working in the agriculture sector possesses, the higher her level of satisfaction, till a certain point.

Equalizing the indirect utility of 10 with the indirect utility of 12 we can establish that \( V(y) = V_x(y) = \frac{1}{3 \cdot 2^{7/2}} R^6 y = V_y(y) = \frac{1}{3 \cdot 2^{7/2}} (Rp)^6 y \Rightarrow p = 1 \). Therefore, exactly like the analysis of the second stage, in autarky, the relative price of the agriculture sector is equal to unity. Hence, when \( p = 1 \) the indirect utility of every individual in a country, with optimal skill levels, is: \( V(y) = \frac{1}{3 \cdot 2^{7/2}} R^6 y \ \forall y > 0 \). The indirect utility of an individual with optimal skill levels is a continuous and linear increasing function of early education levels.

What about international trade? With perfect information, we get the same Ricardian trade story as described at the end of stage two. In other words, in a two country model, if a country has a relatively better early education system, ceteris paribus, it will not engage in international trade, even though it has a relatively
more skilled labor force. This is related to the fact that the pattern of trade between these two countries is independent of the allocation of skills within each country.

4. Imperfect Labor Contracts

Let us now move closer to reality and suppose that in the industrial sector, worker effort levels are not observable and verifiable. The same stands for the output of the firms in the industrial sector. Put differently, the labor markets are imperfect, or there exists imperfect information about workers’ efforts and distortions, and the firm’s output. Therefore, it is difficult to base the contract directly on the firm’s output and workers’ effort and distortion levels because it may be difficult for a court to measure the above due to the assumption of imperfect information. However, the firm’s manager is able to measure the level of a worker’s performance that is partially related to her efforts. In the same way, the worker may be able to measure the performance of the firm that is related to a degree to its output levels. As a result, because of the imperfect monitoring, the managers might offer contracts as described in the third section by the equations 4A and 4B. Thus, the degree of labor contract perfectibility is proxied by the degree of the quality of the institutions $\theta$. With imperfect contracts, the higher the quality of institutions ($\theta$), the higher the effort levels ($a_w$) put by a worker in the team production process because the less the distortion levels ($d_w$) exerted by the same worker. This is because the higher the quality levels of institutions in a country, the higher the verifiability of distortion levels of a worker and therefore the higher the levels of effort exerted by the same worker in the industrial sector. On the other hand, in the agriculture sector, there only exist individual firms. Thus, every individual working in this sector is a capital owner and has no incentives whatsoever to exert any positive level of distortion in her production process. Consequently, with imperfect labor contracts, the analysis of the last four stages is exactly the same as the analysis with perfect labor markets in the agriculture sector. This is not true for the industrial sector. We examine the five stages of the industrial sector with imperfect information following the same procedure as we did with perfect information. In the subsection 4.1 we analyze the five stage game, where managers offer contracts according to the equation 4A, while in subsection 4.2 we analyze the same five stage game, but in this case managers offer contracts according to the equation 4B.

4.1.1 Stage 5. The Decisions on Effort and Distortion Levels

We find the optimal levels of effort and distortions exerted by individuals (workers and managers) working in the industrial sector subject to their training and skill levels. An individual operating in the industrial sector endowed with $t_i$ units of training and with skill level $q_i$ has homothetic preferences generated by the indirect utility: $V_x(a_i, d_i, t_i, q_i) = RL_x - \frac{1}{2t_m} (a_m^2 + d_m^2) - \frac{1}{2t_w} (a_w^2 + d_w^2) - \frac{1}{2q_m} t_m^2 - \frac{1}{2q_w} t_w^2$. However, now we have to distinguish between the indirect utilities of a worker and that of a manager since the latter has no incentive to exert any levels of distortion because she cares only about maximizing the firm’s profit. With imperfect information the manager’s profits are $\Pi = x - wK = x - w \left[ a_w + \left( \frac{e^{1-\theta}-1}{q_m} \right) d_w \right]$. Hence, a manager
matched with a worker by forming a firm in the industrial sector has homothetic preferences given by the indirect utility: 

\[ V_m(a_m, d_m, t_m, q_m) = RL_m^{a_m} - \frac{1}{2t_m} (a_m^2 + d_m^2) - \frac{1}{2q_m} t_m^2 = R \left[ x - w \left[ a_m + \left( \frac{\theta - 1}{q_m} \right) d_m \right] \right] - \frac{1}{2t_m} (a_m^2 + d_m^2) - \frac{1}{2q_m} t_m^2. \]

On the other hand, a worker matched with a manager in the production team has homothetic preferences described by the indirect utility:

\[ V_w(a_w, d_w, t_w, q_w) = RwK - \frac{1}{2t_w} (a_w^2 + d_w^2) - \frac{1}{2q_w} t_w^2. \]

The manager and the worker choose the optimal levels of effort and distortion by maximizing the above indirect utilities.

The optimal distortion levels for the manager are equal to zero \((d_m = 0)\). This is related to the fact that the manager will exert zero distortion levels because she wants to maximize firm’s profits, which are related positively to her effort levels.

The optimal distortion levels for the worker are \(d_w = Rt_w \frac{e^{\theta - 1}}{q_m}. \) This shows that the levels of distortions decrease with an increase in \( \theta \) and an increase in \( q_m \). This follows our intuition that higher levels of quality institutions and more skilled managers will increase the verifiability of workers’ performance and therefore give an incentive to the workers to reduce their levels of distortion.

The optimal effort levels for a worker are \( a_w = Rtw. \) This shows that the optimal levels of a worker’s effort in the \( x \) sector are a monotonically increasing function of the levels of her training and her efficient wage. It makes sense for a worker to exert higher effort levels in the team production process when her efficient wage is higher because she might be afraid of losing her job. It also makes sense for a worker to exert higher levels of effort in the team production when she is better trained because, in this case, it is easier for a worker (due to relatively higher levels of training) to provide higher levels of effort as compared with her levels of distortions.

The optimal effort levels for a manager are \( a_m = R (tw_m)^{\frac{1}{3}}. \) Thus, the higher the wage and training levels of the worker, or/and the manager, the higher the effort levels exerted by the manager of the firm. Substituting, the optimal distortion and effort levels of a worker into her indirect utility function, we can obtain the indirect utility with optimal distortion and effort levels, for a worker who is employed in the industrial sector. This is represented by:

\[
V_w(w, t_w, q_i) = \frac{1}{2} (Rw)^2 t_w \left[ 1 + \left( \frac{\theta - 1}{q_m} \right)^2 \right] - \frac{t_w^2}{q_w} \] (13)

In an analogous way, the indirect utility with optimal distortion and effort levels for a manager who is employed in the agriculture sector can be written as:

\[
V_m(w, t_m, q_i) = \frac{3}{2} R^2 t_m (wt_m)^{\frac{1}{3}} - (Rw)^2 t_m \left[ 1 + \left( \frac{\theta - 1}{q_m} \right)^2 \right] - \frac{t_m^2}{q_m} \] (14)
4.1.2 Stage 4. The Decisions on Training Levels

In the fourth stage, managers and workers choose their optimal levels of training. Workers maximize (13) over the choice of their training. Thus, the optimal level of training for a worker who is employed in the industrial sector, with imperfect labor contracts, is

$$t_w = \frac{1}{2} R (Rw)^2 q_w \left[ 1 + \left( \beta \frac{e_1 - \theta - 1}{q_m} \right)^2 \right]^{\frac{1}{2}}.$$ 

One easily can conclude that each worker’s optimal training level is strictly increasing in her skill levels. So, the most skilled workers obtain the highest levels of optimal job training. This is related to the fact that it is easier for the more skilled workers to invest more in their job training choices.

Managers maximize (14) over the choice of their training. Hence, the optimal level of training for a manager who runs her own firm in the industrial sector is

$$t_m = \frac{1}{2} R^2 w^2 (q_m q_w)^{\frac{1}{2}} \left[ 1 + \left( \beta \frac{e_1 - \theta - 1}{q_m} \right)^2 \right]^{\frac{1}{2}}.$$ 

A manager’s optimal training level is increasing in her own and her employee’s skill levels. Substituting worker’s optimal training levels into (13) we obtain the indirect utility, with optimal training levels, of a worker employed in the industrial sector. This is described by:

$$V_w(q_l) = \frac{1}{8} (Rw)^4 q_w \left[ 1 + \left( \beta \frac{e_1 - \theta - 1}{q_m} \right)^2 \right]^{\frac{1}{2}}.$$ 

(15)

In an analogous way, the indirect utility of a manager, with optimal training levels, who runs a firm in the industrial sector can be written as:

$$V_m(q_l) = \frac{5}{8} R^4 w^{\frac{12}{5}} q_m^{\frac{1}{5}} q_w^{\frac{4}{5}} \left[ 1 + \left( \beta \frac{e_1 - \theta - 1}{q_m} \right)^2 \right]^{\frac{4}{5}} \left[ 1 - \frac{1}{2} (Rw)^4 q_w \left[ 1 + \left( \beta \frac{e_1 - \theta - 1}{q_m} \right)^2 \right]^{\frac{1}{2}} \right]^{\frac{4}{5}}.$$ 

(16)

4.1.3 Stage 3. The Choice of Matching

In the third stage, the optimal production team is chosen by the managers and the workers. Here, the workers and the managers match together by forming a firm. The manager presents the efficient wage to the worker after observing the worker’s levels of training and the worker decides on whether to accept it by focusing on the manager’s training levels. Since, there is imperfect information on the labor contract market, the optimal contract that a manager will offer to a worker now is related to the quality of institutions and her training levels. A manager (worker) maximizes her profits (income) by designing (accepting) such an efficient wage that corresponds to her levels of training, subject to the quality of institutions that exist in a country. This is how the efficient wage is determined in a country with imperfect information. Qualitatively, in order to find the efficient wage, we maximize the aggregate post training utilities of a worker and a manager who work together in a team, when they put their optimal levels of effort, distortion and training. This indirect utility is given by:

$$\Lambda(w, q_l) = \frac{5}{24} R^4 w^{\frac{12}{5}} q_m^{\frac{1}{5}} q_w^{\frac{4}{5}} \psi^{\frac{4}{5}} \psi^{-2} - \frac{1}{8} (Rw)^4 q_w \psi^{-2}.$$ 

(17)
We follow the work of Vogel (2007) and define \( \Psi \equiv \left[ \frac{q_{m}^{\frac{1}{3}}}{q_{n}^{\frac{1}{3}} + (e^{1-\theta-1})^{\frac{1}{2}}} \right] \) where \( \Psi \) shows the quality of the monitoring ability of a manager with skills \( q_{m} \) in a country with quality level of institutions \( \theta \). Thus, the higher the quality of institutions in a country, the higher the quality of the monitoring ability of a manager. Maximizing 17 over the wage, we find that the efficient wage is:

\[
w = (q_{m} \frac{1}{3})^{\frac{1}{3}} \Psi^{\frac{1}{4}}
\]

(18)

The efficient wage in a country with imperfect labor contracts is related positively to the manager’s quality performance measure and to her skill level, but is negatively related to the skill level of the worker who works with a manager in the same firm. Substituting the efficient wage 18 into 17 we get the aggregate post training utilities of a worker matched with a manager with optimal levels of effort and distortion, and optimal efficient wage, which is given by:

\[
\Lambda(t_{i}) = \frac{1}{12} R^{4} \Psi \sqrt{q_{m} q_{w}}
\]

(19)

Therefore, each member of the team is maximizing 19 by choosing the right partner, where the worker’s and manager’s levels of effort are not perfectly known and not perfectly verified in this labor market. However, both members of the team have perfect information on their skill levels, on the quality of the measure performance of the manager, and on the quality of institutions in a country. Solving 19 in terms of the efficient matching process, we find that it is optimal for each manager to match with a worker who obtains the same level of skills: \( q_{m} = q_{w} = q \). This result is efficient because it satisfies the assumption of complementary in the production process. This assumption is related to the property of the supermodularity of the post training indirect utility described in 19. Mathematically, the property of supermodularity is satisfied in the post training indirect utility because \( \frac{\partial^{2} \Lambda(q)}{\partial q_{m} \partial q_{w}} > 0 \) \( \forall \ q_{m}, q_{w} > 0 \). This result is associated with the fact that after efficient matching, a manager will team up with a worker who has identical skill levels with the manager.

Consequently, the efficient wage after the efficient matching process is \( w = \Psi^{\frac{3}{4}} \). Equalizing the indirect utility of a worker with that of a manager implies that their skill levels also must be identical after the efficient matching process. Substituting the manager’s and worker’s levels of training and skill, after the choice of the efficient matching, into 15 and 16, we can obtain the indirect utility function of an individual, with optimal efforts, distortions and training levels after the establishment of the efficient wage. This is given by the following indirect utility:

\[
\begin{align*}
V_{x}(q) &= V_{m}(q_{m}) = V_{w}(q_{w}) = \frac{1}{8} R^{2} \Psi q \\
\end{align*}
\]

(20)

This indirect utility is a monotonically increasing function of the quality of the measure of performance of a manager. This means that each individual (worker or manager) working in the industrial sector gets a higher level of satisfaction for higher quality levels of the measure of the performance of the manager, which in turn
depends on the quality levels of the country’s institutions. In other words, the better institutions a country has, the higher the level of satisfaction of the individuals working in the industrial sector.

4.1.4 Stage 2. The Choice over Industry

In the second stage, we find the optimal choice of individuals over the selection of the industry, where they will seek employment. As described in the third section, in this stage we equate the indirect utility of an individual working in the agriculture sector with the indirect utility of an individual working in the industrial sector in a closed economy with two sectors. Combining equation 6 with equation 20 implies that an individual is indifferent when choosing the sector where she will seek employment only in such cases when the following is satisfied:

\[ V_\gamma(q) = V_x(q) = V(q) \quad \text{or} \quad V(q) = \frac{1}{\theta} (Rp)^q q = V_x(q) = \frac{1}{\theta} R^4 \Psi q \]  

(21)

This implies that \( p \neq 1 \). More specifically \( p = \Psi^\frac{1}{\theta} \). Consequently, with imperfect information, in autarky, the relative price of the agriculture good is related positively to the manager’s quality performance level, which in turn is positively related to the quality levels of the country’s institutions.

Equations 6 and 20 also reveal that the indirect utility with optimal training for an individual working in the agriculture sector is linearly increasing with her skill levels, while the indirect utility with optimal training for an individual working in the industrial sector is increasing in her skill level. Moreover, 6 and 16 show that the indirect utility with optimal training of an individual working in the agriculture sector is independent of the quality of institutions in a country. On the other hand, the indirect utility with optimal training for an individual working in the industrial sector is increasing in the quality of the performance measure of a manager and in the quality of institutions of a country.

4.1.5 Stage 1. The Choice over Skill Level

In this stage we analyze the individual decision over their skill levels. As in the perfect information case, we assume that the skill level depends on the early education levels (\( \gamma \)). In the third section we found the optimal levels of skills for an individual working in the agriculture sector. We show that for certain values of individuals’ early educational level, the better the early educational system in a country, the higher the optimal skill levels of individuals who will work in the agriculture sector. Also, we showed that the indirect utility with optimal skill levels of an individual working in the agriculture sector is concave in her skill levels. In this stage, we find the equilibrium level of \( \gamma \). In order to do so, we first must find the optimal level of early education for individuals working in the industrial sector.

4.1.5.1 The Choice over Skill Level in the Industrial Sector

In the industrial sector, a type \( q_x \) individual who obtains \( \gamma \) early level of education and living in a country with \( \theta \) quality levels of institutions, before deciding on working on the industrial sector, optimizes her levels of skill subject to her levels of early education represented by the following indirect utility:

\[ V_x(q_x, \gamma) = \frac{1}{8} R^4 \Psi q_x - \frac{q_x^3}{3\gamma} \]  

(22)
The above indirect utility is constructed by the use of equation 20 and of course by endogenizing the levels of skill. Individuals in \( x \) maximize \( V_x(q_x, \gamma) \) over their choice of \( q \). The optimal level of skill of individuals that will later choose the industrial sector is:

\[
1 = \frac{1}{8} R^4 \left[ \frac{q_x^2 + 3\beta^2(e^{1-\theta} - 1)^2}{(q_x^2 + \beta^2(e^{1-\theta} - 1)^2)^2} \right] \gamma
\]  

(23)

As in the case of the agriculture sector, the optimal skill level of individuals who in the future will work in the \( x \) sector is increasing in \( \gamma \). Thus, the higher the level of early education in a country, the higher the optimal skill levels of individuals who will work in the industrial sector. However, unlike in the agriculture sector, there exists a positive relationship between the skill levels and country’s quality levels of its institutions. One can easily observe from (23) that the optimal level of skill in the industrial sector also depends on the level of early education. It can be shown that \( \frac{\partial \gamma}{\partial q_x} > 0 \). This implies that in the industrial sector, individuals who obtain high levels of early education will obtain high levels of skills or more able individuals are those who obtained high levels of early education.

Substituting the optimal levels of skill (23) into the indirect utility function of (22) we can obtain the indirect utility of an individual in the industrial sector with optimal skill levels. This is given by:

\[
V_x(\gamma) = \frac{1}{12} R^4 \psi^2 q_x
\]  

(24)

Hence, as in the agriculture sector, the indirect utility with optimal skill levels of an individual working in the industrial sector is increasing with her skill levels \( \frac{\partial V_x(\gamma)}{\partial \gamma} > 0 \); with a country’s quality level of institutions \( \frac{\partial V_x(\gamma)}{\partial \theta} > 0 \); and with the quality of the performance measure of a manager \( \frac{\partial V_x(\gamma)}{\partial \psi} > 0 \). In other words, the higher the levels of skill an individual working in the industrial sector possesses, or the better institutions a country has, or the higher the ability of the performance measure of a manager, the higher the individual’s level of satisfaction.

### 4.1.5.2 The Determination of the Equilibrium Level of Early Education

As described in the third section, in this stage we equate the indirect utility, with optimal skill levels, of an individual working in the agriculture sector with the indirect utility, with optimal skill levels, of an individual working in the industrial sector in a closed economy with two sectors. Combining equation 12 with equation 24 implies that the equilibrium level of early education is achieved only when \( V_x(\gamma) = V_x(q) \) or \( V_x(\gamma) = \frac{1}{2\pi/2} (pR)^6 \sqrt{\gamma} = V_x(q) = \frac{1}{12} R^4 \psi^2 q_x \). This implies that \( p^6 = \psi^2 \left( \frac{q_x^2 + 3\beta^2(e^{1-\theta} - 1)^2}{(q_x^2 + \beta^2(e^{1-\theta} - 1)^2)^2} \right)^{\frac{1}{2}} \). Consequently, with imperfect information, in autarky, the relative price of the agriculture good is related positively to the manager’s quality performance level, which in turn is related positively to the quality levels of a country’s institutions. We can summarize the above results with the use of the following three propositions.
**Proposition 1.** In a closed economy with imperfect information $\theta \in [0,1)$ and $\beta \in [1, q_m)$:

There exists a $\gamma^* \in [\gamma_{\text{min}}, \gamma_{\text{max}}]$, such that individuals join the industrial (agriculture) sector, if and only if $\gamma > \gamma^*$; $\gamma < \gamma^*$.

The intuition behind Proposition 1 is related to the fact that under imperfect information all individuals have to make a choice on how much skill they will obtain, even before making the choice of the industry in which they will seek employment. As indicated in equations 23 and 20, the higher the individuals’ early education levels, the higher their optimal skill levels for each industry. The reason behind such a positive relationship has to do with the existence of different incentives of individuals for skill accumulation. Thus, individuals who have higher levels of early education have higher incentives for skill accumulation because it is relatively easier for them to obtain more skills than individuals who possess lower early levels of education.

In few words, a country with a better early education system also will have a more skilled labor force. Consequently, individuals who possess relatively high levels of early education will join the industrial sector. This is because it is more effective for high skilled workers to put high effort levels in the production function and consequently low distortion levels. Therefore, they will optimize by working in the industrial sector. Thus, with an imperfect labor contract market, in a closed economy that obtains a labor force with more skilled workers, there are more individuals working in the industrial sector than in the agriculture sector.

Since there is a positive relationship between individuals’ skill levels and their early education levels independent of their choice over industry, there must exist a unique early level of education threshold. Individuals who possess higher levels of early education than the threshold level will enter into the industrial sector. The uniqueness of $\gamma^*$ is determined by the relationships of optimal skill levels and early education level as represented in equations 23 and 20. Thus, individuals with better early education levels are more satisfied into the industrial sector, while individuals with inferior early education levels are more satisfied into the agriculture sector. The determination of the threshold level of early education is accompanied by the following corollary:

**Corollary 1:** The following inequalities hold: i) $\frac{\partial \gamma^*}{\partial \theta} < 0$; and ii) $\frac{\partial \gamma^*}{\partial \psi} < 0$.

The intuition behind the first part of corollary one is related to the fact that better institutions will increase the incentives of individuals to join the industrial sector since $V_x(\gamma)$ is increasing in the quality of institutions, but $V_y(\gamma)$ is independent of the institutional quality (see equations 12, 23 and 24). Following the same logic, the second part of the above corollary shows that better institutions also will increase the quality of performance measure of a manager in the industrial sector. Therefore more individuals will enter into the x sector.

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5 See Appendix A for the proofs of Proposition 1 and Corollary 1.
From Proposition 1 we know that individuals who obtain relatively high level of skills optimize their efforts and distortions by working in the industrial sectors, while less skilled individuals optimize their effort by working in the agriculture sector. However, the more skilled individuals are those with relatively high early educational level. Therefore, all individuals with higher levels of early education than the threshold level of early education will accumulate relatively high levels of skills. Consequently, these types of individuals work into the industrial sector. In order to clarify this result and to give the intuition on the incentives of individuals who enter in the industrial sector we proceed with Proposition 2:

**Proposition 2.** In a closed economy with imperfect labor contracts’ market $\theta \in [0,1)$ and $\beta \in [1, q_m)$:

1) $\forall \gamma \geq \gamma^*, q_x(\gamma)$ is convex in $\gamma$;

2) $\forall \gamma \geq \gamma^*, q_x(\gamma) > q_y(\gamma)$;

3) $\forall \gamma \geq \gamma^*, t_i(\gamma) > t_y(\gamma)$, where $i \equiv [w, m]$.  

![Graph](image)

**Fig. 2. Skill level as a function of early educational level**

Part 1) and 2) of Proposition 2 show that in a closed economy with imperfect institutions and contracts designed as described in equation 4A, any individual with an initial level of education greater than the threshold level of utility ($\gamma^*$) accumulates a higher level of skills if she enters into the industrial sector as compared with her level of skills if she were to enter into the agriculture sector. In our model, this statement is obvious (as shown in Appendix B) because an individual’s level of skills is a strictly convex function of her early educational levels for all individuals who enter into the industrial sector, $X$, while it is a concave function of her early educational level for all individuals who enter into the agriculture sector, $Y$. We illustrate the statements of part 1) and 2) of Proposition 2 with the help of fig. 2, where in the vertical axes we plot the values of all individuals’ levels of skills $[q_i(\gamma^*)]$ as a function of their initial educational level ($\gamma$). An

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6 See Appendix B for the proof of Proposition 2.
individual optimizes her utility, and therefore enters into the agriculture sector only if her initial educational level is strictly smaller than the threshold level, and enters into the industrial sector if her initial educational level is equal or greater than the threshold level. As one can observe from fig. 2, there is a jump point in levels of skills right at the threshold level of utility. The bold portion of the graph represents the $q_x$ function for all $\gamma \geq \gamma^*$ and the $q_y$ function for all $\gamma \geq \gamma^*$, where $\theta \in [0,1)$ and $\beta \in [1, q_m)$ and $\gamma^* \in [y_{min}, y_{max}]$.

Part 3) of Proposition 2 states that if institutions are imperfect and contracts are designed according to the equation 4A, any individual with more or equal early educational levels than the threshold level ($\gamma^*$) is more talented in the case where she entered into the industrial sector than she would have been had she entered the agriculture sector. This is related with our definition of talent, where the most talented individual is the one who obtains the highest level of job-training. For all individuals, regardless of their industry choice, training is a linear function of their skill level. However, as stated in part 10 and 2) of Proposition 2, an individual’s level of skill is a strictly convex function in her early educational level for all individuals (with $\gamma \geq \gamma^*$) who join the industrial sector, and it is a strictly concave function in her early educational level for all individuals (with $\gamma < \gamma^*$) who join the agriculture sector. This indicates that individuals with greater or equal initial educational levels than the threshold level will be more talented during their entire life, and therefore, will seek employment in the industrial sector as compared to individuals with strictly smaller early educational level than the threshold level, who will seek employment in the agriculture sector.

**Proposition 3.** In a closed economy with imperfect labor contracts’ market $\theta \in [0,1)$ and $\beta \in [1, q_m)$, the income of an individual who works in the industrial sector always is strictly higher than the income of an individual who works in the agriculture sector for all $\gamma > \gamma^*$.

Assuming in a closed economy, that if institutions are imperfect and contracts are designed according to the equation 4A, every individual’s wealth consists of her current income all of which is spent consumption industrial and agriculture goods, $x$ and $y$. Also assume that, in this closed economy, there is no borrowing and no unemployment and an individual’s source of income comes only from her firm’s profits if she runs her individual firm in the agriculture sector, or if she runs her team production firm in the industrial sector, and it comes only from her wage if she works in the industrial sector. Keep in mind that an individual either could work in the agriculture sector or in the industrial sector, but not in both. Moreover, an individual who enters into the industrial sector either is running her own firm or working as an employer with a firm owner only in one firm. Thus, Proposition 3 states that individuals who obtain an equal or greater early educational level than the threshold level ($\gamma^*$) are strictly richer at any point in their life if they enter into the industrial sector than they would have been had they entered the agriculture sector. This is related to the fact that these individuals are more talented because they receive more job training if they enter into the industrial sector. Consequently,

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7 See Appendix C for the proof of Proposition 3.
their utility is equal or higher in the industrial sector as compared to that of the agriculture sector, while their utility cost of obtaining skills, and therefore, of training is strictly higher in the industrial sector. Thus, their income also must be strictly greater in the industrial sector than in the agriculture one.

4.2 The Five Stage Game When $\beta = q_m$

The analysis of the perfect information case and the agriculture sector is exactly the same as before, since in both cases $K = a_w$. So, let us analyze the industrial sector in the case when $\beta = q_m$, which implies that under imperfect labor contracts the managers offer contracts according to equation 4B ($K = a_w + (e^{1-\theta} - 1)d_w$). In stage 5, the indirect utility of a worker is $V_w = R[a_w + (e^{1-\theta} - 1)d_w] - \frac{1}{2t_w}(a_w^2 + d_w^2) - \frac{1}{2q_w}t_w^2$, and the indirect utility of a manager is $V_m = R\left\{ 2\sqrt{a_w a_m} - w[a_w + (e^{1-\theta} - 1)d_w] - \frac{1}{2t_m}(a_m^2 + d_m^2) \right\} - \frac{1}{2q_m}t_m^2$. From the above indirect utilities we can establish that the optimal level of effort for a worker is $a_w = R wt_w$ and her optimal distortion level is $d_w = R wt_w (e^{1-\theta} - 1)$. This indicates that the optimal levels of a worker’s effort in the industrial sector are increasing in her levels of her training and her wage. A worker exerts more effort in the team production process when she is better trained because it is easier for her to provide higher levels of effort. The levels of distortions decrease with an increase in $\theta$. The intuition is that better institutions and more skilled managers will increase the verifiability of the workers’ performances offering an incentive for workers to reduce their levels of distortion. The optimal levels of efforts for a manager are $a_m = R t_m^2 (wt_w)^{1/3}$, while $d_m = 0$. Thus, a manager’s distortions levels are equal to zero since her objective is to maximize the firm’s profit. A manager’s effort are increasing in hers and the worker’s training levels and in the worker’s wage.

Substituting the equations of optimal effort and distortion levels of a worker into her indirect utility function we obtain the following indirect utility of a worker with optimal effort and distortion levels:

$$V_w = \frac{1}{2} (Rw)^2 t_w \left[ 1 + (e^{1-\theta} - 1)^2 \right] - \frac{t_w^2}{q_w}$$

Substituting the equations of optimal effort and distortion levels of a manager into her indirect utility function we obtain the following indirect utility of a manager with optimal effort and distortion levels:

$$V_m = \frac{3}{2} R^2 t_m^2 \left[ 1 + (e^{1-\theta} - 1)^2 \right] - \frac{t_m^2}{q_m}$$

In the fourth stage workers maximize 25 over the choice of their training. The optimal level of training for a worker who is employed in the $x$ sector is $t_w = \frac{1}{2} (Rw)^2 q_w \left[ 1 + (e^{1-\theta} - 1)^2 \right]$. Like in the previous sector, each worker’s optimal training level is strictly increasing in her skill level. The most skilled workers obtain the highest levels of optimal job training.

Managers maximize 26 over the choice of their training. Hence, the optimal level of training for a manager is $t_m = \frac{1}{2} R^2 w^2 \left[ 1 + (e^{1-\theta} - 1)^2 \right]^2$. Again, a manager’s optimal training level is increasing in her
own and her employee’s skill levels. Substituting a worker’s optimal training levels into 25, we can establish the indirect utility, with optimal training levels, of a worker employed in the industrial sector. This is described by:

\[
V_w(q_i) = \frac{1}{8} (Rw)^4 q_w \left[ 1 + (e^{1-\theta} - 1)^2 \right]^2
\]  

(27)

In an analogous way, the indirect utility of a manager, with optimal training levels, who runs a firm in the industrial sector can be described by:

\[
V_m(q_i) = \frac{5}{8} R^4 w \frac{12}{5} q_m^{\frac{4}{5}} q_w^{\frac{4}{5}} \left[ 1 + (e^{1-\theta} - 1)^2 \right]^\frac{2}{5} - \frac{1}{2} (Rw)^4 q_w \left[ 1 + (e^{1-\theta} - 1)^2 \right]^2
\]

(28)

In the third stage, the optimal production team is chosen by the managers and the workers following the same matching process as described in the previous section. Now, the aggregate post skilled indirect utility for a worker with skill \( q_w \) that matches with a manager with skill \( q_m \) can be written as:

\[
\Lambda = \frac{5}{24} R^4 w \frac{12}{5} q_m^{\frac{4}{5}} q_w^{\frac{4}{5}} \left[ 1 + (e^{1-\theta} - 1)^2 \right]^\frac{4}{5} - \frac{1}{8} (Rw)^4 q_w \left[ 1 + (e^{1-\theta} - 1)^2 \right]^2
\]

(29)

Maximizing 17 over the wage, we find that the efficient wage is: \( w = \left( \frac{q_m}{q_w} \right)^\frac{1}{2} \left( \frac{1}{1+e^{1-\theta}} \right)^\frac{3}{2} \). The efficient wage in a country with imperfect labor contracts is related positively to the manager’s skill level, but is negatively related to the skill level of the worker who works with a manager in the same firm. Substituting the efficient wage into 29 we get the aggregate post training utilities of a worker matched with a manager with optimal levels of effort and distortion, and optimal efficient wage, which is given by:

\[
\Lambda(t_i) = \frac{1}{12} R^4 \frac{1}{1 + (e^{1-\theta} - 1)^2} \sqrt{q_m q_w}
\]

(30)

Analogous to the previous section, each member of the team is maximizing 30 by choosing the right partner, where the worker’s and manager’s levels of effort are not known perfectly and not verified perfectly in this labor market. However, both members of the team have perfect information on their skill levels, on the quality of the measure performance of the manager, and on the quality of institutions in a country. Solving 30 in terms of the efficient matching process, we find, again, that it is optimal for each manager to match with a worker who obtains the same level of skills: \( q_m = q_w = q \). This result is efficient because it satisfies the assumption of complementarity in the production process. This assumption is related to the property of the supermodularity of the post training indirect utility described in 30.

This is an important result, since now the managers offer contracts that are independent of their skill level. Remember that, in this case, contracts are designed according to equation 4B. Therefore, one can conclude that under the assumption of complementarity in production the most efficient worker will pair up with the most efficient manager regardless of the initial contract design by managers of the firms.

The efficient wage after the efficient matching process is \( w = \left( \frac{1}{1+e^{1-\theta}} \right)^\frac{3}{2} \). Equalizing the indirect utility of a worker with that of a manager indicates that their skill levels also must be the same after the
efficient matching process. Substituting the manager’s and worker’s levels of training and skill, after the efficient matching, into 27 and 28, we can obtain the indirect utility function of an individual, with optimal efforts, distortions and training levels after the establishment of the efficient wage. This is given by the following indirect utility:

\[ V_{m}(q_m) = V_{w}(q_w) = \frac{1}{8} R^4 \left[ \frac{1}{1 + (e^{1-\theta} - 1)^2} \right] q \]  

(31)

This indirect utility is a monotonically increasing function of an individual’s skill levels. This means that the most skilled individual (worker or manager) working in the industrial sector gets the highest level of satisfaction. Also, the better institutions a country has \( \frac{\partial V_{m}}{\partial \theta} \), the higher the level of satisfaction of the individuals working in the industrial sector.

In the second stage, we equate the indirect utility of an individual working in the agriculture sector with the indirect utility of an individual working in the industrial sector in a closed economy with two sectors. Combining equation 6 with equation 31 implies that an individual is indifferent when choosing the sector where she will seek employment only when the following is satisfied:

\[ V_{p}(q) = V_{x}(q) = V(q) \text{ or } V_{p}(q) = \frac{1}{8} (Rp)^4 q = V_{x}(q) = \frac{1}{8} R^4 q \left[ \frac{1}{1 + (e^{1-\theta} - 1)^2} \right] \]  

(32)

This implies that \( p = \left[ \frac{1}{1 + (e^{1-\theta} - 1)^2} \right]^{\frac{1}{2}} \). Therefore, with imperfect information, in autarky, the relative price of the agriculture good is related positively to the quality levels of country’s institutions.

In the first stage we analyze the individual’s decision over their skill levels. As demonstrated in the previous section, we assume that the skill level depends on the early education levels (\( \gamma \)). The analysis over the skill level decision of individuals working in the agriculture sector again is identical with that described in the third section. In the industrial sector, a type \( q_x \) individual who obtains \( \gamma \) early level of education and living in a country with \( \theta \) quality levels of institutions optimizes her levels of skill subject to her levels of early education represented by the following indirect utility:

\[ V_{x}(q_x, \gamma) = \frac{1}{8} R^4 \left[ \frac{1}{1 + (e^{1-\theta} - 1)^2} \right] q_x - \frac{q_x^2}{3\gamma} \]  

(33)

Individuals maximize \( V_{x}(q_x, \gamma) \) over their choice of \( q \). The optimal level of skill of individuals who later will choose the industrial sector is:

\[ q_x = \frac{R^2}{2\sqrt{2}} \sqrt[3]{\frac{\gamma}{1 + (e^{1-\theta} - 1)^2}} \]  

(34)

The optimal skill level of individuals who will work in the industrial sector is increasing in \( \gamma \). Thus, the higher the level of early education in a country, the higher the optimal skill levels of individuals who will work in the industrial sector. However, unlike in the agriculture sector, there exists a positive relationship between skill levels and country’s quality level of its institutions \( \frac{\partial V_{x}}{\partial \theta} > 0 \). The better institutions a country has, the
more skilled individuals are part of its labor force. One can easily observe from 34 that the optimal level of skill in the industrial sector also depends on the level of early education. This implies that in the industrial sector, individuals who obtain high levels of early education will obtain high levels of skill \( \frac{\partial V_x}{\partial q_x} > 0 \).

Substituting the optimal levels of skill 34 into the indirect utility function of 33 we can obtain the indirect utility of an individual in the industrial sector with optimal skill levels, which is:

\[
V_x(y) = \frac{1}{12} R^x q_x \sqrt{\frac{1}{1 + (e^{1-\theta} - 1)^2}} \quad \text{or} \quad V_x(y) = \frac{1}{24 \sqrt{2}} R^6 \left[ \frac{1}{1 + (e^{1-\theta} - 1)^2} \right]^{3/2} \sqrt{y} \tag{35}
\]

Hence, as in the agriculture sector, the indirect utility with optimal skill levels of an individual working in the industrial sector is increasing with her skill levels \( \frac{\partial V_x}{\partial q_x} > 0 \), with her early educational levels \( \frac{\partial V_x}{\partial \theta} > 0 \), and with a country’s quality level of institutions \( \frac{\partial V_x}{\partial \theta} > 0 \). Thus, the higher the level of skill or early education that an individual working in the industrial sector possesses, or the better institutions a country has, the higher is the individual’s level of satisfaction.

Combining equation 12 with equation 35 implies that the equilibrium level of early education is achieved only when \( V_y(y) = V_x(y) \) or \( V_x(y) = \frac{1}{2^{1/2}} R^6 \sqrt{y} = V_x(q) = \frac{1}{8 \sqrt{2}} R^4 \left[ \frac{1}{1 + (e^{1-\theta} - 1)^2} \right]^{3/2} \sqrt{y} \). This implies that \( \theta = \left[ \frac{1}{1 + (e^{1-\theta} - 1)^2} \right]^{3/2} \). Consequently, with imperfect information, in autarky, the relative price of the agriculture good is related positively to a country’s quality level of its institutions. We can summarize the above results with the use of the following three propositions.

**Proposition 4.** In a closed economy with imperfect information \( \theta \in [0,1) \) and \( \beta = q_m \):

There exists a \( \gamma^* > 0 \), such that individuals join the industrial (agriculture) sector, if and only if \( \gamma > \gamma^* \); \( \gamma < \gamma^* \). \(^8\)

The intuition behind Proposition 4 is similar to that of Proposition 1. All individuals have to make a decision on how much skill they will obtain even before making the choice of the industry in which they will seek employment. Thus, individuals who obtain high early educational levels have high incentives for skill accumulation because it is relatively easier for them to obtain more skills than individuals who possess low early educational levels. More specifically, individuals who have higher early educational levels than the threshold level enter into the industrial sector and those with lower early educational level than the threshold level enter into the agriculture one. Another implication of Proposition 4 is that, with imperfect labor contracts, in a closed economy that obtains a labor force with more skilled workers as a result of a good early educational system, there are more individuals working into the industrial sector than in the agriculture one regardless of

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\(^8\) See Appendix D for the proofs of Proposition 4 and Corollary 2.
the design of the contracts (according to our two methods of contract design as described by equations 4A and 4B). The determination of the threshold level of utility comes with Corollary 2.

**Corollary 2.** In a closed economy with imperfect information $\theta \in [0,1)$ and $\beta = q_m$. the following inequality is always valid $\frac{\partial y^*}{\partial \theta} < 0$.

Corollary 2 is stating that better institutions increase the incentives of individuals to enter into the industrial sector regardless of the contract design. From Proposition 1 we know that all individuals with higher levels of early education than the threshold level will accumulate higher levels of skills as compared to those with lower levels of early education than the threshold level. In order to provide the intuition on the incentives of individuals who enter into the industrial sector we proceed with the following Proposition:

**Proposition 5.** In a closed economy with an imperfect labor contract market $\theta \in [0,1)$ and $\beta = q_m$\forall y > y^*, the inequality $q_x(y) > q_y(y)$ is always true.\(^9\)

![Fig. 3. Skill level as a function of early educational level](image)

Proposition 5 states that in a closed economy with imperfect institutions, where contracts are designed according to equation 4B, any individual with an initial level of education greater than the threshold level of utility ($y^*$) accumulates a higher level of skill if she enters into the industrial sector as compared with her level of skills if she were to enter into the agriculture sector. This statement is true (as demonstrated in Appendix E) because an individual’s level of skill is a strictly concave function of her early educational levels for all individuals independent of their choice over industry. However, the slope of each point of the concave function for individuals who enter into the industrial sector is different from the slope of each point of the concave function for individuals who enter into the agriculture sector, except for the point that corresponds to the threshold level ($y^*$). We illustrate Proposition 5 in fig. 3, where in the vertical axes we plot the values of the skill levels of all individuals [$q_i(y^*)$] as a function of their initial educational level ($y$). An individual optimizes her utility and therefore enters into the agriculture sector only if her initial educational level is

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\(^9\) See Appendix E for the proof of Proposition 5.
strictly smaller than the threshold level, and enters into the industrial sector if her initial educational level is equal or greater than the threshold level. The bold portion of the graph represents the $q_x$ function for all $\gamma \geq \gamma^*$ and the $q_y$ function for all $\gamma \geq \gamma^*$, where $\theta \in [0,1)$ and $\beta = q_m$.

Comparing Proposition 5 with Proposition 2 we can conclude that the accumulation of skills for each level of early education for individuals who enter into the industrial level is related with the design of the contracts. More specifically, for individuals who join the industrial sector, if $\beta \in [1, q_m)$ an individual’s level of skills is a strictly convex function in her early educational level, while if $\beta = q_m$ it is a strictly concave function in her early educational level.

**Proposition 6.** *In a closed economy with an imperfect labor contract market $\theta \in [0,1)$ and $\beta = q_m$, the income of an individual who works in the industrial sector always is strictly higher than the income of an individual who works in the agriculture sector for all $\gamma > \gamma^*$.*\(^{10}\)

The intuition behind Proposition 6 is similar to that of Proposition 1. Consequently, Proposition 6 states that individuals who obtain equal or greater early educational levels than the threshold level ($\gamma^*$) are strictly richer at any point in their life if they enter into the industrial sector than they would have been had they entered the agriculture sector. These individuals are more talented because they receive more job training if they enter into the industrial sector.

Comparing Proposition 6 with Proposition 3, one easily can observe that the accumulation of skills for each level of early education, only in the case of individuals who join the industrial sector, is related with the design of the contracts. If contracts are designed according to equation 4A, then the more skills an individual who works in the industrial sector obtains, the richer she is going to be. This is because, in this case, an individual’s skill level is a strictly convex function in her early educational level. However, this is not the case if the contracts are designed according to equation 4B. This is related to the fact that an individual’s skill level is a strictly concave function in her early educational level for all individuals who join the industrial sector. Put differently, if $\beta \in [1, q_m)$ there will be a much higher income inequality between an individual who enters into the industrial sector and one who enters into the agriculture sector than the income inequality between an individual who enters into the industrial sector and one who enters into the agriculture sector if $\beta = q_m$.

5. **The Effects of International Trade in Two Large Economies**

In this section, we examine the pattern of trade between two large countries with imperfect labor contracts. In particular, we associate the existence of a trade pattern with the differences on the distribution of skills in the labor force of each country. But, as we explained in the closed economy case, the allocation of skills in the labor force of a country can be determined by the distribution of the early education levels that individuals

\(^{10}\) See Appendix F for the proof of Proposition 6.
possess, and by the quality of a country’s institutions. Focusing on these two exogenous variables, ceteris paribus, we shall be able to determine which country exports what, in a world that consists of two large countries.

Let’s assume that there are two countries, a developed Country \((H)\), and a developing country \((O)\), that have two sectors each, an agriculture sector and an industrial sector.\(^{11}\) Also, suppose that these two countries are exactly the same in all aspects except the quality of their institutions and the distribution of their citizens’ early education levels. Consequently, there exists a difference in the distribution of skill levels between the two countries.

Let’s first assume that country \(H\) has institutions of identical quality, but more individuals with higher levels of early education as compared to country \(O\). The latter is related to the fact that \(H\) offers a better early educational system than \(O\). If the two countries decide to engage in free trade, we shall be able to determine the distribution of skills of their labor force within each sector, and therefore, predict the pattern of trade. For convenience, suppose that after trade each country is incompletely specialized in the production of both goods. Then, Proposition 7 can be established as follows:

**Proposition 7.** In a developed country [developing country] with imperfect information \(\theta < 1\) and \(\beta \in (0, q_m]\), where \(\theta_j = \theta\) and \(j \equiv (O, H)\), there exists a unique \((\gamma^*)^H, [(\gamma^*)^O]\), such that individuals enter into the industrial sector if and only if

1) \(\gamma^H > (\gamma^*)^H, [\gamma^O > (\gamma^*)^O]\). Consequently, the assumption that \(\gamma^H > \gamma^O\) implies that the following inequalities are always true:

2) \((\gamma^*)^H < (\gamma^*)^O \forall \gamma^j > 0\)

3) \((\gamma^*)^O \geq \gamma_{\min}^O\) and \((\gamma^*)^H \leq \gamma_{\max}^H\)^{12}

Proposition 7 concludes that country \(H\) will export the industrial good to country \(O\) in exchange for imports of the agriculture good from country \(O\). Consequently, the main implication of proposition 7 is that it considers the early education system of a country as a unique, independent source of comparative advantage. This is one of the main contributions of our paper. Country \(H\) will export the industrial good as a result of having a labor force that consists of more talented individuals (as compared to country \(O\)). This is related to the fact that country \(H\) offers a better early educational system that produces more skilled individuals, who in turn obtain higher levels of job training and therefore become more talented as explained by our model in the previous sections.

Let’s now assume that \(H\) and \(O\) differ only on the quality level of their institutions, with the existence of better institutions in the developed country. However, we keep the assumption of the existence of imperfect

\(^{11}\) The reason for denoting \(O\) as the developing country and \(H\) as the developed country is related to the intuition of the next section, where “\(O\)” stands for the origin country of immigrants and “\(H\)” stands for the host country of immigrants.

\(^{12}\) See Appendix G for the proof of Proposition 7.
institutions in both countries. In the remainder of this section we examine the effects of institutional differences, in terms of their quality level, on international trade of goods and services that takes place between two large economies in a free trade world. Thus, Proposition 8 follows:

**Proposition 8.** In a developed country [developing country] with imperfect information $\theta < 1$ and $\beta \in (0, q_m]$, where $\gamma^j = \gamma$, there exists a unique $(\gamma^*)^H$, $[(\gamma^*)^O]$, such that individuals enter into the industrial sector if and only if

1) $\gamma^H > (\gamma^*)^H$, $[(\gamma^*)^O]$. Thus, the assumption that $\theta^H > \theta^O$ implies that the following inequalities are always true:

2) $(\gamma^*)^H < (\gamma^*)^O \forall \gamma^j > 0$

3) $(\gamma^*)^O \geq \gamma_{min}$ and $(\gamma^*)^H \leq \gamma_{max}$.

The intuition behind Proposition 8 is analogous to that of Proposition 7. The only difference between these propositions lies in the source of a country’s comparative advantage. According to Proposition 8 the source of comparative advantage is the quality of institutions in each country. The determination of such a comparative advantage for each country is accompanied with the following corollary:

**Corollary 3.** In a free trade world that consists of two large economies, with imperfect information $\theta < 1$ and $\beta \in (0, q_m]$, where $\gamma^j = \gamma$ and $\theta^H > \theta^O$, the following inequalities always are true:

1) $q_x(\gamma)^H > q_x(\gamma)^O \forall \gamma > (\gamma^*)^H$

2) $t_x(\gamma)^H > t_x(\gamma)^O \forall \gamma > (\gamma^*)^H$

3) $l_x(\gamma)^H > l_x(\gamma)^O \forall \gamma > (\gamma^*)^H$

Therefore, according to Proposition 8 and Corollary 3, country $H$ will export the industrial good to country $O$ in exchange for imports of the agriculture good. Consequently, the main implication of Proposition 8 is that it considers the quality of a country’s institutions as an independent source of comparative advantage. This is another main contribution of our paper. Country $H$ will export the industrial good as a result of having a labor force that consists of more talented individuals (as compared to country $O$). This is related to the fact that country $H$ has better institutions as compared with country $O$. This fact gives more incentives to individuals of country $H$ to accumulate more skills, and seek employment into the industrial sector. This in turn allows them to obtain higher levels of job training and therefore become more talented in order to gain higher income levels as explained by our model in the previous sections.

In summary, both propositions described in this section indicate that in a free trade world that consists of two countries, a country that has a better early educational system and better institutions exports the industrial

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13 See Appendix G for the proofs of Proposition 8 and Corollary 3.
good and imports the agriculture good. We illustrate this situation in fig. 4. In the vertical axes, we plot the values of the relative price of the agriculture good, and in the horizontal axes we plot the values of the production of the agriculture good in terms of the industrial good. Since we assumed that each individual in each country has identical and homothetic preferences, then the relative demand line must be the same for both countries. But, if we consider both countries as closed economies, then there exists two different relative supply curves. The relative supply curve of the O country is higher than that of the H country because the labor force of O consists of less talented workers than H. Therefore, RS^O lies in the right of RS^H. Consequently, the relative autarky price of the agriculture good in O is lower than the relative autarky price of the agriculture good in H.

![Fig. 4. World equilibrium under free trade](image)

According to Proposition 7 and 8, O is exporting the agriculture good to H and importing the industrial good from H. Hence, the world relative supply curve should be to the right of RS^H and to the left of RS^O. Consequently, the world relative price of the agriculture good should be higher than the relative autarky price of the agriculture good in O and lower than the relative autarky price of the agriculture good in H.

### 6. The Effects of Emigration in Two Large Economies

In this section, we investigate the pattern of labor movements in a free trade world that consists of two large economies with imperfect labor contracts. We associate the individual’s decision to emigrate with her income difference, subject to her skill level, which exists between her country of origin and the host country of immigrants. We assume that there exist certain fixed costs of migration, such as language and culture barriers.
The fixed costs are the same for every individual who decides to move permanently from one country to the other. Let’s assume that there are no illegal immigrants in any country. Skills are considered perfectly substitutable among individuals who obtain the exact same level of skills, but are citizens of different countries. Suppose that these two countries exactly are the same in all aspects except the quality of their institutions and the distribution of their citizens’ early education levels.

Let’s first assume that country $H$ has better institutions than country $O$, but has identical system of early education. Consequently, in a free trade world according to Proposition 8, $H$ will export the industrial good to $O$ and $O$ will export the agriculture good to $H$. If the two countries decide to engage in a free movement of their respective labor force, we must be able to predict the pattern of immigration. For convenience, let’s keep the assumption that after the free movement of labor between countries, each country is incompletely specialized in the production of both goods. Then, Proposition 9 can be established as follows:

**Proposition 9.** Assume that countries $O$ and $H$ allow free movements of their labor force.

1) In country $j \equiv (O, H)$ with $\theta < 1$ and $\beta \in (0, q_m]$, where $y^j = \gamma$ and $\theta_H^j > \theta^O$, there exists a unique $(\gamma^*)^j$, such that individuals enter into the industrial sector if and only if $y^j > (\gamma^*)^j$.

2) $(\gamma^*)^O \geq \gamma_{\min} \text{ and } (\gamma^*)^H \leq \gamma_{\max}$.

3) $(\gamma^*)^H < (\gamma^*)^O \forall y^j > 0 \text{ and } l^H(\gamma^*)^H > l^O(\gamma^*)^O \forall \gamma > \gamma^*$.

4) Country $H$ will be the host country of immigrants and country $O$ will be the origin country of immigrants only if there exists a $\gamma \geq \gamma^*$ such that $l^H(\gamma^*)^H > [l^O(\gamma^*)^O - c] \forall \gamma > \gamma^*$, where $c \equiv$ costs of immigration.14

Part 1), 2) and 3) of Proposition 9 replicate Proposition 8 and Corollary 3 but for open labor markets. Proposition 9 reinforces the fact that considers institutions of a country as an independent source of comparative advantage. Label $\gamma$ as the migration threshold level. The main implication of Proposition 9 is related to part 4) that states that early educational system acts as an independent source of the establishment of the direction of the labor movement. This is an important contribution of our paper. The intuition is that Country $H$ will continue to export the industrial good as a result of having an even larger labor force that consists of more talented individuals than country $O$. This is related to the fact that country $H$ has better institutions than country $O$. This fact gives more incentive to individuals of country $H$ for skill accumulation, and makes them to seek employment into the industrial sector; as a result they become more talented in order to gain higher income levels. Thus, individuals who have the exact same level of skills but work in different countries obtain different levels of income. Since the quality of institutions in a closed economy is not related to the income of individuals who work in the agriculture sector, their income will be the same independent of their firm’s location. Consequently, there will be no migration of any individual who works in the agriculture sector.

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14 See Appendix H for the proof of Proposition 9.
sector. On the other hand, an individual who works in the industrial sector in country $H$ obtains a higher income than an individual who works in the same sector in country $O$ despite the fact that they have the same level of skills. Therefore, all individuals of $O$ with early educational levels ($\gamma$) greater than the migration threshold level ($\bar{\gamma}$) have an incentive to move to $H$ because of the difference in income. This is represented by Proposition 10:

**Proposition 10.** In a world that consists of two large economies with free movement of goods and labor, and with imperfect information $\theta < 1$, and $\beta \in (0, q_m]$, where $\gamma^I = \gamma$ and $\theta^H > \theta^O$.

1) Only the most talented individuals of country $O$ will immigrate in country $H$.

2) There exists a $\bar{\gamma}$ that corresponds to a $\bar{\xi}$ such that it provides incentive to some individuals, with $\bar{\gamma} < \gamma^O < (\gamma^*)^O$, from $O$ to immigrate in $H$ in order to enter into the industrial sector. These individuals would never have entered into the industrial sector if immigration was prohibited in $H$.

Part 2) of Proposition 10 states that because country $H$ has better institutions, the income of an individual who works in the industrial sector in $H$ is strictly higher than the income of an individual who possesses an identical skill level to the former and who works in the industrial sector in $O$. Thus, such an individual of country $O$ has an incentive to immigrate in $H$ only if her difference of income because of immigration exceeds the cost of immigration. This is the same intuition as in Proposition 9, but with a new ingredient in the mix, the talent development. With the opening of the labor markets, there will be an increase in production of the industrial good. This is related to the fact that in the free labor market world, some individuals from $O$ [those with $\bar{\gamma} < \gamma^O < (\gamma^*)^O$] will immigrate in $H$ and enter into the industrial sector. These individuals would have joined the agriculture sector in $O$ if countries did not allow the international free movement of labor.

Since, there is an increase in the production of the industrial good in the world, then the world relative price of the agriculture good will increase. This in turn will increase the income of the individuals who enter into the agriculture sector independent of their job location. We illustrate part 2) of Proposition 10 in fig. 5. We borrow the world relative demand and supply from fig. 4. Thus, the world relative price of the agriculture good in a world with free movement of goods but not labor is represented by $p^W$. According to Proposition 10, in a world with free movements of goods and labor, the number of talented individuals will increase, implying a boost in the production of the industrial good. Hence, the world relative supply curve should shift to the left of $RS^W$, in $(p, Y/X)$ space, when we move in a free international labor market. Consequently, since the world relative demand does not change, the world relative price of the agriculture good should be higher than before. This is indicated by $(p^W_E > p^W)$ in our graph. Thus, another important implication that originates from Proposition 10 is the fact that, through the price effect, immigration increases the income of individuals who work in the agriculture sector.

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15 See Appendix H for the proofs of Proposition 10 and Corollary 4.
Now, let’s consider the case where country \( H \) has identical quality level of institutions, but obtains more individuals with higher levels of early education as compared to country \( O \). The latter is related to the fact that \( H \) offers a better early educational system than \( O \). Therefore, in a free trade world according to Proposition 7, \( H \) exports the industrial good to \( O \) and imports the agriculture good from \( O \). If both countries decide to engage in a free movement of their respective labor force, we must be able to predict the pattern of immigration, if any. In this case, there will be no individual who will have an incentive to immigrate. The intuition is related to the fact that there are no differences in income among individuals with identical skill levels who work in different a location of the industrial sector because institutions are the same in both countries. Consequently, there will be no movement of labor between both countries when they operate in a free movement of labor world despite the fact that \( H \) has a better system of early education than \( O \). Does this mean that the quality of the early educational system plays no role in the development of talent through human migration? The answer is no. We can consider a scenario where the early educational system of the developing country \( (O) \) plays a powerful role in the development of more talent individuals in the world as a result of an open international labor force. This is developed in the following scenario.

Let’s consider a third scenario where \( H \) has better institutions and a better early educational system than \( O \). In such a case, the quality of early educational system in \( O \), the quality of institutions in \( H \), and the costs of immigration play crucial roles on the volume of immigrants, on the development of talent, and on the income of individuals who work in the agriculture sector. Let’s assume that the government of the developing country cares only about the income of the majority of its citizens. Also assume that the cost of improving the quality of its institutions is strictly higher than the cost of improving its early educational system. According to this scenario, the government of \( O \) can use the improvement of her early educational system as a mechanism in
order to promote the development of talent through emigration in a world of free movement of goods and labor, where immigration costs are low enough. This situation is described by Corollary 4.

**Corollary 4.** *In a world that consists of two large economies with free movement of goods and labor, and with imperfect information* \( \theta < 1, \text{and } \beta \in (0, q_m] \), *where* \( \gamma^H > \gamma^O \) *and* \( \theta^H > \theta^O \), *the government of* \( O \) *ameliorates the income of most of its labor force by improving its system of early education since the latter will encourage more emigration of its citizens towards* \( H \).

Corollary 4 states that a government of a developing country can promote the development of more talented individuals in the world simply by improving the quality of its early educational system. The lower the costs of immigration (\( c \)), or/and the higher the difference of the quality of institutions between the two countries (\( \theta^H - \theta^O \)), the more talented individuals will emigrate from \( O \) towards \( H \). This will increase the intensity of talent development in the world, and also increase the efficiency of the government of \( O \) in achieving its goal. In other words, the improvement of the early educational system will increase the relative price of the agriculture good, increasing the income of all individuals that work in the agriculture sector. Since \( O \) is exporting the agriculture good, most of its labor force will enjoy higher income as a result of the emigration of its most talented individuals towards \( H \).

One could wonder if the scenario described in Corollary 4 is valid when countries can not affect the world price. It should be obvious to the reader that Corollary 4 fails to hold in the case of small open economies, since the price of each good will not be affected by trade or immigration. Thus, in such a case the improvement of the early educational system in the origin country of immigrants only will increase the volume of its emigrants for sufficient low immigration costs and will not affect the income of the individuals who work in the agriculture sector. Consequently, in this scenario, the only way for the government of the origin country to ameliorate the income of its citizens is to encourage the development of its institutions.

### 7. Conclusions

In this paper, we have analyzed a simple general equilibrium model with imperfect labor contracts, between two large economies that are incompletely specialized in two sectors. In the industrial sector, there exist only firms that produce a homogeneous industrial good through a team production process. In the agriculture sector, there exist only firms that consist of one employer, the owner, and that produce a homogeneous agriculture good. The heart of our study lies in the determinants of skill distribution in the labor force of each country. In our model the distribution of skills is endogenously determined by each individual subject to her early educational level and the quality of a country’s institutions.

We have described individuals’ decisions on their level of skills, and therefore, on their choice of the sector where they will seek employment, by developing a five-stage game similar to the four-stage game
developed in Vogel (2007). We have shown that the most talented individuals prefer to work in the industrial sector, where the most talented workers match with the most talented managers in the team production process. The most talented individuals have higher incentives to seek employment in the industrial sector because there they gain a higher level of income subject to their skill level. The remaining individuals with less talent join the agriculture sector.

Countries differ in their distribution of their labor force since their early educational system and institutions have different quality levels. It is shown that in a free trade world, the country with the best system of early education, or/and quality of institutions, obtains a labor force that consists mainly of talented individuals. Consequently, it exports the industrial good and imports the agriculture good.

In a two large economies world with free movements of goods and labor, it is shown that the country which exports the industrial good is the host country of immigrants, while the country that exports the agriculture good is the origin country of immigrants. Also we have demonstrated that only the most talented individuals prefer to emigrate towards the host country because there they capture higher incomes to their level of skill, if they can afford the fixed costs of immigration, such as language barriers.

Finally, we have shown that the economic progress of the origin country of immigrants is related to its ability to improve its quality of institutions in order to prevent its most talented individuals from emigrating. We have also described a scenario where the government of the origin country can promote the development of more talented individuals in the world simply by improving the quality of its early educational system. The latter is shown to increase the intensity of talent development in the world because of immigration. This in turn, causes a raise in the relative price of the agriculture good, therefore increasing the income of all individuals who work in the agriculture sector. Thus, since the host country is exporting the agriculture good, most of its labor force will enjoy higher income as a result of the emigration of its most talented individuals towards the host country of immigrants. Consequently, it is argued that immigration influences the individuals’ income via an indirect effect on their incentive to invest in their skill level and a direct effect on the goods’ prices.

It is fair to admit that our model has certain limitations that are related with some of our assumptions. For instance, in our model the efficient matching process that concludes that the most talented worker pairs up with the most talented individual relies on the assumption of complementarities in the production of the industrial good consistent with Kremer’s O-Ring theory of production (Kremer, 1993). Consequently, our definition of talent that the most talented individual is the one with the highest level of optimal training also depends on the latter assumption. Thus, a possible extension of the model is to solve the five stage game developed here under the assumption of substitutabilities in the production of the industrial good. Under this assumption it might be optimal for the most talented managers to pair up with the least talented workers in the efficient matching process. Another possible interesting extension of the model is to include certain spillover effects associated with the availability of the most talented individuals in a country.
Appendix A

In this appendix we provide the proof of Proposition 1 and Corollary 1.

Proof of Proposition 1:
Proposition 1 is proven with the help of the following two steps.

In the first step, we show that if there exists a $y^*$, where $V_q(y^*) = V_p(y^*)$, then this $y^*$ is unique. In the second step, we prove the existence of $y^* \in [y_{\text{min}}, y_{\text{max}}]$.

Step 1.

From equation 24 we know that $V_q(y) = \frac{1}{12} R^4 \Psi^2 q_x$ and from equation 12 we know that $V_p(y) = \frac{1}{3^{2/7} (R p)^{6/7} \sqrt{y}}$. Let's assume that $y^*$ exists. Hence, when $V_q(y^*) = V_p(y^*)$, there exists a $y^*$, such that for any $y > y^*$, $V_q(y^*) \geq V_p(y^*)$, this $y^*$ is unique.

With the help of equations 12 and 24, $V_q(y^*) \geq V_p(y^*)$ can be written as:

$$23/2 \Psi^2 q_x \geq R^2 p^{6/7} \sqrt{y}$$

We can write the optimal skill level of an individual working in the industrial sector as $q_x = \frac{1}{23/2} R^2 \Psi^2 \sqrt{\frac{q_x + 3 \beta^2 (e^{1-p}-1)^2}{\left[q_x + \beta^2 (e^{1-p}-1)^2\right]^2}}$. This expression is determined from the skill level first-order condition for utility maximization in the case of an individual who works in the industrial sector. Substituting this into the above inequality we obtain the following:

$$\frac{q_x + 3 \beta^2 (e^{1-p}-1)^2}{\left[q_x + \beta^2 (e^{1-p}-1)^2\right]^2} \geq p^6$$

Let $L \equiv \frac{q_x + 3 \beta^2 (e^{1-p}-1)^2}{\left[q_x + \beta^2 (e^{1-p}-1)^2\right]^2}$ and $D \equiv p^6$. Then, $\frac{\partial L}{\partial y} > 0$ and $\frac{\partial D}{\partial y} = 0$. This implies that the left hand side of the inequality $A-1$ is increasing in the early educational levels, while the right hand side of $A-1$ is constant in the early educational levels. Consequently, $y^*$ is unique.

Step 2.

Here I start with the proof of $y^* \leq y_{\text{max}}$. Let's assume that $y^* > y_{\text{max}}$. In terms of inequality $A-1$, this implies that $V_q(y_{\text{max}}) > V_p(y_{\text{max}}) \forall y^* \in [y_{\text{min}}, y_{\text{max}}]$. This indicates that no individual will be employed in the industrial sector, which implies that the relative price of the agriculture good approaches zero ($p \to 0$). This implies that $y^* < y_{\text{max}}$. But, this contradicts our assumption that $y^* > y_{\text{max}}$. Hence, $y^* \leq y_{\text{max}}$. In an analogous way, one can show that $y^* > y_{\text{min}}$. This concludes the proof of Proposition 1.

Proof of Corollary 1.

In order to prove both parts of corollary one we must find an expression for $y^*$: From the proof of proposition one, we know that $V_q(y^*) = V_p(y^*)$ at $y^*$. We also know that $q_x = \frac{1}{8} R^4 q_x \frac{q_x + 3 \beta^2 (e^{1-p}-1)^2}{\left[q_x + \beta^2 (e^{1-p}-1)^2\right]^2}$ from equation 23.

Substituting this into $V_q(y^*) = V_p(y^*)$ and rearranging it, we obtain an expression for $y^*$ as indicated below:

$$y^* = \frac{1}{4} \left(\frac{p^3}{R^2 \Psi^2}\right)^{\frac{1}{4}} \frac{\left[q_x + \beta^2 (e^{1-p}-1)^2\right]^2}{\left[q_x + 3 \beta^2 (e^{1-p}-1)^2\right]^2} \geq p^6$$

i) $\frac{\partial y^*}{\partial \theta} = -\left(\frac{p^3}{R^2 \Psi^2}\right)^{\frac{1}{4}} \frac{\left[q_x + \beta^2 (e^{1-p}-1)^2\right]^2}{\left[q_x + 3 \beta^2 (e^{1-p}-1)^2\right]^2} \left(\frac{q_x + 3 \beta^2 (e^{1-p}-1)^2}{q_x + \beta^2 (e^{1-p}-1)^2}\right)^2 < 0 \forall \theta \in [0,1], \beta \in [1, q_m]$.

ii) $\frac{\partial y^*}{\partial y} = -32 \left(\frac{p^3}{R^2 \Psi^2}\right)^{\frac{1}{4}} \frac{1}{q_x} < 0 \forall \theta \in [0,1], \beta \in [1, q_m]$. This concludes the proof of Corollary one.

Appendix B

Proof of Proposition 2.

I prove part one and part two of proposition two with the help of two lemmas. Then part three of proposition two follows.

Lemma 1. $\forall \theta \in [0,1]$ and $\beta \in [1, q_m]$ there exists a $y(\theta)$ where $q_x$ is convex in $\theta$ if and only if $y > y(\theta)$.

Lemma 2.

1) There exists a $y_0 \in [y_{\text{min}}, y_{\text{max}}]$, such that $q_x(y_0) = q_x(y_0)$

2) $\forall \theta \in [0,1]$ and $\beta \in [1, q_m]$ the following inequality is always valid: $y_0 < y^*$

Proof of Lemma 1.

We know from equation 23 that the optimal level of an individual working in the industrial sector is $q_x = \frac{1}{8} R^4 q_x \frac{q_x + 3 \beta^2 (e^{1-p}-1)^2}{\left[q_x + \beta^2 (e^{1-p}-1)^2\right]^2}$. Dividing both sides with $q_x$ we get: $f(q_x, y) = 1 = \frac{1}{8} R^4 \frac{q_x + 3 \beta^2 (e^{1-p}-1)^2}{\left[q_x + \beta^2 (e^{1-p}-1)^2\right]^2} \sqrt{y}$. From the implicit theorem, we know that $\frac{\partial q_x}{\partial y} = -\frac{\partial f}{\partial y}$. $\frac{\partial q_x}{\partial y} = \frac{1}{8} R^4 \frac{q_x + 3 \beta^2 (e^{1-p}-1)^2}{\left[q_x + \beta^2 (e^{1-p}-1)^2\right]^2} \sqrt{y}$ and $\frac{\partial q_x}{\partial \theta} = -\frac{1}{4} R^4 q_x \frac{q_x + 3 \beta^2 (e^{1-p}-1)^2}{\left[q_x + \beta^2 (e^{1-p}-1)^2\right]^2} \sqrt{y}$. This implies that:
This implies that the optimal skill level of an individual working in the industrial sector is increasing in her early educational level $\forall \theta \in [0,1]$ and $\beta \in [1,q_{\theta}]$.

Let $g(q_{x},\gamma) \equiv 0 = \frac{q_{x}^{2} + 4q_{x}^{2}\beta^{2}(e^{1-\theta} - 1)^{2} + 3\beta^{2}(e^{1-\theta} - 1)^{4}}{q_{x}(q_{x}^{2} + 5\beta^{2}(e^{1-\theta} - 1)^{2})^{2}} A$ where $A$ is a constant. Then using the implicit theorem, we know that $\frac{\partial g}{\partial q_{x}} = \frac{\partial^{2} g}{\partial q_{x}^{2}}$. Then, $\frac{g}{\partial q_{x}} = -\frac{q_{x}^{2} + 4q_{x}^{2}\beta^{2}(e^{1-\theta} - 1)^{2} + 3\beta^{2}(e^{1-\theta} - 1)^{4}}{q_{x}(q_{x}^{2} + 5\beta^{2}(e^{1-\theta} - 1)^{2})^{2}} \frac{1}{2} \text{ and } \frac{\partial g}{\partial q_{x}} = \frac{q_{x}^{2} + 11q_{x}^{2}\beta^{2}(e^{1-\theta} - 1)^{2} - 15\beta^{2}(e^{1-\theta} - 1)^{4}}{q_{x}^{2}(q_{x}^{2} + 5\beta^{2}(e^{1-\theta} - 1)^{2})^{2}} \frac{1}{2}$. This implies that:

$$\frac{\partial^{2} g}{\partial q_{x}^{2}} = \frac{1}{2} \frac{q_{x}^{2} + 4q_{x}^{2}\beta^{2}(e^{1-\theta} - 1)^{2} + 3\beta^{2}(e^{1-\theta} - 1)^{4}}{q_{x}^{2} + 11q_{x}^{2}\beta^{2}(e^{1-\theta} - 1)^{2} - 15\beta^{2}(e^{1-\theta} - 1)^{4}} \frac{1}{2} \frac{q_{x}^{2} + 5\beta^{2}(e^{1-\theta} - 1)^{2}}{q_{x}^{2} + 11\beta^{2}(e^{1-\theta} - 1)^{2}}$$

Hence, $\frac{\partial^{2} g}{\partial q_{x}^{2}} > 0$ only if $q_{x} > \gamma(\theta)$, and $\frac{\partial^{2} g}{\partial q_{x}^{2}} < 0$ only if $q_{x} < \gamma(\theta)$, where $\gamma(\theta) \equiv \frac{\beta(e^{1-\theta} - 1)^{2} \gamma^{0.5}}{q_{x}} \sqrt{\frac{3q_{x}^{2} + 5\beta^{2}(e^{1-\theta} - 1)^{2}}{q_{x}^{2} + 11\beta^{2}(e^{1-\theta} - 1)^{2}}}$ One can easily observe that $\lim_{q_{x} \to 0} \gamma(\theta) = 0$ and $\lim_{q_{x} \to \infty} \gamma(\theta) = \infty$. In order to complete the proof of Lemma 1, we have to show the existence of $\gamma(\theta)$. This is done by substituting $\gamma(\theta)$ into the equation 23 of the optimal skill level of an individual working in the industrial sector and putting it into the indirect utility of equation 24. Therefore, $V_{x}[\gamma(\theta)]$ exists and is strictly higher than zero.

Since, we know that the indirect utility with optimal skill levels is strictly convex in individual skill level, and since $\lim_{q_{x} \to 0} \gamma(\theta) = 0$ and $\lim_{q_{x} \to \infty} \gamma(\theta) = \infty$, then $\gamma(\theta)$ is strictly convex in $\gamma(\theta)$ and $q_{x}$ is strictly concave in $\gamma(\theta)$. This concludes the proof of Lemma 1.

**Proof of Lemma 2**

Let’s start by proofing the second part of Lemma 2. Let’s suppose that the first part of Lemma 2 is true. Then, there must exist a $\gamma_{0} \in \{\gamma_{\text{min}}, \gamma_{\text{max}}\}$ such that $q_{x}(\gamma_{0}) = q_{x}(\gamma_{0})$. From equation 23 we know that the optimal skill level of an individual working in the industrial sector is $q_{x} = \frac{1}{6} \beta^{4} q_{x} \frac{q_{x}^{2} + 4\beta^{2}(e^{1-\theta} - 1)^{2}}{q_{x}^{2} + 5\beta^{2}(e^{1-\theta} - 1)^{2}} \gamma$. We also know that the optimal skill level of an individual working in the agriculture sector is $q_{x} = \frac{1}{2\gamma^{2}} R^{3/2} \sqrt{3}$. Hence, from setting $q_{x}$ equal with $q_{x}$, we can write $\gamma_{0}$ as:

$$\gamma_{0} = 8 \left(\frac{R}{2}\right)^{6} \frac{1}{\frac{q_{x}^{2} + 5\beta^{2}(e^{1-\theta} - 1)^{2}}{q_{x}^{2} + 3\beta^{2}(e^{1-\theta} - 1)^{2}}}$$

We know from equation A-2 (See the proof of Corollary one) that $\gamma = \left(\frac{R}{2}\right)^{6} \frac{1}{\frac{q_{x}^{2} + 5\beta^{2}(e^{1-\theta} - 1)^{2}}{q_{x}^{2} + 3\beta^{2}(e^{1-\theta} - 1)^{2}}}$. Therefore, substituting the values of $\gamma_{0}$ and $\gamma$, the inequality $\gamma_{0} < \gamma$ can be written as: $\Psi^{4} < p^{6}$, which is equivalent to $\Psi^{12} < p^{6}$. From A-1 (See the proof of proposition one), we know that at $\gamma = \gamma^{*} \Rightarrow p^{6} = \frac{\Psi^{5}}{\sqrt{\frac{q_{x}^{2} + 3\beta^{2}(e^{1-\theta} - 1)^{2}}{q_{x}^{2} + 3\beta^{2}(e^{1-\theta} - 1)^{2}}}}$. Hence, $\Psi^{12} < p^{6}$ can be written as: $\frac{\Psi^{12}}{\sqrt{\frac{q_{x}^{2} + 3\beta^{2}(e^{1-\theta} - 1)^{2}}{q_{x}^{2} + 3\beta^{2}(e^{1-\theta} - 1)^{2}}}} > \Psi^{19}$. We know that $\Psi = \frac{q_{x}^{2} + 3\beta^{2}(e^{1-\theta} - 1)^{2}}{q_{x}^{2} + 3\beta^{2}(e^{1-\theta} - 1)^{2}}$ by definition. Thus, the above inequality is equivalent with the following inequality:

$$\frac{q_{x}^{2} + 3\beta^{2}(e^{1-\theta} - 1)^{2}}{q_{x}^{2} + 3\beta^{2}(e^{1-\theta} - 1)^{2}} > \frac{q_{x}^{2} + \beta^{2}(e^{1-\theta} - 1)^{2}}{q_{x}^{2} + \beta^{2}(e^{1-\theta} - 1)^{2}}$$

(B - 1)

B-1 is always valid since the left hand side is always higher than one, while the left hand side is always lower than one $\forall \theta \in [0,1); \beta \in [1,q_{\theta}]$. This concludes the proof of the second part of Lemma 2.

Let us now conclude the proof of Lemma 2 by providing the proof of the first part of Lemma 2. This proof consists of two steps. In the first step, we proof the uniqueness of $\gamma_{0}$, and in the second step we proof the existence of $\gamma_{0}$.

**Step 1.**

If there exists $\gamma_{0} \in \{\gamma_{\text{min}}, \gamma_{\text{max}}\}$ such that $q_{x}(\gamma_{0}) = q_{x}(\gamma_{0})$, then $\gamma_{0}$ is unique. The inequality $q_{x}(\gamma_{0}) \geq q_{x}(\gamma_{0})$ can be written as:

$$\frac{1}{2\gamma^{2}} R^{3/2} \sqrt{\frac{q_{x}^{2} + 3\beta^{2}(e^{1-\theta} - 1)^{2}}{q_{x}^{2} + \beta^{2}(e^{1-\theta} - 1)^{2}}} \geq \frac{1}{2\gamma^{2}} R^{3/2} \sqrt{\frac{q_{x}^{2} + 3\beta^{2}(e^{1-\theta} - 1)^{2}}{q_{x}^{2} + \beta^{2}(e^{1-\theta} - 1)^{2}}} > \Psi^{19}$$

The left hand side of the above inequality comes from skill level first-order condition of utility optimization of individuals working in the industrial sector. The right hand side of the above inequality is the optimal skill level of an individual working in the agriculture sector. The above inequality is equivalent with the following:

$$\frac{1}{2\gamma^{2}} R^{3/2} \sqrt{\frac{q_{x}^{2} + 3\beta^{2}(e^{1-\theta} - 1)^{2}}{q_{x}^{2} + \beta^{2}(e^{1-\theta} - 1)^{2}}} \geq \frac{1}{2\gamma^{2}} R^{3/2} \sqrt{\frac{q_{x}^{2} + 3\beta^{2}(e^{1-\theta} - 1)^{2}}{q_{x}^{2} + \beta^{2}(e^{1-\theta} - 1)^{2}}} > \Psi^{19}$$

(B - 2)
Let $L \equiv \left[ p^2 + 2p^2 \left( \frac{e^\gamma - 1}{e^\gamma - 1} \right)^2 \right]^{1/3}$ and $D \equiv p^6$. Then, $\frac{DL}{A} > 0$ and $\frac{D\bar{y}}{A} = 0$. This indicates that the left hand side of the inequality $B-2$ is increasing in $y$, while the right hand side of $B-2$ is constant in $y$. Consequently, $y_0$ is unique.

Step 2.
In order to prove the existence of $y_0$, let’s assume that $y' < y_{\text{min}}$. In terms of inequality $B-2$, this implies that $q_x(y_{\text{min}}) > q_x(y_0)$, where $y \in \left[ y_{\text{min}}, y_{\text{max}} \right]$. Hence, no individual will invest to optimize her skills in the agriculture sector. This shows that no individual will be employed in the agriculture sector, which implies that the relative price of the agriculture sector goes to infinity ($p \rightarrow \infty$). This contradicts the assumption that $y_0 < y_{\text{min}}$. In an analogous way, one can show that $y_0 < y_{\text{max}}$. This concludes the proof of Lemma 2.

**Proof of the first and the second part of Proposition 2**

Now, we are ready to provide the proof of part one and two of Proposition 2. We showed that, $q_x$ is concave in $y$ only when $y < y(\theta)$. One can easily observe from the equation 23 that $\lim_{q_x \rightarrow 0} \frac{1}{\theta} \frac{\partial^2}{\partial q_x^2} q_x - \beta^2 \left( \frac{e^\gamma - 1}{e^\gamma - 1} \right)^2 \cdot y = 0$. Hence, in the region where $q_x$ is concave in $q_x(y) < q_x(y')$, $q_x$ never intersects $q_y$. In Lemma 2, we showed the existence of $y_0$ such that $q_x(y_0) = q_x(y_0)$. Consequently, $q_x$ must be convex in $y$ at $y_0$. Moreover, we showed that $q_x$ is concave in $y \forall y > y_0$. Since, $q_x$ is concave in $y \forall y > 0$, then $q_x > q_y \forall y > y_0$. We showed in the proof of the second part of Lemma 2 that $y_0 < y'$. This implies that $q_x$ is convex in $y \forall y > y'$ and $q_x > q_y \forall y > y'$. This concludes the proof of the first and second part of Proposition 2.

**Proof of the third part of Proposition 2**

We know that the optimal job training level of an individual working in the agriculture sector is linear to her skill level ($t_y = \frac{1}{2} (R_t)^2 q_y$). The optimal training of an individual working in the industrial sector after the efficient matching process, where the most skilled manager pairs up with the most skilled worker, is strictly convex in her skill level. We can prove this by recalling that the optimal level of training for a worker is $t_w = \frac{1}{2} (R_t)^2 q_w$. We also know that the efficient wage after the efficient matching is $w = \Psi^2$ and that $q = q_y = q_x$. Hence, the optimal level of training of a worker employed in the industrial sector is $t_w = \frac{1}{2} R^2 \Psi^2 q_x [1 + \left( \frac{\beta}{q_x} \right)^2]$. One can easily show that $q_x = 0$ and $\frac{\partial t_w}{\partial q_x} > 0$.

Recall that the optimal level of training for a manager who operates her own firm in the industrial sector is $t_m = \frac{1}{2} R^2 \Psi^2 q_m \left[ 1 + \left( \frac{\beta}{q_m} \right)^2 \right]$. Using the same logic as we did in the above case of the worker, we can show that the optimal training of a manager running her own firm in the industrial sector after the efficient matching process is $t_m = \frac{1}{2} R^2 \Psi^2 \left[ 1 + \left( \frac{\beta}{q_m} \right)^2 \right]$. One can easily show that $t_m > 0$.

Hence, we know that $t_x$ is strictly convex in $q_x$, while $t_y$ is strictly linearly increasing in $q_y$. We proved in the first and second part of proposition two that $q_x > q_y \forall y > y'$. Consequently, it is straightforward that $t_x > t_y \forall y > y'$. This concludes the proof of the third part of Proposition 2.

**Appendix C**

Proof of Proposition 3.
We have to prove that $I_y(y) \geq I_y(y') \forall y > y'$. Let’s first find $I_y(y)$ and $I_y(y')$.
In the agriculture sector we assumed that there only exist individual firms. Therefore, in the agriculture sector, the income of each individual is equal to firm’s profit $I_y(a) = p_a$. In the fifth stage, we found the optimal profits for a firm operating in the agriculture sector. Substituting the optimal effort levels as indicated in the fifth stage into $I_y(a)$ we obtain $I_y(t) = p^2 R_t$. In the fourth stage we found the optimal training levels of an individual working in the agriculture sector. Substituting it into $I_y(t)$, we can obtain $I_y(q) = \frac{1}{2} p^3 R^3 q$. In the first stage we found the optimal skill level for an individual working in the agriculture sector. Substituting it into $I_y(q)$, we can obtain the income of an individual working in the agriculture sector with optimal skill levels:

$$I_y(y) = \frac{1}{4 \sqrt{2}} p^6 R^3 \sqrt{y}$$

In the industrial sector we assumed that firms are created by an efficient matching process between workers and managers. We showed that a worker’s income is determined from her wage as stated in the contract. Thus, the worker income is $I_w(a) = wK = w [d_w + \left( \frac{\beta}{\bar{y}^2} \right) d_w]$. In the fifth stage, we found the optimal effort and distortion levels exerted from a worker employed in the industrial sector. Substituting the optimal effort and distortion levels as indicated in the fifth stage into $I_w(a)$ we obtain $I_w(t) = w^2 R_t \left[ 1 + \left( \frac{\beta}{\bar{y}^2} \right) \right]$. In the fourth stage, we determined the optimal level of training obtained by a worker employed in the industrial sector.
Substituting it into \( I_m(q) \) we obtain \( I_m(q) = \frac{1}{2} R^3 \Psi q_m \). In the third stage, we showed that the most skilled managers match with the most skilled workers creating firms. Hence, \( q = q_m = q_m \). This implies that \( I_m(q) = \frac{1}{2} R^3 \Psi q_m \).

A manager’s income is determined from the profit of her firm. Therefore, \( I_m(a) = \pi = 2 \sqrt{a_m a_q} - R^2 \omega \psi_m \Psi^{-1} \). Substituting the optimal effort, distortion, and training levels for a manager who runs her own firm in the industrial sector (in a analogous way with the worker’s case as described above), we can establish that the income of a manager with optimal skill levels after the efficient matching process is \( I_m(q) = \frac{1}{2} R^3 \Psi q_m \). As one can easily observe, \( I_m(q) = I_m(q) = I_s(q) = \frac{1}{2} R^3 \Psi q \). In the first stage we determined the optimal skill level of an individual working in the industrial sector. Substituting it into \( I_s(q) \) we can establish:

\[
I_s(q) = \frac{1}{16} R^3 \Psi q_s \left( \frac{q_s^3 + 3 \beta^2 (e^{1-\theta} - 1)^2}{(q_s^3 + \beta^2 (e^{1-\theta} - 1)^2)^2} \right)^{\frac{3}{2}} \tag{C-2}
\]

Therefore with the help of C-1 and C-2 the inequality \( I_s(q) \geq I_s(q) \forall \Psi > \Psi^* \) now can be written as:

\[
\Psi > \frac{1 - \frac{2 \beta^2 (e^{1-\theta} - 1)^2}{(q_s^3 + \beta^2 (e^{1-\theta} - 1)^2)^2}}{\left( \frac{1}{\Psi^*} \right)^{\frac{3}{2}}} \tag{D-1}
\]

Since we need to show that the above inequality stands for all \( \Psi > \Psi^* \), we can substitute \( \Psi > \Psi^* \) from equation A-2 (see the proof of Corollary one) that \( \Psi^* = \frac{1}{\Psi} \left( \frac{1}{\Psi^*} \right)^{\frac{3}{2}} \). Hence, the above inequality can be written as:

\[
1 > \frac{q_s^3 + \beta^2 (e^{1-\theta} - 1)^2}{(q_s^3 + \beta^2 (e^{1-\theta} - 1)^2)^2} \Rightarrow \beta^2 (e^{1-\theta} - 1)^2 > 0. \text{ This is always valid } \forall \theta \in (0,1); \beta \in (1, q_m). \text{ This concludes the proof of Proposition 3.}
\]

**Appendix D**

In this appendix, we provide the proof of Proposition 4 and Corollary 2.

**Proof of Proposition 4.**

I follow the same strategy with the proof of Proposition 1. Thus, Proposition 4 is proven with the help of the following two steps.

In the first step, we show that if there exists a \( \gamma^* \), where \( V_k(\gamma^*) = V_k(\gamma^*) \), then this \( \gamma^* \) is unique. In the second step, we prove the existence of \( \gamma^* \in (0, +\infty) \).

**Step 1.**

From equation 34 we know that \( V_k(\gamma) = \frac{1}{24 \sqrt{2}} R^6 \left( \frac{1}{1 + (e^{1-\theta} - 1)^2} \right)^{\frac{3}{2}} \sqrt{\Psi} \) and from equation 12 we know that \( V_k(\gamma) = \frac{1}{3 \gamma^2 \pi} (R \rho)^6 \sqrt{\Psi} \). Let’s assume that there exists \( \gamma^* \). Hence, when \( V_k(\gamma^*) = V_k(\gamma^*) \), there exists a \( \gamma^* \), such that for any \( \gamma > \gamma^* \), \( V_k(\gamma^*) \geq V_k(\gamma^*) \), this \( \gamma^* \) is unique.

With the help of equations 12 and 34, \( V_k(\gamma^*) \geq V_k(\gamma^*) \) can be written as:

\[
\frac{1}{1 + (e^{1-\theta} - 1)^2} \geq \frac{1}{(1 + \frac{e^{1-\theta} - 1)^2}{2} \geq (D-1)
\]

Let \( L \equiv \frac{1}{1 + (e^{1-\theta} - 1)^2} \), and \( D \equiv (D-1) \). Then, \( \frac{\partial L}{\partial \gamma} > 0 \) and \( \frac{\partial D}{\partial \gamma} = 0 \). This implies that the left hand side of the inequality \( D-1 \) is increasing in the early educational levels, while the right hand side of \( D-1 \) is constant in the early educational levels. Consequently, \( \gamma^* \) is unique.

**Step 2.**

Let’s assume that \( \gamma^* < 0 \). In terms of inequality \( D-1 \), this implies that \( V_k(0) > V_k(0) \). This indicates that no individual will be employed in the agriculture sector, which implies that the relative price of the agriculture sector goes to infinity (\( p \to \infty \)). This contradicts the assumption that \( \gamma^* < 0 \). This concludes the proof of proposition 4.

**Proof of Corollary 2.**

From equation 34 we know that \( V_k(\gamma) = \frac{1}{24 \sqrt{2}} R^6 \left( \frac{1}{1 + (e^{1-\theta} - 1)^2} \right)^{\frac{3}{2}} \sqrt{\Psi} \) and from equation 12 we know that \( V_k(\gamma) = \frac{1}{3 \gamma^2 \pi} (R \rho)^6 \sqrt{\Psi} \). Then it is obvious that \( \frac{\partial \gamma}{\partial \theta} < 0 \) and \( \frac{\partial \gamma}{\partial \rho} = 0 \). while \( \frac{\partial \gamma}{\partial \theta} = \left( \frac{1}{24 \sqrt{2}} R^6 \left( \frac{1}{1 + (e^{1-\theta} - 1)^2} \right)^{\frac{3}{2}} \sqrt{\Psi} \right) e^{1-\theta} (e^{1-\theta} - 1) \sqrt{\Psi} > 0 \). This concludes the proof of Corollary 2.

**Appendix E**

**Proof of Proposition 5.**

The proof of proposition 5 requires the introduction and the proof of the following Lemma:

**Lemma 3.**

1) There exists a \( y_0 \in (0, +\infty) \), such that \( q_s(y_0) = q_q(y_0) \)

2) \( \forall \theta \in (0,1), \beta = q_m \), the following equality is satisfied \( y_0 = \gamma^* \)

**Proof of Lemma 3.**

Let’s start by proving the second part of Lemma 2. Let’s suppose that the first part of Lemma 3 is true. Then, there must exists a \( y_0 \in (0, +\infty) \), such that \( q_s(y_0) = q_q(y_0) \). From equation 34 we know that the optimal skill level of an individual working in the industrial sector is \( q_s = \frac{R^3}{24 \sqrt{2}} \sqrt{\frac{\Psi}{1 + (e^{1-\theta} - 1)^2}} \). We also know that the optimal skill level of an individual working in the agriculture sector is
Hence, setting \( q_x(y_0) = q_y(y_0) \) we get the following \( \frac{1}{1 + (e^{1-\theta} - 1)^2} \geq p^4 \). Thus, \( q_x(y_0) \geq q_y(y_0) \) can be written as:

\[
q_y = \frac{1}{\sqrt{2}} (\sqrt{p})^2 \sqrt{\gamma}.
\]

Combining inequality D-1, with inequality E-1, it must be that \( y_0 = \gamma^* \). This concludes the proof of the second part of Lemma 2.

Let us now, conclude the proof of Lemma 2 by providing the proof of the first part of Lemma 2. This proof consists of two steps. In the first step, we prove the uniqueness of \( y_0 \), and in the second step we prove the existence of \( y_0 \).

Step 1.

If there exists a \( y_0 \in (0, \infty) \), such that \( q_x(y_0) = q_y(y_0) \), then \( y_0 \) is unique. We know form E-1 that the inequality \( q_x(y_0) \geq q_y(y_0) \) can be written as:

\[
\frac{1}{1 + (e^{1-\theta} - 1)^2} \geq p^4 \quad (E - 1)
\]

Let \( L \equiv \frac{1}{1 + (e^{1-\theta} - 1)^2} \) and \( D \equiv p^4 \). Then, \( \frac{\partial L}{\partial y} > 0 \) and \( \frac{\partial D}{\partial y} = 0 \). This indicates that the left hand side of the inequality B-2 is increasing in \( y \), while the right hand side of E-1 is constant in \( y \). Consequently, \( y_0 \) is unique.

Step 2.

In order to proof the existence of \( y_0 \), let’s assume that \( \gamma^* \leq 0 \). In terms of inequality B-2, this implies that \( q_x(0) > q_y(0) \). Hence, no individual will invest to optimize her skills in the agriculture sector. This shows that no individual will be employed in the agriculture sector, which implies that the relative price of the agriculture sector goes to infinity \( (\rho \to \infty) \). This contradicts the assumption that \( y_0 \leq 0 \). This concludes the proof of Lemma 2.

Since we showed that \( \gamma^* = y_0 \) where at \( y_0 \), \( q_x(y_0) = q_y(y_0) \), and from proposition 4 we know that individuals will enter into the industrial sector for all \( y > \gamma^* \), then it must be that \( q_x(y_0) > q_y(y_0) \) for all \( y > \gamma^* \). This concludes the proof of proposition 5.

### Appendix F

**Proof of Proposition 6.**

We have to prove that \( I_x(\gamma) \geq I_y(\gamma) \) \( \forall \gamma > \gamma^* \). Let’s first find \( I_x(\gamma) \) and \( I_y(\gamma) \).

In the agriculture sector \( I_x(\gamma) \) is the same regardless of contract design. We know from equation C-1 that \( I_x(\gamma) = \frac{1}{4\sqrt{2}} p^3 R^3 \sqrt{\gamma} \) (See Appendix C).

In the industrial sector with \( \beta = q_m \), a worker’s income is \( I_w(a) = wK = w[a_w + (e^{1-\theta} - 1)d_w] \). In the fifth stage, we found the optimal effort and distortion levels exerted by a worker employed in the industrial sector. Substituting the optimal effort and distortion levels as indicated in the fifth stage into \( I_w(a) \) we obtain \( I_w(t) = w^2 R t w_s [1 + (e^{1-\theta} - 1)^2] \). In the fourth stage, we determined the optimal level of training obtained by a worker employed in the industrial sector. Substituting it into \( I_w(t) \) we obtain \( \omega_q(a) = \frac{w^4}{2} R^2 [1 + (e^{1-\theta} - 1)^2] q_w \). In the third stage, we showed that the most skilled managers match with the most skilled workers creating firms. Hence, \( q = q_w = q_m \). We also determined the efficient wage. Substituting them into \( I_w(q) \) implies that \( I_w(q) = \frac{1}{2} R^3 \frac{q}{1 + (e^{1-\theta} - 1)^2} \).

A manager’s income is determined from the profits of her firm. Therefore, \( I_m(a) = \pi = 2\sqrt{a_m a_w} - w[a_w + (e^{1-\theta} - 1)d_w] \). Substituting the optimal effort, distortion and training levels for a manager who runs her own firm in the industrial sector (in a analogous way with the worker’s case as described above), we can establish that the income of a manager with optimal skill levels after the efficient matching process is \( I_m(q) = \frac{1}{2} R^3 \frac{q}{1 + (e^{1-\theta} - 1)^2} \). As one can easily observe, \( I_m(q) = I_w(q) = I_x(q) = \frac{1}{2} R^3 \frac{q}{1 + (e^{1-\theta} - 1)^2} \). In the first stage, we determined the optimal skill level of an individual working in the industrial sector. Substituting it into \( I_x(q) \) we can establish

\[
I_x(\gamma) = \frac{1}{4\sqrt{2}} R^3 \left[ \frac{1}{1 + (e^{1-\theta} - 1)^2} \right] \sqrt{\gamma} \quad (F - 1)
\]

Therefore with the help of C-1 and F-1 the inequality \( I_x(\gamma) \geq I_y(\gamma) \) \( \forall \gamma > \gamma^* \) can now be written as:

\[
\frac{1}{1 + (e^{1-\theta} - 1)^2} \geq p^4
\]

The above inequality is exactly the same as the inequality D-1. Thus, for all \( I_x(\gamma) \geq I_y(\gamma) \) \( \forall \gamma > \gamma^* \). This always is valid \( \forall \theta \in [0,1] \), \( \beta = q_m \). This concludes the proof of Proposition 6.

### Appendix G

In this appendix, we provide the proof of propositions 7, 8 and Corollary 3.

**Proof of Proposition 7.**

The proof of uniqueness and existence of \( (\gamma^*)^0 \), is exactly the same as the proof of uniqueness and existence of \( (\gamma^*)^0 \) in a closed economy of Proposition 1 (see the first and second steps of the proof of Proposition 1 in appendix A).

The proof of the second part of Proposition 7 is straightforward. \( (\gamma^*)^0 < (\gamma^*)^0 \) since \( \frac{\partial \gamma^*}{\partial \gamma} \) regardless of the country index (see equation A-2 in the proof of Corollary 1 provided in Appendix A). But, \( (\gamma^*)^0 > (\gamma^*)^0 \), by assumption. This implies that \( q^M > q^0 \forall \gamma > (\gamma^*)^0 \). The argument for the existence of the latter inequality comes directly from the first and second part of Proposition 2.

**Proof of the third part of Proposition 7.**

Let’s start with the proof of the inequality \( (\gamma^*)^0 \geq \gamma_{\text{min}} \). Assume that \( (\gamma^*)^0 < \gamma_{\text{min}} \). In the above paragraph we proved that \( (\gamma^*)^0 < (\gamma^*)^0 \). This implies that \( (\gamma^*)^0 < \gamma_{\text{min}} \) and \( (\gamma^*)^0 < (\gamma^*)^0 \). But, if both \( (\gamma^*)^0 \) and \( (\gamma^*)^0 \) are strictly lower than \( \gamma_{\text{min}} \), then no one enters.
into the agriculture sector, which implies that the relative price of the agriculture good \((p)\) goes to infinity (see inequality A-1 in Appendix A). This indicates that both \((y^*)^R\) and \((y^*)^0\) must be strictly higher than \(y_{min}\). Consequently, \((y^*)^R \geq y_{min}\). One can show that \((y^*)^R \leq y_{max}\) following an analogous proof as the one provided above. This concludes the proof of Proposition 7.

**Proof of Proposition 8.**

The proof of uniqueness and existence of \((y^*)^l\), is identical to the uniqueness and existence of \(y^*\) of Proposition 1 (see Appendix A).

**Proof of the second part of Proposition 8.**

\((y^*)^R < (y^*)^0\) since \(\frac{\sigma_y}{\sigma_b} < \frac{\sigma_y}{\sigma_b}\) regardless of the country index (see the proof of Corollary 1 provided in Appendix A). But, we assumed that \(\theta^R > \theta^0\). Hence, \((y^*)^R < (y^*)^0\).

The proof of the third part of Proposition 8 is exactly the same as the one of the third part of proposition 7. This concludes the proof of Proposition 8.

**Proof of Corollary 3.**

The proofs of all inequalities of Corollary 3 are exactly analogous with the proofs of Propositions 2 and 3 (see Appendix B and C).

### Appendix H

In this appendix, we provide the proof of propositions 9, 10 and Corollary 4.

**Proof of Proposition 9.**

The proof of part 1 and 2 of Proposition 9 is analogous to that of Proposition 8, and the proof of part 3 of Proposition 9 is analogous to that of Corollary 3 (see Appendix G).

The proof of part 4 of Proposition 9:

We drop the superscript \((j)\) when necessary for notation simplicity. We first prove that \(y^*\) exists. Then, we show that there also exists a \(\bar{y}\) such that \(I_x(\bar{y})^R > [I_x(\bar{y})^0 - c] \forall y > \bar{y}\).

The proof of the existence of \(y^*\):

We know from Proposition 1 (see Appendix A) that \(V_x(y) \geq V_x(y) \forall y > y^*\). Moreover, according to part 3 of Proposition 9 \((y^*)^R < (y^*)^0 \forall y > 0\) because \(\theta^0 < \theta^R\) and \(\frac{\partial(y^*)^R}{\partial y} < 0\). This implies that \(I_x(y^*)^R > I_x(y^0) \forall y > (y^*)^0\). Since \(y^*\) exists, then \(\bar{y}\) must also exist for low enough values of \(c\).

We know that \(I_x(\bar{y})^R > I_x(\bar{y})^0 \forall y > (y^*)^0\). Since \(V_x(\bar{y})^R\) is positive, then there must exist a \(\bar{y}\) such that for any positive \(c\), \(V_x(\bar{y})^R = [V_x(\bar{y})^0 - c] \forall y > \bar{y}\). Hence, \(V_x(\bar{y})^R > [V_x(\bar{y})^0 - c] \forall y > \bar{y}\) From Proposition 3 we know that \(I_x(y) \geq I_x(y) \forall y > y^*\). Hence, there exists a \(\bar{y}\) such that \(I_x(\bar{y})^R > [I_x(\bar{y})^0 - c] \forall y > \bar{y}\) and \(I_x(y) \geq I_x(y) \forall y > \bar{y}\). Since country \(H\) is exporting \(Y\) and country \(O\) is exporting \(X\), then the individuals who obtain the highest level of income are those who work in the industrial industry from \(H\). Thus, the flow of labor movement will be from \(O\) to \(H\). This completes the proof of Proposition 9.

**Proof of Proposition 10.**

**Proof of part 1) of Proposition 10:**

From Proposition 9 we know that \(I_x(\bar{y})^R > [I_x(\bar{y})^0 - c] \forall y > \bar{y}\) since \(\theta^R > \theta^0\). Hence, only those individuals of \(O\) with \(y > \bar{y}\) have an incentive to seek employment in \(H\). No individual who works in the agriculture sector has an incentive to emigrate in either country since \(I_x(\bar{y})^R = I_x(\bar{y})^0 \forall y > 0\). This is related to the fact that \(\frac{\partial I_x(\bar{y})^R}{\partial y} = 0 \Rightarrow \frac{\partial I_x(\bar{y})^0}{\partial y} = 0\). Therefore, only individuals who work in the industrial sector have an incentive to emigrate in \(H\) because \(\frac{\partial I_x(\bar{y})^R}{\partial y} > 0 \Rightarrow \frac{\partial I_x(\bar{y})^0}{\partial y} > 0\). However, not all individuals of \(O\) that work or will seek employment in the industrial sector will emigrate to \(H\). There would be some of them whose income difference because of immigration is lower than the fixed costs of immigration. These individuals will work in the industrial sector in \(O\). The early education level of such individuals is \(y^* < y^0 < \bar{y}\). Hence, all individuals of \(O\) with \(y > \bar{y}\) will emigrate in \(H\). These are the most talented individuals of \(O\) will choose to emigrate in \(H\) because \(\frac{\partial y^*}{\partial x} \geq \frac{\partial y}{\partial x} \geq 0 \forall y > y^*\) and moreover \(\forall y > \bar{y}\).

**Proof of part 2) of Proposition 10:**

We know from part 3) of Proposition 9 that \((y^*)^R < (y^*)^0\). This implies that \(y^* < (y^*)^0\). This in turn implies that \(I_x(y^*)^R > I_x(y^0) \forall y > y^*\). Also from part 4) of Proposition 9, we know that \(y^* < \bar{y}\) and \(I_x(y^*)^R > [I_x(y)^0 - c] \forall y > \bar{y}\). But, when \(c\) approaches zero then \(I_x(y)^R > [I_x(y)^0 - c] \forall y > \bar{y}\), which also is true for \(\forall (y^*)^0\). Hence, there must exist a threshold \(\bar{y}\), such that \(I_x(y)^R > [I_x(y)^0 - c] \forall y > \bar{y}\), where \((y^*)^R < \bar{y} \geq \bar{y}\). This completes the proof of Proposition 9.
Proof of Corollary 4.
The proof of Corollary 4 is straightforward. In Proposition 10 we showed that \( I_\gamma(y)^H > [I_\gamma(y)^O - \varepsilon] \forall y > \varepsilon \). Then according to fig. 5 the relative price of the agriculture good increases from the world increase in production of the industrial good because of emigration of individuals of \( O \), with \( y > \varepsilon \), towards \( H \). Assuming that the following inequality \( \Psi \frac{\alpha_1 \gamma \alpha_1}{\varepsilon^2} \geq p^0 \) is still valid after immigration, then \( \frac{\partial \gamma(y)}{\partial p} > 0 \) because \( \frac{\partial \gamma(y)}{\partial p} > 0 \). Since \( I_\gamma(y)^I = I_\gamma(y)^H = I_\gamma(y)^O \) and most of the individuals who already work or seek employment in the agriculture sector are in \( O \) and no one of them has an incentive to emigrate in \( H \), then their income will keep increasing as their government improves the early education system. This completes the proof of Corollary 4.

References


