Costly Evidence Production and the Limits of Verifiability

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Revised September 2006

Abstract

This paper explores the limits of “verifiability” induced by the process of costly evidence production in contractual relationships of complete information. I study how the cost of providing evidence (disclosing documents) influences the set of enforceable contracts. I show that evidence cost can be both beneficial and detrimental with regard to enlarging the set of settlement outcomes that can be implemented. Further, I study how what can be considered verifiable is influenced by parties’ incentives to produce evidence and by the particular evidence cost structure. My analysis includes the opportunity for contracting parties to renegotiate (or settle) prior to the enforcement phase. I also study how the availability of redundant documents expands the set of enforceable contracts, and discuss the relevance of my findings to the design of legal institutions.

JEL Classification: C70, D74, K10.

In this paper, I present a game theoretic model to show how verifiability — that a court can observe a given aspect of a contractual relationship — depends on evidence production costs. In practice, courts do not simply observe a given aspect of a contractual relationship. Instead, court action depends upon the actual evidence that is presented to the court by the contracting parties, and this evidence production is costly. Thus, verifiability depends on both the cost of producing evidence and on the parties’ incentives to produce evidence.

This paper builds from the analysis of Bull and Watson (2004), who provide a foundation for verifiability by explicitly modeling evidence disclosure and contract enforcement in contractual relationships of complete information. There, evidence is costless to disclose when it is available, and simply cannot be disclosed when it is not.

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available. They treat evidence cost as binary in that presenting evidence has a cost of zero when it is available, and an infinite amount when it is not available. However, in practice, there may be a moderate cost associated with disclosing available evidence. For example, it may be costly to locate available documents in one’s files, to transport physical objects to the external enforcer, or to present documents or testimony. This paper’s key departure from that analysis is that here evidence production is *costly* in a moderate sense.\(^1\)

Here, players interact over five periods of time. They first agree to a contract. In the second period, they engage in productive interaction. They have the opportunity to renegotiate the original contract in the third period. Renegotiation can be viewed as pretrial settlement. Players simultaneously produce costly evidence in the fourth period, and, in the fifth period, the court imposes transfers on the basis of evidence presented. As the enforcement phase (the fourth and fifth periods) typically involves the costly production of evidence, settlement is jointly beneficial to the players. The surplus of this renegotiation is the evidence production costs that would be incurred if external enforcement were to occur with the original contract in place.\(^2\) It is the outcome of settlement, which depends upon the actual productive outcome that is realized, that shapes players’ incentives during the productive phase of interaction. Thus, my analysis focuses on the settlement outcomes that can be implemented.

One important result from this model is that evidence cost can be both beneficial and detrimental with regard to enlarging the set of settlement outcomes that can be implemented. Clearly, if two different states are to have different settlement outcomes, then at least one player must have the incentive to produce different evidence in one state than in the other. This implies a different evidence production cost in one state than in the other. However, when the players renegotiate, this cost difference is divided between them. So depending upon the particular cost, it may be possible that either more or less can be implemented than in the costless setting. My main technical result is to characterize the implementable settlement outcomes. I show that the set of enforceable contracts is sensitive to both the manner by which states can be distinguished from each other and the cost of doing so.

One feature of the evidence environment that has implications for institutions are, what I call, *redundant documents*. Two documents are redundant if they are available in exactly the same contingencies. I show that redundant documents are quite useful in that they provide the flexibility to tailor the cost of equilibrium evidence production to fit the desired outcome of settlement.\(^3\) Further, the study of redundant documents forms the basis for my analysis of two legal rules. The first rule deals with the

\(^1\)Another difference is that I restrict attention to contracts between two parties, while the analysis of Bull and Watson (2004) applies to \(n\)-player settings.

\(^2\)This is because players can avoid the cost of producing evidence by settling prior to litigation.

\(^3\)That is, though the available documents and the cost of producing them is exogenous, it is possible to design contracts that induce different documents to be disclosed and different costs to be incurred.
feature of the adversarial legal system known as the “discovery” process. I show that allowing parties to request evidence from each other expands the set of implementable settlement outcomes by creating redundant documents. I also study a legal rule which requires that the court not condition upon which player provided a redundant document. This is motivated by the court ruling based upon the facts of the case. I show that this rule reduces the set of settlement outcomes that can be implemented. I then explore the impact of both rules being imposed and discuss ways of reducing the impact of the second legal rule.

Since I study complete information settings, this model is, in a sense, one of Nash implementation with renegotiation where the renegotiation takes place before costly evidence is disclosed. However, it is not a standard mechanism-design model because it specifies inalienable (and costly) evidentiary decisions and state-contingent evidence sets. In terms of mechanism design, this model has fixed message spaces, state-dependent constraints on what players can say, and sending messages involves a cost (which may be state-dependent). Green and Laffont (1986) previously studied a model of this type without moderate cost. However, the focus and results here are not related. Instead, the results here rely on the nature of “public actions” in my model in which the court only compels transfers. I discuss why this additional structure is somewhat realistic. Evans (2006) consider costs of sending messages in a mechanism design framework, but those costs do not depend on the state and the focus of his analysis differs from that here. Bull (2006) presents a model of mechanism design with moderate evidence cost in a complete information setting.

My focus is on contracting parties’ opportunities to present documented evidence, and the cost of doing so. As such, this model differs from other models of court decision making and evidence production. Some more recent papers have analyzed some
settings with state-dependant evidence production cost. Sanchirico (2000) models the court’s decision as depending upon the evidence presented at trial. In his model, the cost of producing evidence depends upon the state, and evidence can be forged (at a greater cost). The analysis there focuses on a tort setting with two possible states, and is geared towards providing an explanation of the English transition toward a more passive fact-finding jury. Sanchirico and Triantis (2004) consider a contract setting in which a single player presents different levels of evidence that are costly to produce, and all evidence can be forged at a greater cost. They present optimal contracts that induce evidence forgery. Deneckere and Severinov (2001) consider a principal-agent setting where the agent communicates with the principal over multiple periods and can send any, and only one, of his messages each period. These messages are basically the same as the documents presented here. In one part of their analysis, they consider a setting where the agent incurs a cost of lying, but no cost of truth telling. In my model, evidence disclosure by the players occurs simultaneously. This is consistent with much of the literature on evidence production, and I would suggest is an appropriate starting point for understanding the effects of moderate evidence production cost.7

The paper is organized as follows. In Section 1, I describe the model of costly evidence production. In Section 2, I discuss the relationship between evidence production costs and implementable outcomes of settlement. There I specifically demonstrate how the additional feature of evidence production being costly changes the set of implementable outcomes. I provide a characterization of implementable outcomes in Section 3. I begin to explore implications for institutional design in Sections 4 and 5. I study the flexibility allowed by the redundancy of documents in Section 4. Further, I discuss how contracts and evidence rules can be structured to take make use of redundant documents. In Section 5, I explore the implications of two rules of evidence when evidence is costly to produce. Appendix A contains proofs not found in the text. Appendix C briefly examines the no settlement case.

1 A Model of Contract, Evidence Disclosure, External Enforcement, and Settlement

I consider a contractual relationship between two players (also called agents) who interact over five periods of time. In the first period, the players form a contract. This contract has an externally enforced component $m$ which specifies monetary transfers to be compelled by the court in period 5, conditional on evidence presented to the court in period 4. The contract also has a self-enforced component, which specifies the players’ individual behavior in the contractual relationship. The interaction of players is described in Figure 1.

7My plans for future research include analyzing dynamic mechanisms in a setting of moderately costly evidence production. Bull (2006) is one project in that direction.
In the second period, productive interaction occurs, leading to an outcome $a$ which I call the state of the relationship. The state is commonly observed by the players. I let $A$ denote the set of possible states and I assume $A$ is finite. Players receive an immediate payoff given by $u : A \to \mathbb{R}^2$. My analysis focuses on how the state influences the subsequent settlement outcome.\(^8\)

In the third period, the players have the opportunity to renegotiate $m$. At the end of period 3 the players’ renegotiation puts $m'$ in place. If the players do not renegotiate, then $m' \equiv m$. Renegotiation can be interpreted as pretrial settlement.\(^9\) I do not explicitly model the procedure by which players renegotiate. Instead I assume that the outcome of renegotiation is given by the generalized Nash bargaining solution. Players commonly know the state $a$ and available documents which may be provided in court. Thus, the outcome of external contract enforcement of the initial contract $m$ in court defines the disagreement point for their bargaining. I denote player $i$’s bargaining weight by $\pi_i$.

In period 4 the players simultaneously and independently disclose documents which are presented to the court.\(^10\) The production of evidence is costly in two respects. First, each document may exist in one state but not in another (infinite cost in the latter state). Denote by $D_i(a)$ the set of documents that can be presented by player $i$ in state $a$. Since not all documents may be available in all states, $D_i(a) \neq D_i(b)$ is typically, but not necessarily, the case. Let $D_i \equiv \bigcup_{a \in A} D_i(a)$ denote the set of documents available to player $i$ over all states. I assume $D_i$ is finite. I also assume $D_1$ and $D_2$ are disjoint sets. That is, each player’s documents are unique. I let $D \equiv D_1 \cup D_2$ and $D(a) \equiv D_1(a) \cup D_2(a)$ for each $a$. For any set of documents $E \subset D$, I write $E_i$ as those documents produced by player $i$. The feasible sets of produced documents are given by $\mathcal{D} \equiv \{ E \mid E \subset D(a), \text{ for some } a \in A \}$. Note that the empty set (no documents disclosed) is an element of $\mathcal{D}$.

The second kind of cost is a more moderate cost the players must pay as a function of the evidence set. An example of this type of cost is that if a party is to produce

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\(^8\)A detailed treatment of how the behavior later in the game influences implementable productive actions is found in Bull and Watson (2004).

\(^9\)Of course, the surplus maximizing outcome will specify a constant $m'$ and no evidence will be produced.

\(^10\)Of course, renegotiation will optimally lead to a constant transfer $m'$, and no documents will actually be disclosed.
a canceled check, it may be necessary to locate it in her files. I denote the cost of gathering the set of documents \( E \) to player \( i \) as \( \gamma_i(E) \), where \( \gamma_i : D \rightarrow R_+ \). This allows for player \( i \)'s evidence production cost being influenced by those documents produced by player \( j \). Documents represent evidence on which the court conditions transfers.

In period 5 the court imposes the transfer \( m' \) as a function of the documents disclosed by the players.\(^{11}\) I assume \( \sum_{i \in N} m'_i \leq 0 \). Formally, \( m' : D \rightarrow R^2 \), so for any evidence set \( E \in D \), \( m'_i(E) \) is the monetary transfer made to player \( i \). Thus, player \( i \)'s total payoff in the contract game, if the players go to court, is \( u_i(a) + m_i(E) - \gamma_i(E) \). Remember that \( m \) is jointly selected by the players in period 1.

In practice courts generally cannot impose fines (which impose that a player pay a transfer to a third party) in contract cases.\(^ {12} \) That is, the court can only impose transfers between litigants. This justifies restricting attention to balanced externally-enforced contracts, which are functions of the form \( m : D \rightarrow R^2_0 \), where \( R^2_0 \equiv \{ x \in R^2 \mid \sum_{i=1,2} x_i = 0 \} \).\(^ {13} \)

I apply the term disclosure rule to any function \( \beta : A \rightarrow D \), satisfying \( \beta(a) \subset D(a) \) for each \( a \in A \). This function describes how the players behave in period 4, conditional on the state. For example, in state \( a \) the players disclose documents \( \beta(a) \). Let \( \beta_i(a) \) denote the documents presented by player \( i \) in state \( a \). I refer to the interaction of players in period 4 and the court imposition of transfers as an evidence production game.

Renegotiation in period 3 leads to a state-contingent continuation value function. This is found by working backward from the enforcement outcome with \( m \) in place. The players can avoid the cost of evidence disclosure anticipated with \( \beta \) and \( m \) (in the evidence production game) by renegotiation. This implies, given \( a \), that players negotiate over how to divide the joint surplus of \( \gamma_1(\beta(a)) + \gamma_2(\beta(a)) \). Given state \( a \) the generalized Nash bargaining solution implies that the state-contingent continuation value of player \( i \) is given by

\[
m_i(\beta(a)) - \gamma_i(\beta(a)) + \pi_i[\gamma_1(\beta(a)) + \gamma_2(\beta(a))].
\]

I refer to the state-contingent continuation value as a value function and denote it by \( g : A \rightarrow R^2 \). The definition of a value function and the possibility of renegotiation between periods 4 and 5 (which justifies restricting attention to balanced externally enforced transfers) justify restricting attention to balanced value functions. That

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\(^{11}\)The model applies equally well to other external enforcement systems.

\(^{12}\)See, for example, Barnett (1999).

\(^{13}\)Further, this constraint may reflect the players’ ability to renegotiate between periods 4 and 5. Suppose the players can renegotiate the externally-enforced contract \( m' \) (or \( m \) if no renegotiation outcome is reached) between periods 4 and 5. If their outstanding contract is such that \( \sum_{i=1,2} m_i(E) < 0 \) for some \( E \), then following disclosure \( E \) the players would re-specify \( m \) before the court compels transfers.
is, I restrict attention to value functions of the form \( g : A \rightarrow \mathbb{R}^2_0 \). An externally-enforced contract \( m \) and a disclosure rule \( \beta \) imply the value function \( g \). The following terminology will be useful.

**Definition 1** A value function \( g \) is implemented by externally enforced contract \( m \) and disclosure rule \( \beta \) if
\[
g_i(a) = m_i(\beta(a)) - \gamma_i(\beta(a)) + \pi_i[\gamma_1(\beta(a)) + \gamma_2(\beta(a))]
\]
for every \( a \in A \), for \( i = 1, 2 \).

The notion is that for a given \( m \), \( \beta \), and \( a \), the players know what will be the outcome of going to court. Their renegotiation (pretrial settlement) results in the value \( g(a) \). I require that players use disclosure rules that induce Nash equilibrium in the evidence production game for all contingencies. Given \( m \), I call \( \beta \) an equilibrium disclosure rule if \( \beta \) specifies an equilibrium of the evidence production game for every \( a \); that is,
\[
m_i(\beta(a)) - \gamma_i(\beta(a)) \geq m_i(E_i \cup \beta_j(a)) - \gamma_i(E_i \cup \beta_j(a))
\]
for any \( E_i \subset D_i(a) \), for all \( a \in A \), and for each \( i = 1, 2 \). I describe the value functions that can be implemented given equilibrium behavior in the evidence production game as follows.

**Definition 2** A value function \( g \) is called implemented by equilibrium disclosure rule \( \beta \) if and only if there exists an externally enforced contract \( m \) such that
\[
(i) \ \beta \text{ is an equilibrium disclosure rule and (ii) } g \text{ is implemented by } m \text{ and } \beta.
\]

My analysis focuses on the implementable value functions as these describe the payoffs players expect to receive conditional on the state (which summarizes behavior in the production phase of the relationship).\(^{14}\) I consider whether a value function \( g \) can be implemented given a disclosure rule \( \beta \) since the particular \( \beta \) is crucial in determining which states can be differentiated. That is, the continuation values may differ under different equilibrium disclosure rules because both the cost of equilibrium evidence production and the equilibrium enforced transfers may differ.

### 2 Illustrative Examples

In this section, I begin to explore the relationship between evidence costs and the value functions that can be implemented. My aim is to better understand when additional equilibrium evidence cost is useful and when it is a hindrance. This is relevant for the design of legal institutions with regard to evidence admissibility laws. By placing more weight on certain types of evidence, it may be possible for the legal institution to influence the cost of equilibrium evidence production in a beneficial

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\(^{14}\)Thus, the value function shapes players’ incentives in the production phase. For a full treatment of the relationship between productive outcomes that can be induced and implementable value functions, see Bull and Watson (2004).
way. Both the available evidence and the cost structure influence the value functions that can be implemented. I begin by showing, through two examples, how the setting of costly evidence differs from that of costless evidence.

**Example 1: Costly Evidence Restricts Implementability**

Consider the following somewhat stylized scenario of a sale of goods contract. The seller $S$ can either deliver the product (state $a$) or not deliver the product (state $b$). The $a$ and $b$ can be thought of as “alright” and “bad.” These are the only two states. When the product has been delivered the buyer $B$ can document that it is in his possession by presenting the good. This is represented by the buyer disclosing document $d$. However, when it has not been delivered, the buyer has nothing to present for evidence. The seller possesses no evidence in either state. Assume that value of delivery of the good to the buyer is much more than the cost savings to the seller of non-delivery, implying that it is efficient for the good to be delivered.

Here disclosure of $d$ is considered *positive evidence* of state $a$ since the buyer can produce this document in state $a$ but not state $b$. Further, *non-disclosure* of $d$ is considered *negative evidence* of $b$ because the buyer cannot disclose $d$ in state $b$ but can disclose the document in state $a$. I say that player $i$ can *positively distinguish* state $a$ from state $b$ when $i$ possesses a document in state $a$ that she does not possess in state $b$. In this example, the buyer can positively distinguish $a$, delivery has occurred, from $b$, non-delivery.

Consider first the case where the production of $d$ is costless. That is, it costs the buyer nothing to produce the document $d$ and it costs the seller nothing when the buyer produces $d$, or $\gamma_B(\{d\}) = \gamma_S(\{d\}) = 0$. Remember that the buyer possesses the document. Hence the buyer must be given the incentive to produce $d$ if the transfers are to be different in each state. This requires $m_B(\{d\}) \geq m_B(\emptyset)$. As there is no cost of evidence production here, $g_B(a) = m_B(\{d\})$ and $g_B(b) = m_B(\emptyset)$. Thus, it is possible to implement any $g$ such that $g_B(a) \geq g_B(b)$. When evidence production is costless, the value function must “favor” a player who can positively distinguish one state from another. Here this means that the buyer must be made better off by producing document $d$, and since there is no cost, this is reflected by $g_B(a) \geq g_B(b)$. This implies, in this example, there does not exist a contract that implements the efficient outcome of delivery of the good. Note that in the costless setting, it makes no difference whether players can settle prior to trial because the surplus from renegotiating is zero.

Suppose, instead, that the buyer’s production of $d$ has a cost of 10 to the buyer and no cost to the seller. That is, $\gamma_B(\{d\}) = 10$ and $\gamma_S(\{d\}) = 0$. This is represented

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15 This is a stylized example that is intended to provide some concreteness to the simplest analytical example that makes my points concerning evidence cost. Thus, I abstract from the possibility of the seller requiring that the buyer sign for the good and other steps that may be taken in reality.

16 This notion is developed fully in Bull and Watson (2004).

17 This is because with no evidence production cost the surplus from pretrial settlement is zero. Thus, $m' = m$. 

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in Figure 2. This is more realistic as it is in fact costly for the buyer to present the good. The cost to each player is indicated where the document is available (with the buyer’s cost first), and a dash indicates where the document is not available. I assume equal bargaining weights; that is, $\pi_B = \pi_S = 1/2$. If the gain in the buyer’s transfer of disclosing $d$ is less than 10, she will not produce the document (under $m$) in state $a$. That is, the document is never produced when $m_B(\{d\}) - m_B(\emptyset) < 10$. This is because the buyer receives a lower overall payoff by producing $d$ (and bearing the cost of 10) than she would by just not producing it. When this is the case the buyer will not distinguish state $a$ from state $b$, which implies $g_B(a) = g_B(b)$.

Now suppose that $m$ is set so that $m_B(\{d\}) - m_B(\emptyset) \geq 10$. This implies that the buyer would actually disclose $d$ if the evidence production stage were reached. Recall the definition of a value function. Here this implies that

$$g_B(a) = m_B(\{d\}) - \gamma_B(\{d\}) + \pi_B[\gamma_B(\{d\}) + \gamma_S(\{d\})],$$

or

$$g_B(a) = m_B(\{d\}) - 10 + 5 = m_B(\{d_B\}) - 5.$$  

Rearranging implies $g_B(a) + 5 = m_B(\{d\})$. Since there is no cost associated with withholding a document, it must be that $g_B(b) = m_B(\emptyset)$. Substituting these two expressions into $m_B(\{d\}) - 10 \geq m_B(\emptyset)$ yields $g_B(a) \geq g_B(b) + 5$. Thus, any value function $g$ where $g_B(a) \geq g_B(b) + 5$ can be implemented by a suitably chosen contract $m$. This suggests that there must be a “gap” between $g_B(a)$ and $g_B(b)$ (unless $g_B(a) = g_B(b)$). Clearly, the larger is $\gamma_B(d)$, the larger is the gap. Thus, in this example, the cost of evidence production exacerbates the difficulty of implementing the efficient outcome where the good is delivered.

Note that the renegotiation between the production phase and evidence production phase, in addition to the costly evidence production, leads to this gap. If there were no renegotiation, then the buyer’s overall continuation payoff following productive interaction could be made arbitrarily small by setting $m_B(d)$ close to $\gamma_B(\{d\})$, giving the buyer a small incentive to disclose $d$. However, without renegotiation, value functions are not balanced, and this type of $m$ does not, in general, prevent the seller’s value function from having a gap.

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18I address the setting of no renegotiation in Appendix C.
Example 2: Costly Evidence Expands Implementability

Consider the following example. Suppose that the owner of a factory hires a contractor to improve the factory machinery. A properly performed job by the contractor allows the production line to operate at a faster rate that is more efficient. For simplicity, assume that there are only two possible rates when the improvements have been made properly. These are the faster rate and the old rate at which the inefficient machinery operated. Further, assume that if improperly improved, the machinery can operate only at the slower rate.\textsuperscript{19} Suppose that improperly improving the machinery yields the contractor a cost savings of 4, but that having the more efficient machinery yields the factory a gain of 100. Clearly, it is efficient for the improvements to be made.

Denote the “high efficiency” state by $H$, and the “low efficiency” state by $L$. Assume that the contractor possesses no documents in either state. The factory owner potentially possesses two documents $d_f$ and $d'_f$. In state $H$ she can produce both $d_f$ and $d'_f$, but in state $L$ she can only produce $d_f$. The factory owner can produce $d_f$ in either state at a cost of 16, while production of $d'_f$ in state $H$ costs her 4. Assume that the contractor incurs no evidence production costs regardless of the documents produced by factory owner. Formally, this is $D(H) = \{d_f, d'_f\}, D(L) = \{d_f\}, \gamma_f(\{d_f\}) = 16$, $\gamma_f(\{d'_f\}) = 4$, and $\gamma_c(\{d_f\}) = \gamma_c(\{d'_f\}) = 0$. This is described in Figure 3. Costs are additive, implying that $\gamma_f(\{d_f, d'_f\}) = 20$. Again, assume $\pi_f = \pi_c = 1/2$. Clearly, if there were no cost of evidence production, the set of implementable value functions comprises all value functions satisfying $g_f(H)\geq g_f(L)$. This is much like above. Without evidence production costs, $g(H)$ must favor the player who can positively distinguish $H$ from $L$. In this example, the efficient productive outcome cannot be implemented in a setting of costless evidence production.

With costly evidence production, it is possible to implement $g_f(H) < g_f(L)$. To see this, consider an evidence disclosure rule $\beta$ such that $\beta(H) = \{d'_f\}$ and $\beta(L) = \{d_f\}$. Suppose that $m$ is defined such that $m_f(\{d_f\}) = 13$, $m_f(\{d'_f\}) = 2$, $m_f(\emptyset) = -4$, and $m_f(\{d_f, d'_f\}) = 15$. Note that given this $m$, $\beta$ is an equilibrium disclosure rule. So

$$g_f(H) = m_f(\{d'_f\}) - \gamma_f(\{d'_f\}) + \pi_f \gamma_f(\{d'_f\}) = 2 - 4 + 2 = 0.$$  

Also,

$$g_f(L) = m_f(\{d_f\}) - \gamma_f(\{d_f\}) + \pi_f \gamma_f(\{d_f\}) = 13 - 16 + 8 = 5.$$  

Thus, $m$ and $\beta$ implement $g$ such that $g_f(H) = 0 < g_f(L) = 5$. Further, if $\gamma_f(\{d'_f\})$ is held constant, the larger $\gamma_f(\{d_f\})$ is, the larger the difference between $g_f(H)$ and $g_f(L)$ can be made. This shows that, in some settings, additional cost actually allows

\textsuperscript{19}This example is similar in spirit to an example of Okuno-Fujiiwara, Postlewaite, and Suzumura (1990). However, the emphasis here is on which documents exist in particular contingencies and the cost of producing those documents. This is some what stylized. It assumes that the contractor cannot have the machinery run. This is revisited below.
more value functions to be implemented. In this example, the presence of evidence production costs allows the efficient productive outcome, the improvements being made to the factory, to be implemented.

**Implications**

I let $F_D(\gamma)$ denote the set of value functions that are implementable given a cost structure $\gamma$. I denote by $F_D(0)$ the set of value functions that can be implemented when $\gamma_i(\beta(a)) = 0$ for all $a \in A$, for $i = 1, 2$. As the above examples show, neither set contains the other, in general.\(^{20}\) I state this formally as follows.

**Lemma 1** In general, $F_D(\gamma) \not\subset F_D(0)$ and $F_D(0) \not\subset F_D(\gamma)$.

The relationship between the two is made clearer by precisely characterizing those value functions that can be implemented.

### 3 Characterization

My main technical result characterizes the value functions that can be implemented given a particular evidence cost structure. Recall that $\beta$ must specify an equilibrium disclosure rule in every contingency. This equilibrium behavior in the evidence production phase provides the disagreement point for the bargaining problem in the pretrial settlement phase. Thus, each disclosure rule may implement a different set of value functions. In comparing legal institutions, as well as specific contracts, it is essential to compare the set of implementable outcomes associated with each alternative.

For each $i$, I define

$$
\Lambda_i(\beta, E) \equiv \{a \in A \mid E_i \subseteq D_i(a) \text{ and } E_j = \beta_j(a)\}.
$$

Thus, $\Lambda_i(\beta, E)$ represents the set of states for which player $i$ can unilaterally reach $E$ when player $j$ discloses documents as prescribed by $\beta$. Note that when $\Lambda_i(\beta, E) = \emptyset$ it must be that player $j$ deviated from $\beta$ (though possibly player $i$ deviated as well).

\(^{20}\)I discuss settings where both players can produce documents in Section 4. In Appendix B, I present an example where a player’s evidence production imposes a cost upon her opponent.
It is useful to consider how equilibrium behavior in the evidence production game is related to a given value function \( g \). The definition of \( g \) implies
\[
g_i(a) - \pi_i[\gamma_1(\beta(a)) + \gamma_2(\beta(a))] = m_i(\beta(a)) - \gamma_i(\beta(a)).
\]
I define the following upper bound on \( m_i(E) \).
\[
\hat{z}_i(E; \beta, g) \equiv \begin{cases} 
\min_{a \in \Lambda_i(\beta, E)} [g_i(a) - \pi_i[\gamma_1(\beta(a)) + \gamma_2(\beta(a))] + \gamma_i(E) & \text{if } \Lambda_i(\beta, E) \neq \emptyset \\
\infty & \text{if } \Lambda_i(\beta, E) = \emptyset 
\end{cases}
\]

I characterize the set of implementable value functions as follows.

**Theorem 1 (Characterization)** Take as given a disclosure rule \( \beta \) and a value function \( g \). There exists an externally enforced contract \( m \) such that (i) \( g \) is implemented by \( \beta \) and \( m \), and (ii) \( \beta \) is an equilibrium disclosure rule, if and only if
\[
\sum_{i=1,2} \hat{z}_i(E; \beta, g) \geq 0, \text{ for every } E \in \mathcal{D}.
\]

The intuition is as follows. Consider any \( E \in \mathcal{D} \). If \( \Lambda_i(\beta, E) \neq \emptyset \), since \( \beta \) is an equilibrium disclosure rule and the \( m \)'s are balanced, there is an upper bound on \( m_i(E) \) of
\[
\min_{a \in \Lambda_i(\beta, E)} [g_i(a) - \pi_i[\gamma_1(\beta(a)) + \gamma_2(\beta(a))] + \gamma_i(E).
\]
This is an upper bound since in equilibrium player \( i \) must not wish to deviate from any \( \beta(a) \). If \( \Lambda_i(E, \beta) = \emptyset \), then it must be that player \( j \) deviated. When player \( j \) has deviated to reach \( E \), equilibrium disclosure imposes no upper bound on player \( i \)'s transfer at \( E \). This implies that player \( j \), having deviated, can be punished as harshly as needed to induce equilibrium disclosure. A given \( g \) can be implemented by \( \beta \) and some \( m \) only when the sum of the upper bound on players' transfers is non-negative at every \( E \in \mathcal{D} \). This characterization applies to a particular \( \beta \). The set of value functions that one disclosure rule can implement may differ from those that another disclosure rule will implement. This is because different disclosures may distinguish between between states differently, and may involve different evidence production costs.

Thus, \( g \in F^D(\gamma) \) if and only if \( g \) satisfies the condition of Theorem 1 for some \( \beta \). So finding the set of implementable value functions \( F^D(\gamma) \) requires that one use the characterization of Theorem 1 for every possible \( \beta \), and then take the union of the set of \( g \)'s that each \( \beta \) can implement.

Returning to the theme of the above examples, Theorem 1 has implications for the value functions that can be implemented when one player has additional documents available in one state that she does not disclose in another and her opponent’s disclosure does not distinguish these states from each other. This is as follows.
Corollary 1 Suppose that a value function \( g \) is implemented by an equilibrium disclosure rule \( \beta \). If \( \beta_i(b) \subset D_i(a) \) and \( \beta \) specifies that player \( j \) disclose the same documents in either state \( (\beta_j(a) = \beta_j(b)) \), then it must be that \( g_i(a) - \pi_i[\gamma_1(\beta(a)) + \gamma_2(\beta(a))] \geq g_i(b) - \pi_i[\gamma_1(\beta(b)) + \gamma_2(\beta(b))] \).

The intuition is that, with this type of disclosure rule and evidence environment, player \( i \) can pretend that state \( b \) has occurred when state \( a \) has. So she must be motivated to disclose \( \beta_i(a) \). Note that Corollary 1 has implications both on the basis of player \( i \)'s disclosure rule and available evidence. When \( \beta_i(b) \subset \beta_i(a) \), it must be that \( \beta_i(b) \subset D_i(a) \). Further, \( D_i(b) \subset D_i(a) \) implies \( \beta_i(b) \subset \beta_i(a) \). When evidence is costly, it is \( g_i(a) - \pi_i[\gamma_1(\beta(a)) + \gamma_2(\beta(a))] \) that must favor the player who distinguishes \( a \) from \( b \) when her opponent does not. To see this recall that the definition of the value function implies that \( g_i(a) - \pi_i[\gamma_1(\beta(a)) + \gamma_2(\beta(a))] = m_i(\beta(a)) - \gamma_i(\beta(a)) \). That is, player 1’s payoff from distinguishing that it is state \( a \) must be higher than her payoff from acting like it is state \( b \).

Consider the implications of Corollary 1 for the examples above. In Example 1 when \( \gamma_1(\{d_1\}) = 10 \) this implies that, for \( \beta \) such that \( \beta(a) = \{d_1\} \), \( g_i(a) - 5 \geq g_i(b) \). For Example 2, consider \( \beta \) such that \( \beta(a) = \{d'_1\} \) and \( \beta(b) = \{d_1\} \). Substituting into the inequality of Corollary 1 implies \( g_i(a) - 2 \geq g_i(b) - 8 \) or \( g_i(a) + 6 \geq g_i(b) \). This, of course, is consistent with the described \( \gamma \) yielding \( g_i(a) = 4 < g_i(b) = 9 \). In each of these examples, player 1 receives a higher payoff by disclosing \( \beta(a) \), and showing that the state is \( a \), than she does by disclosing \( \beta(b) \).

4 Redundant Documents

In practice there are often multiple documents that exist in the same states. For example in a sale of goods contract, the buyer may be able to prove that she has paid the seller by producing a receipt issued by the seller or by producing a canceled check. How these documents are treated by the court and how costly they are to produce may have implications for those value functions that can be implemented. When two documents exist in the same states, I refer to these documents as being redundant. I state this formally as follows.

Definition 3 Documents \( d \) and \( d' \) are called redundant if \( d \in D(a) \) if and only if \( d' \in D(a) \).

When there are redundant documents, it may be possible to implement a larger class of value functions than when there is only a single document.

Example 3: Redundant Documents and Implementability

The following example illustrates one setting where redundant documents expand the set of implementable value functions. Suppose there are two states, \( a \) and \( b \). Only player 1 can produce documents. In state \( a \) player 1 can produce documents \( d_1, d'_1 \),
and $d''_1$. In state $b$ she can produce documents $d_1$ and $d''_1$. Each document costs 4 for player 1 to produce, regardless of the state. I assume that costs are additive. So, for example, $\gamma_1(\{d_1, d'_1\}) = 8$. The availability of documents and the cost per document are represented in Figure 4. Player 1’s production of evidence imposes no cost on player 2. I assume $\pi_1 = \pi_2 = 1/2$.

Here, $d_1$ and $d''_1$ are redundant. To see the value of having a redundant document, consider first the disclosure rule that does not specify the disclosure of $d_1$ or $d''_1$ in any state, but does differentiate between states $a$ and $b$. Suppose that $\beta$ specifies that $\beta(a) = d'_1$ and $\beta(b) = \emptyset$. Then it must be that

$$g_1(a) - \pi_1 \gamma_1(d'_1) \geq g_1(b).$$

Note that other deviations can easily be prevented by making the transfer conditional only on whether $d'_1$ is disclosed. This implies that $g_1(a) \geq g_1(b) + 2$.

Next consider $\beta$ such that $\beta(a) = \{d'_1\}$ and $\beta(b) = \{d_1\}$. Here, player 1 produces evidence in each contingency as opposed to just indicating when state $a$ has occurred. This requires that

$$g_1(a) - \pi_1 \gamma_1(\{d'_1\}) \geq g_1(b) - \pi_1 \gamma_1(\{d_1\}) \geq m_1(\emptyset).$$

Other deviations are easily dealt with here, as well. Thus it must be that $g_1(a) \geq g_1(b)$. Here the additional document cost is useful in that it reduces the gap between $g(a)$ and $g(b)$. Of course, this is specific to this cost structure and bargaining weights. But, in this example, any value function such that $g_1(a) \geq g_1(b)$ can be implemented by a disclosure rule that never specifies the disclosure of both redundant documents (meaning $d''_1$ is never disclosed). This is the same as could be implemented without costs.

Now consider use of a disclosure rule that specifies the disclosure of both redundant documents. Suppose that $\beta$ is such that $\beta(a) = \{d'_1\}$ and $\beta(b) = \{d_1, d''_1\}$. This requires that

$$g_1(a) - \pi_1 \gamma_1(\{d'_1\}) \geq g_1(b) - \pi_1 \gamma_1(\{d_1, d''_1\}) \geq m_1(\emptyset).$$

As before, other deviations are easily prevented. The set of value functions that can be implemented by this $\beta$ are characterized by $g_1(a) + 2 \geq g_1(b)$. So here the redundancy is useful, and it allows a wider range of value functions to be implemented.

<table>
<thead>
<tr>
<th></th>
<th>$d_1$</th>
<th>$d'_1$</th>
<th>$d''_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
</tr>
<tr>
<td>$b$</td>
<td>4.0</td>
<td>-</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Figure 4: Example – evidence redundancy.
than in the costless disclosure setting. The intuition is that the redundancy allows for increasing the cost of \( \gamma_1(\beta(b)) \) (should the evidence production game be reached) and this strengthens the incentive to disclose \( \beta(a) \) when \( \beta(a) \) is available. This allows for decreasing the gap between \( g_1(a) \) and \( g_1(b) \). In the example the redundancy allows the players to contract in a way so that a relatively large cost is incurred when \( \beta(a) \) is not disclosed.\(^{21}\) However, as the examples of Section 5 demonstrate, it is not necessary that a document exist in all states for its cost to be useful in this type of manner. In general, having redundant documents provides the flexibility for the players to adjust the cost of equilibrium disclosure to suit a particular value function. Legal institutions can be designed exploit this feature.

Deneckere and Severinov (2001) study a similar notion in a principal-agent, mechanism design model where communication is costly. One setting they study is such that stating the truth about the state is costless to the agent, but lying is costly.\(^{22}\) The principal can design a contract requiring the agent stating the state \( n < \infty \) times that induces the agent to truthfully report the state. This is similar in spirit to the above example in which the redundant documents are used to make it more expensive for player 1 to avoid disclosing the distinguishing document.

**Aside: Message Game Phenomena**

In certain settings if two states have the same documents, say \( D(a) = D(b) \), and there are redundant documents, different value functions can be implemented in each of those states. To see this, consider the following example. Suppose \( D_1(a) = D_1(b) = \{d_1, d'_1\} \) and \( D_2(a) = D_2(b) = \{d_2, d'_2\} \). Let \( \gamma \) be such that \( \gamma_1(\{d_1\}) = 6, \gamma_1(\{d'_1\}) = 2, \gamma_2(\{d_2\}) = 2, \) and \( \gamma_2(\{d'_2\}) = 6 \). This is represented in Figure 5-(a). Further, assume that player \( i \)'s cost is additive and is only influenced by those documents player \( i \) produces. So, for example \( \gamma_1(\{d_1, d'_1\}) = 8 \). If the players agree to a contract where the self-enforced component implies \( \beta(a) = \{d_1, d_2\} \) and \( \beta(b) = \{d'_1, d'_2\} \) and the relevant externally enforced component is given by \( m_1(\{d_1, d_2\}) = 12, m_1(\{d'_1, d'_2\}) = 4, \) and \( m_1(\{d_1 d_2\}) = m_1(\{d'_1, d'_2\}) = 8 \). Note that any disclosure such that \( E_i = \{d_i, d'_i\} \) or \( \emptyset \) can be prevented by specifying \( m \) so that player \( i \) is punished severely if player \( i \) makes such a disclosure when player \( j \) discloses either \( d_j \) or \( d'_j \). This implies \( E_i = \{d_i, d'_i\} \) and \( E_i = \emptyset \) are dominated for \( i = 1, 2 \). Thus the relevant portion of the evidence production game is represented in Figure 5-(b). If the evidence production game is reached in state \( a \), the players disclose \( \{d_1, d_2\} \). This implies \( g_1(a) = m_1(\{d_1, d_2\}) - \gamma_1(\{d_1\}) + \pi_1[\gamma_1(\{d_1\}) + \gamma_2(\{d_2\})] = 12 - 6 + 4 = 10 \). Similarly, if the evidence production game is reached in state \( b \), the players disclose \( \{d'_1, d'_2\} \). So \( g_1(b) = m_1(\{d'_1, d'_2\}) - \gamma_1(\{d'_1\}) + \pi_1[\gamma_1(\{d'_1\}) + \gamma_2(\{d'_2\})] = 4 - 2 + 4 = 6 \).

\(^{21}\)In some sense, this can be viewed as the players being able to commit to “burn money” when \( \beta(a) \) is not disclosed.

\(^{22}\)I believe that it is straight forward to extend my Theorem 1 to allow the cost of producing a particular evidence set to be state dependant. This would allow for studying the forging of documents or lying, but the focus of this paper is on how the existence and non-existence of documents and
This is interesting as $D(a) = D(b)$, but $g(a) \neq g(b)$. In some sense this is a “message game phenomenon” as the same documents are available in each state, but settlement results in different transfers in each state. I emphasize that this is an anomaly, and I think of little practical interest. The intuition of this example is that for the right cost structure and externally enforced component, the evidence production game may have multiple equilibria that have different payoffs, and the players can use the self-enforcing component of the contract to coordinate on an equilibrium given the state. The difference in payoffs implies, since the total cost of the two equilibrium disclosures specified by $\beta$ are the same, that $g(a) \neq g(b)$.

This differs from the usual message game type results in two key ways. The first is that documents are not cheap here. I emphasize that evidence is not the same as the cheap message that is central to the message game literature. Evidence actually conveys information. If there were no cost associated with producing documents (as is the case in the message game literature), this would be a zero-sum game. It is well known that in a two player, zero-sum game that all Nash equilibria result in the same payoffs. Thus no cost implies $g(a) = g(b)$ when $D(a) = D(b)$. The second, and more important, distinction is that this type of phenomena does not hold in any sort of general manner. This is because a very limited type of cost structure is needed in order for this type of phenomena to hold.

5 The Implications of Two Legal Rules

There is a sense in the law of the United States that litigants ought to have equal access to the available evidence, and that the ruling of the court should be based on the evidence put before it without regard to which litigant presented what evidence. The former notion is more clearly embedded in the law. I briefly explore how these two notions influence those value functions that can be implemented.

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23Bull and Watson (2004) show that when coalitions are possible in an $n$-player setting of costless, but state-dependant, evidence production that message game phenomena are not possible.
Enforced Discovery Requests

In the adversarial system of fact-finding, litigants are typically permitted to request evidence from each other in the process known as “discovery”. Recall that I have assumed that players have common knowledge of the availability and content of documents. Here I consider a setting where the legal institution enables each litigant to request and actually receive with certainty documents from her opponent. That is, a player can force her opponent to produce and disclose a desired document. I term this a setting of enforced discovery requests. It may be that when a player requests a document that she incurs a request cost in addition to her cost associated with production of the document. As the cost structure is not crucial here, I defer discussion of the exact nature of the request cost until below.

I am interested in studying how each player having access to all producable evidence influences the implementable value functions. I take as given that these requests are in fact enforced. Here, I do not study the possibility of the suppression of evidence. Certainly, in practice enforced discovery requests may be difficult to implement. Players may have the incentive to destroy or suppress evidence. Further, litigants may have different incentives in the discovery process. Cooter and Rubinfeld (1994) characterize an efficient level of discovery requests. Practical measures to address the problem of excessive discovery requests are suggested in Cooter and Rubinfeld (1995). My analysis does not address these issues, but does suggest that further attention should be devoted to improving the workings of the discovery process. Much of literature has focused on cases of asymmetric information. In these settings discovery allows parties to become informed. Here discovery allows parties to be able to convey similar information.

I capture the setting of enforced discovery requests by assuming that, each player has a redundant document for each of the documents in the possession of the other player. Formally, I assume \( D = \{D^1, D^2, \ldots, D^K\} \), where \( D^k \equiv \{d^k_1, d^k_2\} \) and \( D_i \equiv \{d^1_i, d^2_i, \ldots, d^K_i\} \) such that \( d^k_1 \) and \( d^k_2 \) are redundant for each \( k \). Thus, if one player can positively distinguish \( a \) from \( b \), then her opponent can do so as well.

Denote by \( F^D \) the set of value functions implemented by equilibrium disclosure rules for any \( \gamma \). That is, \( F^D \) is the set of value functions that are not sensitive to the actual cost structure. Note that \( g \in F^D \) if and only if \( g \) satisfies the condition of Theorem 1 for some \( \beta \). Consider an arbitrary partition \( P \) of the state space. For any state \( a \), \( P(a) \) denotes the element of the partition containing \( a \); that is, \( b \in P(a) \) if and only if \( a \) and \( b \) are in the same element of the partition \( P \). I denote by \( G(P) \)

\[ 17 \]
the set of value functions that are measurable with respect to \( P \). Let \( P^D \) denote the partition of \( A \) induced by the notion of distinguishing between states. Formally \( P^D \) is defined so that \( a \in P^D(b) \) if and only if \( D(a) = D(b) \). The following definition is useful.

**Definition 4** The full disclosure rule \( \beta \) is defined by \( \beta(a) \equiv D(a) \) for all \( a \in A \).

With full disclosure, each player submits all of the documents in his possession in every state. In a setting of enforced discovery requests, \( F^D \) is not constrained by the failure of positive evidence.

**Theorem 2** Suppose that the legal rule of enforced discovery requests is in effect. Then, regardless of \( \gamma \), the difference between positive and negative evidence is not critical, and full disclosure rules can induce \( G(P^D) = F^D \).

The intuition is that in state \( a \) players are induced to disclose \( \beta(a) \) because for each of player \( i \)'s documents player \( j \) has a document, and if player \( i \) does not disclose a document for which \( j \) discloses his corresponding document, \( i \) can be punished arbitrarily harshly. Thus, the punishment for unilaterally deviating from a full disclosure equilibrium can be made large enough to offset any evidence production costs associated with full disclosure.

It is interesting that this setting is one where evidence production is costly, but all implementable value functions can be characterized by considering only those disclosure rules that involve disclosure of all available documents in each contingency. When evidence production is costless this result is quite intuitive. One player will always have the incentive to disclose any available document, and given that she is disclosing that document, her opponent will be indifferent about disclosing his corresponding document. However, when evidence production is costly this result relies upon “forcing” the player for whom a given document is not beneficial to incur the associated cost and disclose it. This is done by the court being able to differentiate the transfers imposed based upon whether a player has disclosed the redundant documents corresponding to those disclosed by his opponent.

In many legal settings this is not a reasonable assumption. Often the court is viewed as applying the rules of law to the facts of the case. Obviously, the facts of the case are proven by the evidence disclosed, but who disclosed that evidence may not be relevant to the facts of the case. Thus, in many settings it is appropriate to constrain the transfers to reflect this. Further, the meaning of simultaneous evidence production is unclear.

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\(^{29}\)Bull and Watson (2004) show, when evidence production is costless, that all implementable value functions can be characterized by constraining attention to only full disclosure rules. They consider “transfer functions” which result from external enforcement. In a costless setting these are equal to the value functions described here.
Transfers That Do Not Distinguish Between Redundant Documents

I now depart from the assumption of enforced discovery requests, and first consider the case where the court does not differentiate transfers on the basis of redundant documents. This is motivated by the legal argument that court action should be based upon what the evidence proves and not on who discloses it. See, for example, Tepley and Whitten (2000) for a discussion of rules of civil procedure. The court is to apply the rules of law to the facts of the case before it. The facts are established by the disclosure of evidence. Notions of fairness may suggest that the same transfers be imposed whether the plaintiff or the defendant presented the convincing evidence. This differs from much of the treatment in the mechanism design literature, which allows for conditioning on who sends messages. However, when evidence exists in some contingencies and not in others, there is greater sense of what constitutes the facts of the case.

Let

\[ J(E) \equiv \{ d \in D \mid \text{there exists } d' \in E \text{ such that } d, d' \text{ are redundant} \}. \]

I reflect this constraint on transfers by the requirement that \( m(E) = m(E') \) whenever \( J(E) = J(E') \). Note that this differs slightly from the case where transfers do not condition on which player requested a particular document.\(^{30}\)

The set of evidence sets that must have the same transfer is denoted by

\[ \Omega(E) \equiv \{ E' \in D \mid J(E') = J(E) \}. \]

I define the upper bound on \( m_i(E) \) to be

\[ \psi_i(E; \beta, g) \equiv \min_{E' \in \Omega(E)} \hat{z}_i(E'; \beta, g). \]

Given this constraint on transfers, I characterize those value functions that can be implemented as follows.

**Theorem 3 (Characterization)** Suppose that the court cannot condition on which player produces a redundant document \((m(E) = m(E') \text{ whenever } J(E) = J(E'))\). Take as given a disclosure rule \( \beta \) and a value function \( g \). There exists an externally enforced contract \( m \) such that (i) \( g \) is implemented by \( \beta \) and \( m \), and (ii) \( \beta \) is an equilibrium disclosure rule, if and only if

\[ \sum_{i=1,2} \psi_i(E; \beta, g) \geq 0, \text{ for every } E \in D. \]

---

\(^{30}\)As I discuss below this may actually be the more appropriate constraint. However, here I consider a setting that does not have enforced discovery requests. So the request notion is not particularly relevant.
The intuition is similar to that of Theorem 1. As before $\hat{z}_i(E; \beta, g)$ represents an upper bound on $m_i(E)$. However, now the constraint that $m(E) = m(E')$ whenever $J(E) = J(E')$ implies that the relevant upper bound on $m(E)$ is the minimum $\hat{z}_i(E'; \beta, g)$ for any $E'$ such that $J(E) = J(E')$.

I denote by $F_D^D(\gamma)$ the set of implementable value functions under this constraint on $m$. This reduces the set of implementable value functions.

**Corollary 2** If the court cannot condition on which player produces a redundant document ($m(E) = m(E')$ whenever $J(E) = J(E')$), then the set of implementable value functions is reduced, so $F_D^D(\gamma) \subset F_D^D(\gamma)$.

Note that with this constraint message game phenomena, as discussed in the example in Section 4, do not exist.

**Enforced Discovery Requests with Constrained Transfers**

Now I study the implication of having both of these legal rules in place. The set of implementable value functions under both legal rules will be somewhere between those under each rule individually. Note that if enforced discovery requests are introduced in a setting that requires $m(E) = m(E')$ whenever $J(E) = J(E')$, the set of implementable value functions will be enlarged.

**Corollary 3** In a setting where the court cannot condition on which player produces a redundant document ($m(E) = m(E')$ whenever $J(E) = J(E')$), allowing enforced discovery requests always (weakly) expands the set of implementable value functions. That is, $F_D^{D_1} \subset F_D^{D_1 \cup D_2}$.

This is because any $m$ and $\beta$ that implement a value function without enforced requests can also be used with enforced requests. How much the setting of enforced discovery requests increases the set of implementable value functions depends upon the particular evidence environment.

To illustrate the relationship between the evidence environment and the set of implementable value functions I consider a series of examples. These examples assume a common cost structure, in which both parties’ costs of producing and requesting documents are the same.\textsuperscript{31} Further, I represent enforced discovery requests as previously described on page 17. I assume that if a player desires that a particular document be produced, that she must pay a request cost. Further, I assume that the request cost is the same across players and documents. I describe this more formally as follows. Let $\theta^k \in \mathbb{R}_+^2$ denote the cost of producing a document of type $k$. For $E \in \mathcal{D}$ let

$$C(E) \equiv \{ k \mid d_i^k \in E \text{ for some } i = 1, 2 \}.$$

\textsuperscript{31}This common cost structure is not necessary to make the points concerning the evidence environment and implementable value functions under these two legal rules. However, it captures the notion of discovery quite well and facilitates the exposition.
Let $\eta_i(E) \equiv \# \{k \mid d^i_k \in E \}$. I denote the cost of requesting a single document by $\varepsilon$. Thus,

$$
\gamma_i(E) \equiv \sum_{k \in C(E)} \theta^k + \varepsilon \eta_i(E).
$$

Note that this cost structure implies that if both players request a document of type $k$, each incurs $\theta^k + \varepsilon$.

**Example 4: Enforced Discovery Increases Implementability**

Suppose there are two states, $a$ and $b$. In state $a$, a document can be produced. In state $b$ no document can be produced. Suppose there are no enforced discovery requests and player 1 possesses the document $d_1$. From the above assumptions on cost, $\gamma_1(\{d_1\}) = \theta + \varepsilon$ and $\gamma_2(\{d_1\}) = \theta$. Then by Corollary 1 any value function such that either $g_1(a) - \pi_1[2\theta + \varepsilon] \geq g_1(b)$ or $g(a) = g(b)$ can be implemented. A practical motivation for this example would be quite similar to that of the sale of goods contract in Example 1.

Now suppose that enforced discovery requests are allowed, meaning that player 2 possesses document $d_2$ in state $a$. In the context of the sale of goods contract in Example 1, this means that both players can cause the final product to be disclosed. For $\beta$ such that $\beta(a) = \{d_1\}$, it is possible to implement $g_1(a) - \pi_1[2\theta + \varepsilon] \geq g_1(b)$. This is represented in Figure 6. For $\beta$ such that $\beta(a) = \{d_2\}$, it is possible to implement $g_2(a) - \pi_2[2\theta + \varepsilon] \geq g_2(b)$ or $g_1(a) + (1 - \pi_1)[2\theta + \varepsilon] \leq g_1(b)$. Thus, the set of implementable value functions is enlarged by allowing enforced discovery requests. However, as it must be that $m(\{d_1\}) = m(\{d_2\}) = m(\{d_1, d_2\})$ no equilibrium disclosure will involve both $d_1$ and $d_2$ being disclosed. Further, there must always be a gap between $g_1(a)$ and $g_1(b)$ unless $g_1(a) = g_1(b)$. This gap, of course, is due to the requirement that transfers not differentiate on the basis of how state $a$ was distinguished.

**Example 5: Document Overlap**

Suppose there are three states, $a$, $b$, and $c$. Assume there are two types of documents. The first is available in state $a$. The second is available in both states $a$ and $b$. This is represented in Figure 7. If both documents are only available to player $i$, then there must be a gap between $g_i(a)$ and $g_i(c)$. Clearly, if only the first type of
Figure 7: Example – the availability of a non-distinguishing document helps close the gap.

document were available to both players (that is \(d_1\) and \(d_2\)), then there would have to be a gap as was described in Example 4.

However, here the difference between \(g_1(a)\) and \(g_1(c)\) is unconstrained. To see this, consider \(\beta\) such that \(\beta(a) = \{d_1, d'_2\}\), \(\beta(b) = \{d'_2\}\), and \(\beta(c) = \{\emptyset\}\). Inducing player 2 to disclose \(d'_2\) requires that \(g_2(b) - \pi_2[2\theta' + \varepsilon] \geq g_2(c)\) or \(g_1(b) + (1 - \pi_1)[2\theta' + \varepsilon] \leq g_1(c)\).

Note that \(m_2(\{d_1\})\) can be set as harshly as is needed to induce player 2 to disclose \(d'_2\) in state \(a\). For player 1 to disclose \(d_1\) in state \(a\), we need

\[
g_1(a) - \pi_1[2\theta + \theta' + \varepsilon] \geq g_1(b) - \pi_1[2\theta' + \varepsilon]
\]

or

\[
g_1(a) \geq g_1(b) + \pi_12\theta.
\]

As the two constraints only require that both \(g_1(a)\) and \(g_1(c)\) be sufficiently larger than \(g_1(b)\), the gap between \(g_1(a)\) and \(g_1(c)\) is unconstrained. The notion is that the availability of an extra document that does not distinguish state \(a\) from the other states may be useful if it is available to both players.

**Example 6: A Different Document Overlap**

Now consider the three state, two document type example described in Figure 8. Suppose, for comparison, that neither document type is available to player 2. Then by straightforward application of Corollary 1, it is easy to see that any disclosure such that \(\beta(a) \neq \beta(b)\) requires \(g_1(b) > g_1(a)\). However, when both documents are available to both players, this gap can be reduced. To see this, consider \(\beta\) such that \(\beta(a) = \{d_1\}, \beta(b) = \{d_1, d'_2\}\), and \(\beta(c) = \{d'_2\}\).

To induce player 1 to disclose \(d_1\) in state \(b\) it must be that

\[
g_1(b) - \pi_12[\theta + \theta' + \varepsilon] \geq g_1(c) - \pi_1[2\theta + \varepsilon].
\]

In state \(a\) it requires be that \(g_1(a) - \pi_1[2\theta + \varepsilon] \geq m_1(\emptyset)\). To induce player 2 to produce \(d'_2\) in state \(b\) requires

\[
g_2(b) - \pi_22[\theta + \theta' + \varepsilon] \geq g_2(a) - \pi_2[2\theta + \varepsilon].
\]
As the $g$’s are balanced, this implies $g_1(b) \leq g_1(a) - (1 - \pi_1)[2\theta' + \varepsilon]$. That player 2 discloses $d'_2$ in state $c$ requires $g_2(c) - \pi_2[2\theta' + \varepsilon] \geq m_2(\emptyset)$, or $g_1(c) - (1 - \pi_1)[2\theta' + \varepsilon] \leq m_1(\emptyset)$. Combining these yields

$$g_1(a) + \pi_1[2\theta + \varepsilon] \leq g_1(b) \leq g_1(a) - (1 - \pi_1)[2\theta' + \varepsilon].$$

Thus, the gap between $g_1(a)$ and $g_1(b)$ is closed. Here again a document that is only slightly related to the states of concern is quite useful.

**Implications For Legal Institutions**

The clearest implication of this research for legal institutions is that enforced discovery is beneficial because it expands the set of implementable value functions. However, the effectiveness of enforced discovery requests is reduced when it is combined with the restriction on transfers that prevents the court from conditioning on who produced which documents. Certainly, this legal rule is not followed as systematically as that of discovery. To be sure, there is a tension between this legal principle and the ability to implement a large set of value functions.

As the above examples concerning the overlap of documents suggest, the negative impact of this restriction may be lessened by the use of slightly related evidence that does not distinguish between the two states in question. This may in fact be consistent with the popular view of litigation. A typical complaint of people who have been involved in litigation is that attorneys tend to ask questions or present evidence concerning things which are only tangentially related. It may be that these attorneys are simply using these extra documents to their fullest.

Another possible way to lessen the impact of this type of restriction on transfers is to simply require that transfers cannot condition on which player requested a particular document rather than requiring the same transfers not differentiate between redundant documents. This would allow for greater flexibility from the use of redundant documents. Returning to the sale of goods example in Section 4, this would imply that the court would treat the buyer disclosing the canceled check the same as the seller forcing the disclosure of the canceled check, but it could treat these events differently from the buyer disclosing the receipt. As the discussion of Section 4 sug-
gests, redundant documents allow for more flexibility of the value functions, which implies a larger set of enforceable contracts.

6 Conclusion

I have presented a model of contract enforcement in which verifiability depends upon litigants’ costly production of evidence. The model treats evidence as documents, the availability of which are state dependant. My analysis has focused on the setting where players can settle their dispute prior to litigation and thus avoid the cost of actually producing evidence. I have characterized how incentives in the production of evidence influence those outcomes of settlement that can be implemented, which determines the contracts that parties will agree to initially. The presence of evidence production costs both allows outcomes to be implemented that could not be implemented in a costless setting, and prevents the implementation of some outcomes which can be implemented in a costless setting. My results show that contract enforcement is sensitive to both the manner in which states can be distinguished from each other and the cost of doing so. Further, I have studied the flexibility that redundant documents provide.

In the productive interaction, players are motivated by the value function that is induced by their contract. Thus, enlarging the set of implementable value functions can increase the set of implementable productive outcomes. The legal institution can influence the set of implementable value functions by influencing evidence production cost through evidence admissibility laws. As the results of this paper suggest, in some evidence environments having available additional evidence of the appropriate cost will expand the set of implementable outcomes. This suggests that legal institutions ought to consider the cost and availability of various documents when determining rules that influence the level of proof as well when setting evidence admissibility standards.

My approach differs from that which is typically taken in the law and the law and economics literatures. Here, evidence has informative content on the basis of which documents are available in some states and not in others, and evidence is costly to produce. I have briefly examined some implications for legal institutions, and see this as a promising direction for further research. This would include the more detailed comparison of institutions on the basis of default rules, evidence admissibility rules, and burden of proof issues. These are particularly interesting issues to study since the institutional design may be able to influence the cost associated with producing evidence. Other interesting directions for future research include (a) allowing for the sequential disclosure of evidence, (b) studying settings in which players have private information about the availability of documents, (c) permitting settings which have $n > 2$ players, and (d) examining settlement in a more detailed model.
A Proofs Not in the Text

Proof of Theorem 1

(Necessity) Suppose $\beta$ is an equilibrium disclosure rule, but

$$\sum_{i=1,2} \hat{z}_i(E; \beta, g) < 0,$$

for some $E \in \mathcal{D}$.

Equilibrium disclosure requires $m_i(\beta(a)) - \gamma_i(\beta(a)) \geq m_i(E_i \cup \beta_j(a)) - \gamma_i(E_i \cup \beta_j(a))$, for $i = 1, 2$, for any $E_i \subset D_i(a)$, for all $a \in A$. But if $\sum_{i=1,2} \hat{z}_i(E; \beta, g) < 0$, for some $E \in \mathcal{D}$, then (since $g_i(a) = m_i(\beta(a)) - \gamma_i(\beta_i(a)) + \pi_i[\gamma_1(\beta(a)) + \gamma_2(\beta(a))]$ and $\sum_{i=1,2} m_i = 0$) for some $\alpha, \beta$ is not an equilibrium disclosure.

(Sufficiency) Take any $E \in \mathcal{D}$. That $\sum_{i=1,2} \hat{z}_i(E; \beta, g) \geq 0$, for every $E \in \mathcal{D}$ implies the existence of an enforced contract $m : \mathcal{D} \rightarrow \mathbb{R}_0^2$ such that $g_i(a) - \pi_i[\gamma_1(\beta(a)) + \gamma_2(\beta(a))] + \gamma_i(\beta(E)) \geq m_i(E)$ for all $a \in \Lambda_i(\beta, E)$, for every $E \in \mathcal{D}$, and each $i = 1, 2$. This implies $m_i(\beta(a)) - \gamma_i(\beta(a)) \geq m_i(E_i \cup \beta_j(a)) - \gamma_i(E_i \cup \beta_j(a))$ for any $E_i \subset D_i(a)$, for all $a \in A$, and for each $i = 1, 2$. Q.E.D.

Proof of Theorem 2

Consider disclosure rules which specify $\overline{\beta}(a) = D(a)$ for all $a \in A$. Take any $g$ and $E \in \mathcal{D}$. First consider the case where $E$ is such that for $l(a) \in \{1, 2\}$ it is the case that $D_{l_i}(a) = E_{l_i}(a)$ and $E_{l_i}(a) \subset D_{l_i}(a)$ (strictly) for some $a \in A$. I proceed by finding all $a$’s such that for some $l(a) \in \{1, 2\} D_{l_i}(a) = E_{l_i}(a)$ and $E_{l_i}(a) \subset D_{l_i}(a)$ (strictly). Let $\tilde{A}$ denote the set of such $a$’s. It can be shown that $\bigcap_{a \in \tilde{A}} - l(a) \neq \emptyset$.

As $\bigcap_{a \in \tilde{A}} - l(a) \neq \emptyset$, and the court can always tell that at least one player has deviated at any such $E$. This implies that for any such $E$, it is always the case that $\sum_{i=1,2} \hat{z}_i(E; \overline{\beta}, g) \geq 0$.

Next, consider $E$ such that $E = \overline{\beta}(a)$ for some $a \in A$. It must be that $g_1(a) - \pi_1[\gamma_1(\beta(a)) + \gamma_2(\beta(a))] + \gamma_1(\beta(a)) + g_2(a) - \pi_2[\gamma_1(\beta(a)) + \gamma_2(\beta(a))] + \gamma_2(\beta(a)) = 0$. Q.E.D.

Proof of Theorem 3

(Necessity) Suppose $\beta$ is an equilibrium disclosure rule, but

$$\sum_{i=1,2} \psi_i(E; \beta, g) < 0,$$

for some $E \in \mathcal{D}$.

Equilibrium disclosure requires $m_i(\beta(a)) - \gamma_i(\beta(a)) \geq m_i(E_i \cup \beta_j(a)) - \gamma_i(E_i \cup \beta_j(a))$, for $i = 1, 2$, for any $E_i \subset D_i(a)$, for all $a \in A$. But if $\sum_{i=1,2} \psi_i(E; \beta, g) < 0$, for some $E \in \mathcal{D}$, then (since $g_i(a) = m_i(\beta(a)) - \gamma_i(\beta_i(a)) + \pi_i[\gamma_1(\beta(a)) + \gamma_2(\beta(a))]$ and $\sum_{i=1,2} m_i = 0$) for some $\alpha, \beta$ is not an equilibrium disclosure.

(Sufficiency) Take any $E \in \mathcal{D}$. That $\sum_{i=1,2} \psi_i(E; \beta, g) \geq 0$, for every $E \in \mathcal{D}$ implies the existence of an enforced contract $m : \mathcal{D} \rightarrow \mathbb{R}_0^2$ where $m(E) = m(E')$.
whenever \( J(E) = J(E') \) such that \( g_i(a) - \pi_i[\gamma_1(\beta(a)) + \gamma_2(\beta(a))] + \gamma_i(E) \geq m_i(E) \) for all \( a \in \Lambda_i(\beta, E) \), for every \( E \in D \), and each \( i = 1, 2 \). This implies \( m_i(\beta(a)) - \gamma_i(\beta(a)) \geq m_i(E_i \cup \beta_j(a)) - \gamma_i(E_i \cup \beta_j(a)) \) for any \( E_i \subset D_i(a) \), for all \( a \in A \), and for each \( i = 1, 2 \). Q.E.D.

B Costly Evidence – Two Sided Cost

Here I consider a setting like Example 1, but now player 1’s evidence production imposes a cost on player 2. A stylized example of this might be a setting where a construction contractor has installed a piping system in the yard of a homeowner and only the contractor knows where the piping is located. Perhaps the contractor can prove the quality of the piping, but the contractor must be who produces the evidence. This scenario implies that the contractor’s digging up the piping in the yard to produce the evidence imposes a cost on the homeowner — the homeowner must suffer the yard being made less attractive for some period of time.

Consider the specific parameters represented in Figure 9. That is, there are two states, \( a \) and \( b \). In state \( a \) player 1 can produce document \( d_1 \), but in state \( b \) she can produce no documents. Player 2 never possesses any documents. Here the production of \( d_1 \) has a cost of 10 to player 1 and a cost of 4 to player 2. That is, \( \gamma_1(\{d_1\}) = 10 \) and \( \gamma_2(\{d_1\}) = 4 \). I assume \( \pi_1 = \pi_2 = 1/2 \). As in Example 1, if the gain in player 1’s transfer of disclosing \( d_1 \) is less than 10, she will not produce the document in state \( a \). When this is the case, \( g_1(a) = g_1(b) \).

However, if \( m_1(\{d_1\}) - m_1(\emptyset) \geq 10 \), player 1 would actually disclose \( d_1 \) if the evidence production stage were reached. Recall that

\[
g_1(a) = m_1(\{d_1\}) - \gamma_1(\{d_1\}) + \pi_1[\gamma_1(\{d_1\}) + \gamma_2(\{d_1\})],
\]

or

\[
g_1(a) = m_1(\{d_1\}) - 10 + \frac{1}{2}[14] = m_1(\{d_1\}) - 3.
\]

Rearranging implies \( g_1(a) + 3 = m_1(\{d_1\}) \). Since there is no cost of withholding a document, it must be that \( g_1(b) = m_1(\emptyset) \). Substituting these two expressions into \( m_1(\{d_1\}) - 10 \geq m_1(\emptyset) \) yields \( g_1(a) \geq g_1(b) + 7 \). Thus, any value function \( g \) such that \( g_1(a) \geq g_1(b) + 7 \) can be implemented. Here the additional cost to player 2 increases the gap between \( g_1(a) \) and \( g_1(b) \). Note that Corollary 1 applies here as well. For \( \beta \) such that \( \beta(a) = \{d_1\} \), substitution yields \( g_1(a) - 7 \geq g_1(b) \).
C No Settlement

My analysis has assumed that players can freely renegotiate in period 3. In reality, not all disputes are resolved prior to litigation.\textsuperscript{32} It is likely that litigants in these disputes make some attempt to settle, but are unable to do so. In some contractual relationships, parties may know that if they have a dispute then they will end up in court. I take a somewhat simplistic approach here, and assume that parties who will be unable to reach a settlement know this is in period 1.\textsuperscript{33} For these types of players, this reduces to studying the case where renegotiation in period 3 is not possible.

When renegotiation in period 3 is not possible the non-settlement value function is given by \( g_i(a) = m_i(\beta(a)) - \gamma_i(\beta(a)) \). I continue to require that \( \sum_{i=1,2} m_i = 0 \).\textsuperscript{34} However, this does not generally imply that \( \sum_{i=1,2} g_i(a) = 0 \). It implies that \( \sum_{i=1,2} g_i(a) = -[\gamma_1(\beta(a)) + \gamma_2(\beta(a))] \). Given \( \beta \), I define the upper bound on \( m_i(E) \) as

\[
\tilde{z}_i(E; \beta, g) \equiv \begin{cases} 
\min_{a \in \Lambda_i(\beta, E)} g_i(a) + \gamma_i(E) & \text{if } \Lambda_i(\beta, E) \neq \emptyset \\
\infty & \text{if } \Lambda_i(\beta, E) = \emptyset 
\end{cases}
\]

I characterize the value functions that can be implemented given \( \beta \) as follows.

**Theorem 4 (Characterization)** Take as given a disclosure rule \( \beta \) and a (non-settlement) value function \( g \). There exists an externally enforced contract \( m \) such that (i) \( g \) is implemented by \( \beta \) and \( m \), and (ii) \( \beta \) is an equilibrium disclosure rule, if and only if

\[
\sum_{i=1,2} \tilde{z}_i(E; \beta, g) \geq 0, \text{ for every } E \in \mathcal{D}.
\]

Proof: (Necessity) Suppose \( \beta \) is an equilibrium disclosure rule, but

\[
\sum_{i=1,2} \tilde{z}_i(E; \beta, g) < 0, \text{ for some } E \in \mathcal{D}.
\]

\textsuperscript{32}Further, renegotiation, when possibly, may be costly. See Brennan and Watson (2001), and Schwartz and Watson (2001) for thorough discussions of these ideas.

\textsuperscript{33}A more complete way to model this would be to assume that with some positive probability renegotiation in period 3 breaks down or is not possible.

\textsuperscript{34}Recall that this is due to the potential for renegotiation between periods 4 and 5, or the rules of the court.
Equilibrium disclosure requires $m_i(\beta(a)) - \gamma_i(\beta(a)) \geq m_i(E_i \cup \beta_j(a)) - \gamma_i(E_i \cup \beta_j(a))$, for $i = 1, 2$, for any $E_i \subset D_i(a)$, for all $a \in A$. But if $\sum_{i=1,2} \tilde{z}_i(E; \beta, g) < 0$, for some $E \in \mathcal{D}$, then (since $g_i(a) = m_i(\beta(a)) - \gamma_i(\beta_i(a))$ and $\sum_{i=1,2} m_i = 0$) for some $a, \beta$ is not an equilibrium disclosure.

(Sufficiency) Take any $E \in \mathcal{D}$. That $\sum_{i=1,2} \tilde{z}_i(E; \beta, g) \geq 0$, for every $E \in \mathcal{D}$ implies the existence of an enforced contract $m : \mathcal{D} \rightarrow \mathbb{R}_0^2$ such that $g_i(a) - \gamma_1(\beta(a)) + \gamma_i(E) \geq m_i(E)$ for all $a \in \Lambda_i(\beta, E)$, for every $E \in \mathcal{D}$, and each $i = 1, 2$. This implies $m_i(\beta(a)) - \gamma_i(\beta(a)) \geq m_i(E_i \cup \beta_j(a)) - \gamma_i(E_i \cup \beta_j(a))$ for any $E_i \subset D_i(a)$, for all $a \in A$, and for each $i = 1, 2$. Q.E.D.

The intuition here is much like that of Theorem 1. Given $\beta$, the upper bound on $m_i(E)$ is given by $\tilde{z}_i(E; \beta, g)$. When $\sum_{i} \tilde{z}_i(E; \beta, g) \geq 0$ for a given $E$, it means that a transfer at $E$ can be specified so that $\beta$ is an equilibrium disclosure rule and $g$ is implemented.

The important distinction of the setting where settlement cannot occur is that the gap of the renegotiated case is not present. This is because players actually receive their payoffs from the evidence production game. However, this implies a new type of gap since (non-settlement) value functions are generally not balanced. Thus, it is possible for the players to discard resources. This allows a greater range of value functions to be implemented than in the costless setting. Of course, the usefulness of this feature depends upon the specific evidence environment and cost structure.  

References


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