The Dance of the Dynamics: The Interplay of Trade and Growth

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Abstract

We study the interaction of endogenous growth and Ricardian trade working solely through comparative advantage. The model is built to be consistent with several facts about technical progress, R&D activity, industrial organization, and trade. We obtain a full characterization of the transition dynamics and trade’s welfare effects. We find that trade affects growth and that growth affects trade in ways previously unexplored. The model can explain in a single framework several phenomena usually analyzed separately. Trade may increase or decrease the balanced growth rate of either country. The possibility of a decrease arises from a growth-related dynamic inefficiency. Trade leads to equal growth rates for some countries but permanently unequal growth rates for others. In general, the world’s distribution of growth rates may become one of “twin peaks” with groups of countries having persistently high or low growth rates. Trade leads to effective technology transfer, with a country’s growth rate being the same as if that country had adopted its trading partner’s R&D technology if no technology transfer ever occurs. Effective technology transfer offers a new interpretation of the evidence on productivity gains from trade. Economic growth can change the trading regime endogenously by moving the world economy across the boundary between the standard Ricardian interior (complete specialization) and corner (incomplete specialization) solutions. Through its effects on economic dynamics, trade may raise or lower social welfare in the short run, the long run, or both. For all results, the model specifies the conditions under which each possible outcome occurs.
1 Introduction

The last decade has seen a resurgence of interest in Ricardian trade theory. Inspired by that work, we examine the effect of Ricardian trade on economic growth and *vice versa*. Our contribution is to use the latest vintage of endogenous growth theory, the Schumpeterian Dynamic General Equilibrium (SDGE) model of fully endogenous growth, to study the interactions among trade, endogenous economic growth, and the economy’s industrial structure. Previous work on trade and growth either has used earlier types of growth models that have been found to have empirical difficulties or has not studied trade *per se* but rather technology transfer, perhaps facilitated by trade. Our analysis shows that trade and growth affect each other in ways not previously described in the literature and provides many results that differ substantially from those in earlier studies, sometimes contradicting them. The analysis also provides a unified explanation of several disparate phenomena in the data that previously have been given unrelated explanations or not even explained at all.

The feature of the model that delivers the novel results is the endogeneity of the economy’s microeconomic market structure, the hallmark of the second-generation SDGE class of models and what distinguishes it from other types of macroeconomic models. Endogeneity of market structure eliminates several empirically unacceptable aspects of the great wave of first-generation models, such as aggregate scale effects on the growth rate and excessive sensitivity of the aggregate growth rate to policy choices. However, endogeneity of market structure has implications far beyond correcting those problems. Entry and exit by firms and competition for market share - staples of microeconomic analysis - have important effects on the dynamic behavior of the aggregate economy. Symmetrically, general equilibrium dynamics affect the evolution of the industrial structure. In the SDGE approach microeconomics and macroeconomics interact in a natural and fruitful way, allowing one to address questions and obtain results that are beyond the scope of other macroeconomic models. We shall see examples of these interactions and their implications in the analysis that follows. The model is quite tractable and completely characterizes not only the economy’s balanced growth path but also its transition dynamics. Having the transition dynamics allows a complete description of the economy’s path. That is interesting in itself and also permits a welfare analysis of trade.

Our results fall into two groups: the effect of trade on growth and the effect of growth on trade.

First, *trade affects growth* purely through comparative advantage. Although that result may seem to be a dynamic analog of the standard static gain from trade, it is not simply a differential version of the static result. The channels through which trade affects growth are interesting and subtle, reflecting the interaction of the economy’s endogenous market structure and its general equilibrium dynamics. Indeed, trade may raise or lower the long-run growth rate of either trading partner, depending on how the two economies’ industrial structures are affected by trade. Trade is determined by comparative advantage in production, whereas growth is determined by R&D. Virtually all private R&D, however, is done in-house by existing firms. Which firms survive an opening to trade thus determines not only which firms produce the traded goods but also which firms remain to do the R&D that delivers growth. The firm whose good has the lowest current quality-adjusted price may or may not be the firm that is the most efficient at R&D. If it is, then trade will increase the growth rate. If it isn’t, trade will reduce the growth rate. This result depends on subtleties of the trading partners’ industrial structures. The possibility that trade reduces long-run growth brings with it the possibility that it also reduces long-run welfare, which means that the world economy may exhibit a type of dynamic inefficiency. To paraphrase Bhagwati (1958), we have the possibility of *immiserizing trade*, though for completely different reasons than those that generated Bhagwati’s immiserising growth. Our theory provides the conditions under which the various possible outcomes hold. It thus offers an explanation for - indeed, a reconciliation of - the sometimes seemingly contradictory evidence on the relation between trade and growth that has fueled a long-running debate in the literature. Trade also can produce a world equilibrium identical

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1 The basic references for the 1st-generation models are Romer (1986, 1990), Aghion and Howitt (1992), and Grossman and Helpman (1991).
to that which would emerge from technology transfer even if no technology transfer actually occurs. Trade thus can deliver “effective technology equalization” in a way that is reminiscent of factor-price equalization.

Second, growth affects trade. In particular, growth can move the world economy from one kind of trading regime to another. This result is important because, as Eaton and Kortum (2012) have noted, one of the major reasons Ricardian theory was not used for empirical work was that it delivered different types of trading regimes, creating a “clumsy taxonomy.” The theory offered no explanation of how the world economy could move from one regime to another, even though such moves can be seen in the data. Our theory overcomes that difficulty by offering a theory of the world’s dynamic adjustment. As in the standard static Ricardian model, our model has two possible trading regimes: an interior solution in which the countries specialize in different sets of goods and a corner solution in which one country specializes but the other continues to produce all types of goods. In static theory, whichever type of solution obtains initially also obtains forever because there is no technical progress. The different types of regimes thus appear to be steady states. In our model, however, technical progress changes quality-adjusted prices and leads to the possibility of moving endogenously from one regime to another. The regimes thus are not necessarily steady states, though they can be. The interior regime is dynamically stable and so is a local attractor for the world system. Once the world economy enters the interior, it converges to a unique balanced growth path and remains in the interior forever. In contrast, the corner regime is saddle path stable. There is a saddle-path-stable steady state inside the corner regime, to which the world economy will converge if it happens to be on the razor-edge dynamic adjustment path that leads there. Otherwise, the world economy either will move out of the corner and into the interior or will move deeper into the corner and go asymptotically to a third steady state where the growth rates of the trading partners are permanently different. This last possibility contrasts sharply with most of the literature, in which the world converges to a balanced growth path with all growth rates the same. To the best of our knowledge, these results on the effects of trade on growth are completely new. Each of the possible outcomes corresponds to observations in the data. For example, the movement to an interior trading equilibrium is consistent with the relation between the world’s old and new industrialized countries, such as Western Europe on the one hand and Japan on the other. The movement toward an equilibrium with permanent growth rate differences is consistent with the relation between the developed world and sub-Saharan Africa or, more generally, Quah’s (1997) famous "twin peaks."

Trade and growth are engaged in a dynamic dance, intricate and beautiful, in which the partners move together and react to each other. Let’s see how they do it.

2 Facts, Modeling Choices, and Related Literature

Many articles that study trade and growth in fact are not about trade per se but rather are about technology transfer, as Feenstra (1996) first noted. In those studies, technology transfer may be facilitated by trade, but trade has no growth effects if there is no technology transfer. Previous articles that do study trade itself use early versions of growth models now known to be inconsistent with various aspects of the data, as we briefly explain in a moment. The modern version of endogenous growth theory - the 2nd-generation SDGE model - solves the empirical problems of the earlier versions of growth theory, but no one has used such a model to study trade and growth. We take that step in the present paper.

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3E.g., Acemoglu and Ventura (2002); Alvarez, Buera, and Lucas (2012).
4See Barro and Sala-i-Martin (1997) for a model of imitation with no trade, Connolly (2003) and Connolly and Valderrama (2005) for sophisticated models of imitation facilitated by trade, and Schafer and Schneider (forthcoming) for a model of intellectual property rights, trade, and growth.
6We are aware of only three papers that use a modern fully-endogenous growth model to study any aspect of openness. Howitt (2000) and Peretto (2003) study technology transfer. Dinopoulos and Syropoulos (2004) study trade’s effect on
Like any economic model, ours requires assumptions for it to be useful and tractable. We base most of our assumptions on several facts about technical progress and what drives it, the industrial structure of the economy, and the types of goods that dominate international trade. We make a few other assumptions for the sake of tractability. In this section we explain the major assumptions that determine our analytical framework and the reasons we make them. We discuss minor assumptions about modeling details in the main analysis below.

We begin with an outline of our major sets of facts and then discuss their implications for our modeling choices. The discussion also explains the relation of our work to the previous literature.

2.1 Some Facts

The first set of facts concerns the endogeneity of the industrial structure. Both the number of firms and firms’ market size are endogenous. These facts seem uncontroversial, but most aggregate models have either an exogenous industrial structure or one with only limited endogeneity, restrictions that have important consequences for the predictions emerging from those models that we explain momentarily. Another aspect of the endogeneity of market structure that is important for our purposes is that whole industries shut down in the face of competition from abroad. Examples from the United States are sewing machines and commercial ship building in the past and textiles and furniture in the present. The sewing machine industry is a particularly stark example. Although the sewing machine was invented in the United States, it no longer is produced there. Firms in Japan, Switzerland, and Norway drove the US firms out of business.

The second set of facts concerns the kinds of firms that do R&D and the kind of R&D that they do. Long-run growth in income per person is driven by technical progress, which in turn is driven by firms’ R&D. R&D is done mostly by incumbent firms. In the US, about 70% of R&D is private, and over 90% of that is done by incumbents. The situation is similar in other countries. Within incumbents, most R&D is devoted to reducing the cost of production (about 13%-26% of total R&D expenditure) and improving the quality of the product (about half the rest), as has been known for a long time. The remainder is mostly devoted to developing new products, most of which simply replace existing goods made by the same firm and so really are another form of quality improvement. Very little R&D expenditure is done by outside firms to develop products that then challenge and displace incumbents’ products. For the most part, outsiders bring to market new products that increase the number of varieties rather than replace existing products. R&D thus is much more of the “creative accumulation”
type than the “creative destruction” type.

The third set of facts concerns the kinds of goods that are traded. First, about three-quarters of international trade is in factors of production, either physical capital or intermediate goods. Second, much technical progress is embodied in capital and intermediate goods, which means that the technical progress directly benefits the buyer of the goods rather than the seller. Furthermore, empirical studies have found that embodied technical progress is important in explaining growth.\textsuperscript{12}

Of course the foregoing are not all the facts, which no single model can address. They are the facts we need to construct our model of trade and growth. As with any model, ours ignores aspects of reality in order to be tractable and have anything to say at all.

2.2 Implications for Modeling Choices

The first set of facts determines the type of growth model we use. Models in which the number of firms is fixed or each firm’s market size is exogenous do not correspond to the realities of industrial organization. More important, they produce counterfactual aggregate predictions precisely because of their inadequate treatment of the economy’s industrial structure. In the 1st-generation endogenous growth models, either the number of firms is fixed (the quality-ladder models), or firms have an exogenous market size equal to the entire economy (the variety expansion models). In either case, their best-known counterfactual prediction is the aggregate scale effect, which says that the economy’s growth rate is positively related to the economy’s size, usually measured by the size of the labor force or the whole population. The solid rejection of that prediction led to much effort to produce models of growth without the scale effect.\textsuperscript{13} Two types of models emerged: first semi-endogenous growth models and then 2nd-generation fully-endogenous (i.e., the SDGE) growth models. Semi-endogenous growth models leave intact the industrial structure of the 1st generation models but add a quantitative restriction on a parameter that introduces diminishing returns to R&D. The 2nd-generation endogenous growth models proceed quite differently, altering the industrial structure of the model by making both the number of firms and firms’ market size endogenous.\textsuperscript{14} Both approaches eliminate the scale effect, but they also produce a host of other testable implications that differ strongly across the two approaches. For example, semi-endogenous growth models predict that the economic growth rate is proportional to the population growth rate, depends on R&D resources, and is insensitive to government policy changes. In contrast, 2nd-generation models predict that the economic growth rate has a non-negative but less-than-proportional (and perhaps non-existent) relation with population growth, depends on R&D resources per firm (not just aggregate resources), and is sensitive to policy choices.\textsuperscript{15} A large literature has tested the two approaches, sometimes individually and sometimes jointly. The upshot is that a large battery of tests using a large variety of data favors the 2nd-generation approach over the semi-endogenous alternative.\textsuperscript{16}

\textsuperscript{12}For evidence on: the types of goods traded, see Miroudot, Lanz, and Ragoussis, (2009); the fraction of R&D devoted to embodied technical progress, see Mansfield (1968) and Griliches (1986); and the importance of embodied progress for growth, see Gort, Greenwood, and Rupert (1999) and Meliciani (2000).

\textsuperscript{13}See Chapters 6 and 7 of Barro and Sala-i-Martin (2004) for introductions to those types of models, Grossman and Helpman (1990, Proposition 2) for a statement of the scale effect, and Backus, Kehoe, and Kehoe (1992) and Gong, Greiner, and Semmler (2004) for two examples of evidence against the scale effect.

\textsuperscript{14}The basic references for semi-endogenous growth models are Jones (1995), Kortum (1997), and Segerstrom (1998) and for the 2nd-generation endogenous growth models they are (Peretto 1999c and Howitt 1999). Sometimes semi-endogenous models also are included as a third subclass of 2nd-generation models, but they are better treated as a refinement 1st-generation endogenous growth models because they use the 1st-generation framework with an added restriction of diminishing returns to R&D. In contrast, 2nd-generation models extend the 1st-generation framework by moving from one dimension of R&D to two, which opens whole new types of behavior not possible with either of the dimensions alone. For a somewhat different view on this classification issue, see Dinopoulos and Thompson (1999). However, the classification issue is entirely pedagogical, not substantive.

\textsuperscript{15}See Dinopoulos and Thompson (1999) and Dinopoulos and Sener (2007) for overviews of how different branches of growth theory deal with the scale effect.

In light of the evidence, we proceed with a 2nd-generation model.

Conducting our analysis in the framework of a 2nd-generation growth model distinguishes our paper from virtually all the previous literature on trade and growth, which has used either the 1st-generation or the semi-endogenous approach (or a neoclassical exogenous growth model in the case of the older literature). Furthermore, in light of the evidence on the nature of R&D and the types of firms that do it, we adopt the analytical framework of the “creative accumulation” subset of 2nd-generation fully-endogenous growth models, in which new firms mostly expand the number of varieties rather than replace incumbents as in the “creative destruction approach.” Because so little R&D is of the creative destruction type, we simplify the model by omitting creative destruction altogether and restricting R&D to incumbent firms. Also, in our model quality improvement and cost reduction are isomorphic, so we simplify further by considering only quality improvement, which as we have seen is the lion’s share of R&D anyway. To explore shutdown, our model must allow it, which requires that we use production functions that have no “love of variety,” something we want to do anyway to exclude love of variety as a motive for trade, as we explain momentarily. We show below that many of our conclusions differ, sometimes radically, from those of the earlier work precisely because our analytical framework allows interaction between an endogenous industrial structure and the general equilibrium dynamics.

On the trade side, most trade is in intermediate goods, so we simplify by restricting all trade to intermediate goods, allowing no trade in final consumer goods.

We make a few other assumptions either to keep the analysis analytically tractable or to isolate comparative advantage as the only motive for trade so that we can study its effects alone. On the tractability side, we restrict attention to the case of two countries and two tradable goods, in contrast to the case of many countries and many goods, such as in Eaton and Kortum (2002) and Shiozawa (2007). Although it obviously would be desirable to extend the analysis to the multi-country, multi-good case, doing so poses a severe curse of dimensionality. We are able to solve our model because the number of active state variables never exceeds three. Once we add either more goods and more countries, the number of state variables rises above three, and analytical results become impossible to obtain. Similarly, we cannot entertain the rich market structure that Melitz (2003) uses to examine which firms enter the international market because once again adding a distribution of productivities (that must change over time in a growth model such as ours) would increase the number of state variables beyond analytical tractability. Thus we must surrender some richness on the trade side in order to gain insight on the growth side. Although we allow trade in produced factors of production, we do not allow migration of labor. To isolate comparative advantage from other motives for trade, we rule out (direct) technology transfer and foreign direct investment. Our utility and production functions do not exhibit love of variety, the crucial element of “new trade theory” (e.g., Krugman, 1979; Melitz, 2003), so trade in our model is driven only by comparative advantage even though imperfect competition and increasing returns to scale are present.

Finally, one other aspect of our analysis is worth noting. We do not set out to explain some set of observations. Our motive instead is simple intellectual curiosity: we would like to know how trade...
and growth interact. Nonetheless, our analysis can explain many observations, some of them seemingly contradictory, so the enterprise has practical value.

The remainder of the paper presents the model structure, the model solution, and a welfare analysis, followed by a brief conclusion. Mathematical derivations are relegated to an Mathematical Appendix available from the authors.

3 Model Structure

A country can produce three types of goods: final, processed, and intermediate. Intermediate goods are combined with labor to make processed goods. Processed goods are used to make final goods. Final goods are used for consumption, as an input for intermediate goods, and as an input for into research and development. Endogenous growth models usually have only final goods and intermediate goods. We add the intervening processed goods sector to facilitate the discussion of trade.

3.1 Final Goods

Identical competitive firms produce a single homogeneous final good $Y$ using two non-durable processed goods $X_1$ and $X_2$ as inputs. The production function for the representative firm is Cobb-Douglas:

$$Y = X_1^{1-\epsilon}X_2{\epsilon}$$  \hspace{1cm} (1)

We take the final good as the numeraire, so $P_Y = 1$. The representative firm’s profit is

$$\pi_Y = Y - P_{X_1}X_1 - P_{X_2}X_2$$  \hspace{1cm} (2)

where $P_{X_1}$ and $P_{X_2}$ are the prices of $X_1$ and $X_2$.

3.2 Processed Goods

The processed goods sector comprises two industries, each producing a single homogeneous good $X_i$, where $i \in \{1, 2\}$. Both industries are competitive in all markets. The representative firms in the two industries use non-durable intermediate goods $G$ and labor $l$ to produce their respective processed goods. Consider first the closed economy. We follow Aghion and Howitt (2005) and Peretto (2007) in specifying the processed goods production functions as

$$X_1 = \int_0^{N_1} G_{1j} \left( Z_{1j}^\delta Z_1^\gamma Z_2^{1-(\delta+\gamma)l_{1j}} \right)^{1-\lambda} dj, \hspace{0.5cm} 0 < \lambda, \gamma, \delta < 1$$  \hspace{1cm} (3)

$$X_2 = \int_0^{N_2} G_{2j} \left( Z_{2j}^\delta Z_2^\gamma Z_1^{1-(\delta+\gamma)l_{2j}} \right)^{1-\lambda} dj, \hspace{0.5cm} 0 < \lambda, \gamma, \delta < 1$$  \hspace{1cm} (4)

where $G_{ij}$ is the amount of the intermediate good produced by firm $j$ and used in industry $i$, $Z_{ij}$ is the quality of good $G_{ij}$, $Z_i \equiv (1/N_i) \int_0^{N_i} Z_{ij}dj$ is the average quality of class-$i$ intermediate goods (explained momentarily), $l_{ij}$ is the amount of labor working with intermediate good $G_{ij}$, and $N_i$ is the number of varieties of intermediate goods used in each industry. There are two classes of intermediate goods, $\{G_{1j}\}_{j=0}^{N_1}$ and $\{G_{2j}\}_{j=0}^{N_2}$, with one class providing inputs for the $X_1$ industry and the other class providing inputs for the $X_2$ industry. The sets of intermediate goods used by the $X_1$ and $X_2$ industries are disjoint and generally have different numbers of elements (i.e., in general $N_1 \neq N_2$). Each intermediate good’s quality $Z_{ij}$ is determined by the R&D that has been done by the firm that produces $G_{ij}$. Labor productivity depends on the quality $Z$ of the intermediate good it works with. To allow for the types of knowledge spillovers usually found in the endogenous growth literature, we let labor productivity in industry $X_1$ depend on both the average quality $Z_1$ of the $\{G_{1j}\}$ goods used in industry $X_1$ and the average quality $Z_2$ of the $\{G_{2j}\}$ goods used in industry $X_2$. Industry $X_2$’s situation
is symmetric. The importance of knowledge spillovers across industries is governed by the parameters $\delta$ and $\gamma$. Setting $\delta + \gamma = 1$ would exclude knowledge spillovers across industries. As we show later, only one of our results depends on the presence of cross-industry spillovers. Everything else in the paper would remain intact if we set $\delta + \gamma = 1$. However, empirically cross-industry spillovers are economically and statistically significant, so we include them.\(^{20}\)

The quality $Z_{ij}$ of intermediate good $G_{ij}$ is embodied in the good but augments labor. The idea is that many kinds of machines have embodied in them characteristics that replace skill. For example, the original textile machines of the Industrial Revolution apparently were like that:

“[W]ith the marvelously perfect and self-acting machinery of today no special skill is required on the part of the attendant. The machinery itself supplies the intelligence.”\(^{[Quoted by Clark (2007), emphasis added.]}\)

With the advent of textile machines, a worker with no weaving skill could produce the identical patterns of a skilled worker but in much larger quantity. The progress embodied in the machines replaced the human capital embodied in the skilled workers, so it should enter the production function in exactly the same way that human capital did - as an augmentation of unskilled labor. Similarly, automobile mechanics no longer need the diagnostic skills they once did because the computer chips in modern cars tell the mechanic what is wrong with the car. Changes in quality constitute technical progress in our model. Labor is the only non-reproducible factor of production, so technical progress must be labor-augmenting if there is to be perpetual growth. As explained in Section 2.2, this labor-augmenting technical progress embodied in intermediate goods is the only kind of continuing technical progress that we allow in the model. The model also has technical progress in the form of variety expansion, but variety expansion comes to an end because of fixed operating cost and so is not a source of long-term economic growth.\(^{21}\)

Two issues concerning our specification of the production functions (3) and (4) require brief comment. First, the functions do not exhibit love of variety, a restriction that helps us in two ways: it isolates the effects of comparative advantage by eliminating an alternative motive for trade (\textit{a la} Krugman, 1979, and Melitz, 2003) and it allows shut-down by preventing the marginal product of an intermediate good from going to infinity when the quantity of that intermediate goes to zero. Recall from section 2.1 that shut-down is one of the facts we want to address. Second, because the functions (3) and (4) do not exhibit love of variety, they are unstable in the Nash sense. Intermediate goods are perfect substitutes in (3) and (4). That means that, if one firm gets slightly ahead of the others in its level of technology, it takes over the market immediately, converting the model from monopolistic competition to monopoly. We avoid the monopoly outcome by following Aghion and Howitt (2005) and Peretto (1999b, 2007) in assuming that all firms start at the same level of technology and that new entrants enter at the industry average level. Those assumptions guarantee that the economy remains in the monopolistically competitive equilibrium because the model solution is symmetric and there are no random shocks to disrupt the equilibrium. Under such circumstances the equilibrium is well-behaved dynamically, as we show below. Intra-industry behavior is not relevant to the issues we discuss, so Nash

\(^{20}\)For evidence on intra-industry productivity spillovers, see Irwin and Klenow (1994). For inter-industry productivity spillovers, see Bernstein and Nadiri (1988), Jaffe (1988), Glaeser, Kallal, Scheinkman, and Shleifer (1992), Nadiri (1993), and Forni and Paba (2011). For cross-country productivity spillovers, see Coe, Helpman, and Hoffmaister (1997), Frantzen (2000), and Schmerring, Fischer, and Reissmann (2007). For learning-by-doing spillovers, see Argote and Epple (1990). Some spillovers are even more transcendent. For example, James Watt used knowledge gained from studying distillation of Scotch whiskey to improve the Newcomen steam engine, which then was used to drain water from mines and subsequently to transform weaving. Johannes Kepler used methods for measuring the amount of wine in wooden barrels to interpret Tycho Brahe’s planetary data and confirm the elliptical orbit of Mars. See Burke (1985) for a brief description of many more such examples, right up to modern times.

\(^{21}\)If the model had population growth, then variety expansion would be a source of growth even with fixed operating costs. None of our conclusions would be affected by including population growth. See Peretto and Connolly (2007) for a full discussion of the implications of fixed operating costs for variety expansion.
instability is not a problem for our analysis. We therefore sacrifice richness in one dimension of the economy’s microeconomic behavior (aspects of intra-industry dynamics) in order to gain insight into other aspects (shut-down induced by international competition). In fact, we are sacrificing very little by using a Nash-unstable model that starts in the Nash equilibrium. Even in models that are Nash-stable, the general equilibrium dynamics are impossible to work out in closed form because there are infinitely many state variables - one for each firm’s quality level. For that reason, even growth models that are Nash-stable always assume that the economy starts in the Nash equilibrium. The economy then stays there because of the symmetry of the firm decisions. The intra-industry dynamics that would arise out of equilibrium thus are ruled out, the problem of infinite dimensionality is side-stepped, and the models become analytically tractable. For examples, see Peretto (1996, 1998a, 1998b, 1998c, 1999a, 1999b) and Howitt (1999). Several of Peretto’s papers have detailed discussions of the assumptions needed to produce Nash stability and symmetric equilibrium. We make the same equilibrium assumption and so side-step the same problem in the same way.

In our model, we allow trade in the intermediate goods $G_{ij}$, so the foregoing processed goods production function must be modified to reflect the broadened array of available intermediate goods. For the industry 1 in the home country, it becomes

$$X_{H1} = \int_0^{N_{H1}} (G_{H1j} - G_{H1j}^E)^\lambda \left[ Z_{H1j}^\delta Z_{H1}^{\gamma} (\tilde{Z}_{H2})^{1-(\delta+\gamma)} l_{H1H} \right]^{1-\lambda} dj$$

$$+ \int_0^{N_{F1}} (G_{F1k}^I)^\lambda \left[ Z_{F1k}^\delta Z_{F1}^{\gamma} (\tilde{Z}_{H2})^{1-(\delta+\gamma)} l_{H1F} \right]^{1-\lambda} dk$$

The subscripts $H$ and $F$ denote the home and foreign countries, respectively, $G_{H1j}^E$ is the amount of $G_{H1j}$ exported, and $G_{F1k}^I$ is the amount of intermediate good $G_{F1k}$ imported from the foreign country. To keep the analysis tractable, we restrict a country either to export all of one class of intermediates or to import all of it, so only one of $G_{H1j}^E$ and $G_{F1k}^I$ can be positive at any time. We assume that $\tilde{Z}_{H2}$ takes the following form:

$$\tilde{Z}_{H2} = \begin{cases} Z_{H2} & \text{only home-produced } G_2 \text{ used} \\ Z_{H2}^{1-\eta} Z_{F2} & \text{both home- and foreign-produced } G_2 \text{ used} \\ Z_{F2} & \text{only foreign-produced } G_2 \text{ used} \end{cases}$$

where $0 < \eta < 1$.

Processed goods firms in industry 1 in the home country choose the combination of intermediate goods 1 to buy from domestic firms and foreign firms to maximize profit:

$$\max \pi_{X_1} = P_{X_1} X_{H1} - \int_0^{N_1} P_{G_{H1j}} (G_{H1j} - G_{H1j}^E) dj - \int_0^{N_{F1}} P_{G_{H1F1}} G_{F1k}^I dk$$

$$- \int_0^{N_1} w l_{H1H} dj - \int_0^{N_{F1}} w l_{H1F} dk$$

Solving for labor demand and plugging into the first-order conditions for $G_{H1j} - G_{H1j}^E$ and $G_{F1k}^I$ yields a bang-bang solution in which processed goods firms buy only $G_{H1j} - G_{H1j}^E$ or only $G_{F1k}^I$, according to which has the lower quality-adjusted price: $P_{G_{H1}}/Z_{H1}^{\delta+\gamma}/(1-\lambda)/\lambda$ and $P_{G_{F1}}/Z_{F1}^{\delta+\gamma}/(1-\lambda)/\lambda$, respectively. The situation for producers of $X_2$ is similar.

### 3.3 Intermediate Goods

The intermediate goods sector comprises two industries distinguished by which processed goods industry buys their products, as explained above. We divide the discussion of the intermediate goods industries into two parts, describing the behavior of incumbents and entrants.
3.3.1 Incumbents

Each intermediate goods industry comprises a continuum of monopolistically competitive firms. A firm produces a single intermediate good $G_{ij}$ unique to that firm and also undertakes R&D to improve the quality $Z_{ij}$ of the good it produces. An increase in quality raises the demand for the good and so raises profit. We follow Peretto (1996, 1998b, 1998c) and assume that all R&D is done by incumbents because the data show that the lion’s share of R&D is done in-house, as explained in section 2.1. As we show below, that fact has important implications for the effect on trade on growth.

Production, technologies, R&D technologies, and fixed costs are the same for all firms within a given industry but differ across industries. The industrial structure thus is one of symmetry within each intermediate goods industry but asymmetry across the industries. Asymmetry is unusual in endogenous growth models, and we regard it as one of the contributions of our analysis. Asymmetry provides a natural division among goods along which comparative advantage may operate. It also is a step toward a more realistic analysis than the usual framework of complete symmetry.

All firms in industry $i$ have a linear technology that converts $A_i^{-1}$ units of the final good into one unit of intermediate good $G_{ij}$ (i.e., the unit cost of $G_{ij}$ is $A_i$)

$$G_{ij} = A_i^{-1}Y_{ij}$$  \hspace{1cm} (5)

where $Y_{ij}$ is the amount of the final good used by firm $j$ in industry $i$. Similarly, the R&D production functions are the same within an industry but differ across the industries. Spending one unit of the final good on R&D in industry $i$ yields $\alpha_i$ units of quality improvement:

$$\dot{Z}_{ij} = \alpha_i R_{ij}$$  \hspace{1cm} (6)

where $R_{ij}$ is amount of the final good $Y$ spent on R&D.\(^{22}\) The firm obtains the resources for $R$ from retained earnings.\(^{23}\)

Firms face a fixed operating cost $\phi_{ij}$ that depends on the average qualities $Z_i$ and $Z_k$ of the firm’s own industry and the other industry, respectively. There are two channels of influence. First, the operating cost depends positively on own industry quality on the assumption that a more sophisticated industry requires more sophisticated inputs. We borrow a page from the adjustment cost literature and assume that fixed operating costs are convex in the level of industry sophistication. Second, operating costs are reduced by knowledge, which in our model is captured by quality. We suppose that both $Z_i$ and $Z_k$ help reduce costs. To keep the analysis tractable, we assume that all firms in a given industry have the same fixed cost function, which takes the analytically convenient form $\phi_{ij} = \theta_i Z_i^3/Z_i Z_k = \theta_i Z_i^2/Z_k$. The cubic term in the numerator captures the convexity of cost, and the two terms in the denominator capture the effect of knowledge in reducing costs. Dependence of cost on industry averages and not the firm’s own quality level is not restrictive because symmetry within an industry makes each firm’s quality equal to the industry average.

The firm pays a dividend of

$$\Pi_{ij} = G_{ij} \left( P_{G_{ij}} - A_i \right) - \phi_i - R_{ij}$$

The value of the firm is the present discounted value $V_{ij}(t)$ of its dividends:

$$V_{ij}(t) = \int_t^\infty \Pi_{ij}(\tau) e^{-\int_t^\tau r(s) ds} d\tau$$  \hspace{1cm} (7)

\(^{22}\)We could add knowledge spillovers in the production of $G$ and in R&D, but they would add no new insight in the model, only different channels for the main effects that the spillovers in the processed goods production functions already capture. Thus for simplicity we exclude them.

\(^{23}\)It would be slightly more precise to distinguish between investment $I$ and retained earnings $R$ because in principle the two need not be the same. However, the requirements of general equilibrium will make them the same, so we keep the notation simple by imposing $I = R$.  

9
The firm chooses the paths of its product price $P_{G_{ij}}$ and its R&D expenditure $R_{ij}$ to maximize its value subject to its demand function, R&D production function, and the average qualities, $Z_1$ and $Z_2$, which the firm takes as given.

Differentiating eq.(7) with respect to time gives the firm’s rate of return to equity (i.e., entry):

$$r^E_{ij} = \frac{\Pi_{ij}}{V_{ij}} + \frac{\dot{V}_{ij}}{V_{ij}}$$

which is the usual profit rate plus the capital gain rate.

### 3.3.2 Entrants

We assume that entry and exit are costless. For simplicity, we refer only to entry, even though exit is always possible. Costless entry implies that $N_i$ is a jumping variable. Whenever the net present value of a new firm $V$ differs from the entry cost of zero, new firms jump in or out to restore equality between the value of the firm and the entry cost. We thus have at all times

$$V_{ij} = 0$$

As a result, we also have $\dot{V} = 0$. Multiplying both sides of eq. (8) by $V$ and imposing $V = 0$ and $\dot{V} = 0$ implies that

$$\Pi_{ij} = 0$$

As mentioned earlier, we follow Aghion and Howitt (2005) and Peretto (1999b, 2007) in assuming that initially all firms have the same level of technology and that new entrants arrive with the industry average level of technology. The model’s equilibrium then is symmetric at all times within each industry, with all firms in an industry making the same decisions on pricing, production, and R&D expenditures. Firms in an industry are all alike, but firms differ across industries, with the firms in the two industries generally making different decisions on everything. This asymmetry is a type of heterogeneity. It differs from the heterogeneity in Melitz (2003) and Baldwin and Robert-Nicoud (2008), in which there is only one industry whose individual firms are heterogeneous. The pattern of production and trade is not determined by comparative advantage in those models but rather by initial endowment of types of goods, i.e., by an Armington (1969) assumption.

The zero profit condition (10) implies that firms pay no dividends but instead retain all earnings for investment in R&D. The household owners of the firm reap their return in the form of increasing consumption as R&D delivers higher quality and raises output. The current-value Hamiltonian for the intermediate good firm is

$$CVH_{ij} = G_i(P_{G_{ij}} - A_i) - \phi_i - R_{ij} + q_{ij}(\alpha_iR_{ij})$$

where $q_{ij}$ is the costate variable. The Maximum Principle gives the necessary condition for the evolution of $q_1$, which we can write as

$$r^{R&D}_{ij} = \frac{\partial F_{ij}}{\partial Z_{ij}} \frac{1}{q_{ij}} + \frac{\dot{q}_{ij}}{q_{ij}}$$

---

24 We explored an extension of the model with costly entry but could not obtain a solution. The problem is that costly entry leads to a gradual approach to the equilibrium number of varieties, and along that transition path all four types of varieties are active, leading to a four-dimensional system of differential equations in $\{N_{H1}, N_{H2}, N_{F1}, N_{F2}\}$ that cannot be solved analytically. See Peretto (2007) for discussions of costly entry in a framework similar to ours.

25 Note that a strategy in which a firm with below-average quality leaves the market and then immediately re-enters with the average quality is not feasible. An incumbent who leaves loses all claim to the niche he vacates. That is the meaning of exit, after all. Upon re-entering the market, he would join the pool of other potential entrants vying for the vacated niche. There are an uncountable number of them, so the probability that the former incumbent will reclaim the vacated niche is zero, so the expected value of the strategy is zero, rendering it unprofitable. For a complete discussion of market equilibrium and its stability in these types of R&D models, see Peretto (1996, 1999b, 2007) and the references cited therein.
This equation defines the rate of return to R&D (i.e., to quality innovation) \( r_{ij} \) as the percentage marginal revenue from R&D plus the capital gain (percentage change in the shadow price). As with intermediate goods prices, the expressions for the rates of return differ across the two industries.

### 3.4 Households

There is a representative household that supplies labor inelastically in a perfectly competitive market and buys corporate equity. We assume for simplicity that there is no population growth.\(^{26}\) The utility function is

\[
U(t) = \int_t^\infty \log(c) e^{-\rho t} \tag{12}
\]

where \( c \) is consumption per person and \( \rho \) is the rate of time preference.

The only assets that the household can accumulate are firms that it owns. The household’s lifetime budget constraint therefore is

\[
0 = \int_0^\infty \left( \int_0^{N_1} \Pi_{1j} dj + \int_0^{N_2} \Pi_{2j} dj + wL - C \right) e^{-\int_t^T r(s) ds} dt \tag{13}
\]

where \( C \) is aggregate consumption and \( L \) is labor supply. The intertemporal consumption plan that maximizes utility is given by the consumption Euler equation

\[
r = \rho + \frac{\dot{C}}{C} \tag{14}
\]

### 3.5 Trade

The trade part of the model is a standard Ricardian specification in most respects. Ricardian models remain popular because they correspond well with the data and because they are useful for theoretical exercises (e.g., Eaton and Kortum, 2002; Alvarez and Lucas, 2007). The main difference between the model used here and the standard model is that the unit costs that determine comparative advantage are adjusted by quality levels that change through quality-improving technical progress. That feature allows growth to affect trade and can lead to endogenous changes in the trading regime, as we show below. The only other contribution to the literature of which we are aware with this latter property is Redding’s (1999) first-generation learning-by-doing model. We restrict attention to two country and two goods. The trade dimension of the model is thus more restricted than some contributions (Dornbusch, Fischer, and Samuelson, 1977; Eaton and Kortum, 2002), but that restriction greatly simplifies the growth part of the model. We have a model of fully endogenous growth, whereas growth is completely absent from the static models of Dornbusch, Fischer, and Samuelson (1977) and Eaton and Kortum (2002), and is purely exogenous in Alvarez, Buera, and Lucas’s (2012) extension of Eaton and Kortum (2002). Merging the two types of models to produce one of multilateral trade in many goods in a framework of fully endogenous growth would be a very useful extension of what we do here.

#### 3.5.1 Assumptions and Overview

There are two countries, home and foreign. They have the same utility functions and the same production functions for \( Y \), \( X_1 \), and \( X_2 \) but different production functions and fixed costs for the intermediate goods. The G-production functions have the same form in the two countries but different values for

\(^{26}\)Positive population growth would induce perpetual entry at the rate of population growth, but no important results obtained below would change. See Peretto (2007) for a related model with population growth.
their productivity parameters $A_1$ and $A_2$, R&D productivities $\alpha_1$ and $\alpha_2$, and fixed operating costs $\phi_1$ and $\phi_2$.

We assume that only intermediate goods are tradable, obviously a simplification but also not a bad approximation given that trade in intermediates constitutes about three-quarters of all trade in OECD countries (Miroudot, Lanz, and Ragoussis, 2009). The particular intermediate goods a country imports or exports are determined by comparative advantage, not imposed \textit{a priori}. In that regard, the model differs from almost all the literature on growth with trade, in which the set of varieties each country can invent and produce is exogenously given and countries always trade all varieties they produce (e.g., Grossman and Helpman 1990 and 1991, Feenstra 1996, Acemoglu and Ventura 2002). As we show below, the endogenous determination of the sets of imported and exported goods leads to several important results on the relation between trade and growth.

We follow Grossman and Helpman (1991, Chapter 7) in treating the two countries as “large” because intermediate goods producers are monopolistically competitive and set prices rather than take them as given. We also assume that both countries do some R&D. For minor technical reasons, that helps make the model analytically tractable, but the assumption also corresponds with the data. Eaton and Kortum (1996) show that among OECD countries most R&D is concentrated in a handful of countries but also that every country takes out at least a few patents, indicating that those countries have active R&D programs. They also note that many inventions are never patented, so total R&D effort is larger than that captured by the patent data. To avoid complications arising from strategic behavior, we suppose that neither country has a government that can act as an agent representing all its firms collectively in bargaining with the other country. Each country comprises a multitude of agents who cannot form a cartel to act as monopolists or monopsonists. The focus of this paper is trade, so to keep the analysis simple and the results sharp, we suppose there is no foreign investment and no direct technology transfer by multinational firms. There is indirect technology transfer through knowledge spillovers as already discussed.

In economies with fixed operating costs, long-run growth is driven by quality improvement, as explained earlier, so if trade is to affect growth, it must affect a country’s rate of quality improvement. That possibility arises in our model. When a country imports a good, it also imports the quality embodied in the good, which has two effects. First, there is a static effect through the knowledge spillover. That type of effect is well-known from trade models where externalities are present. Second, there is a dynamic effect through the growth rate of the good’s quality. A country imports the quality improvement generated by its trading partner. Both the spillover effect and the growth rate effect can be positive or negative. The pattern of trade is determined by quality-adjusted prices. It is possible for a country to have the lowest quality-adjusted price even though it does not have the highest quality good. The country merely needs to be sufficiently low unit cost $A$ for the good. It then becomes the exporter for that good. However, production efficiency in unrelated to R&D efficiency, so a country that is highly efficient in production may be inefficient at R&D. Because R&D is done in house, countries that shut down production of a good and import it also shut down their R&D on that good, accepting whatever R&D is delivered by the exporting country. Because of these effects, opening a country to trade may raise or lower that country’s initial income, growth rate, and welfare. Furthermore, even though there is no direct technology transfer in the model, trade may generate \textit{effective technology equalization} in a manner reminiscent of factor-price equalization. Finally, besides these effects of trade on growth, there are interesting effects in the other direction, of growth on trade. In particular, growth may change trading patterns in ways apparently never previously studied. Except for the static spillover, these results are new to this paper.

3.5.2 Comparative Advantage

International trade takes place if each country has a comparative advantage in selling a good. In our model, comparative advantage means that each country has a lower quality-adjusted relative price for
one class of intermediate goods:

\[
\frac{P_{G_{H1}}}{Z_{H1}^{\frac{(\delta+\gamma)(1-\lambda)}{\lambda}}} \leq \frac{P_{G_{F1}}}{Z_{F1}^{\frac{(\delta+\gamma)(1-\lambda)}{\lambda}}} \quad \text{and} \quad \frac{P_{G_{H2}}}{Z_{H2}^{\frac{(\delta+\gamma)(1-\lambda)}{\lambda}}} \geq \frac{P_{G_{F2}}}{Z_{F2}^{\frac{(\delta+\gamma)(1-\lambda)}{\lambda}}} \tag{15}
\]
or the reverse. The direction of the inequalities determines the trading pattern, i.e., which goods are exported and imported. The direction is inconsequential to our results, so we suppose hereafter that the equalities are as shown in (15). The dependence of the trading pattern on comparative advantage may seem an obvious property for a trade model to have, but in several prominent studies of trade and growth, such as Grossman and Helpman (1990) and Acemoglu and Ventura (2002), comparative advantage plays no role in determining the trade pattern. In those studies, each country is endowed with a set of goods that it produces and that differ from the goods produced in any other country. Each country always trades all types of goods that it produces, so the pattern of trade is exogenous and independent of the terms of trade, which affect only trading volume.

Because the quality-adjusted price does not depend on the quantity bought, a country will buy all of an intermediate good from whomever has the lower quality-adjusted price. In our model, all firms in a given intermediate goods industry charge the same price, so if a country decides to import one good in a given industry, it will import all goods in that industry. Also, a country tends to stop producing the class of intermediate goods it imports and specialize in producing the other class of goods. Countries specialize completely if strict inequality holds in (15), which means under our imposed direction of the inequalities that the home country specializes in intermediate good 1 and the foreign country specializes in good 2. With weak inequality, a country may not fully specialize, meaning that it may import a good but also continue to make it at home. We discuss complete and incomplete specialization in more detail below.

The final good in the home country is the numeraire: \( P_{Y_{H}} = 1 \). The price of the final good in the foreign country is \( P_{Y_{F}} \). The price of the intermediate good equals the monopolistic markup over unit cost. These facts imply that the comparative advantage condition (15) is equivalent to

\[
\frac{A_{H2}}{A_{F2}} \left( \frac{Z_{F2}}{Z_{H2}} \right)^{\frac{(\delta+\gamma)(1-\lambda)}{\lambda}} \geq P_{Y_{F}} \geq \frac{A_{H1}}{A_{F1}} \left( \frac{Z_{F1}}{Z_{H1}} \right)^{\frac{(\delta+\gamma)(1-\lambda)}{\lambda}} \tag{16}
\]

The price \( P_{Y_{F}} \) must be in this closed interval because otherwise condition (15) would be violated and both countries would try to export the same good, implying a market disequilibrium. If we ignore the quality ratios and look at the unit cost ratio only, inequality (16) is the standard trade condition for Ricardian model. In the standard Ricardian model, labor is the only factor of production for tradable goods, and the relative wage across countries must be inside an interval defined by the unit cost ratios. In our model, it is not final goods but rather intermediate goods that are traded, so the relevant interval is defined by the productivity ratios. The difference between (16) and the standard expression is that (16) adjusts the productivity parameters (on which the monopolistic prices depend) by the quality of the respective goods.

It is important in what follows to notice that a country may buy the good with the lower quality if that good’s production cost is low enough (e.g., "Walmart goods from China"). This outcome differs from growth models based on creative destruction, in which the firm with the best quality uses limit pricing to drive out of business all competitors with lower quality. Our model is not one of creative destruction but rather one of creative accumulation, in which new varieties do not supplant an existing variety but rather add to the total number of varieties available.

4 Model Solution

To solve the model, we find the prices and quantities of the final good \( Y \), the processed goods \( X_{1} \) and \( X_{2} \), and the intermediate goods \( G_{1j} \) and \( G_{2j} \). We also must find consumption \( C \), the investment
amounts $I_1$ and $I_2$, the employment levels $l_{1j}$ and $l_{2j}$, the wage $w$, the numbers of firms $N_1$ and $N_2$, and the rates of return $r_{Eij}, r_{R&Dij}$, and $r$. Those allow us to solve for the growth rates of the variables we are interested in. The solution is obtained in the usual way, so the details are relegated to the Mathematical Appendix and only the important results are discussed here.

As usual with monopolistic competition, the optimal values for the prices $P_{Gi,j}$ are constant markups over variable cost:

$$P_{Gi,j} = \frac{A_i}{\lambda} \equiv P_{Gi} \quad (17)$$

The Hamiltonian is linear in R&D expenditure, so the solution for investment expenditure $R_{ij}$is bang-bang with $R_{ij}$ infinite, zero, or positive and finite depending on whether $1/\alpha$ is larger, smaller, or equal to the costate variable $q_{ij}$. We rule out $R_{ij} = \infty$ because it is inconsistent with market equilibrium. We rule out the other corner solution $1/\alpha < q_{ij}$ because it implies no economic growth, and we are interested here in the case where perpetual growth occurs. We thus have the interior solution

$$\frac{1}{\alpha_i} = q_{ij} \quad (18)$$

The left side of eq. (18) is the same for all $j$, so all firms in industry $i$ choose the same level of R&D, which we denote $R_i$. Firms in a given industry have the same unit cost $A_i$ and so also charge the same price, by (17). Those firms thus pay the same dividend and so have the same value, by (7). Because $\alpha_i$ is a constant, we also have $\dot{q}_{ij}/q_{ij} = 0$ in (11).

The solutions for the number of firms, number of employees per firm, and the rates of return for each industry show us why the endogenous market structure of this model kills the aggregate scale effect on the long-run growth rate. The expressions are easiest to understand for the closed economy, but the principles are identical for the open economy. The numbers of firms in the two industries are

$$N_1 = \Omega_{N1}\epsilon L$$
$$N_2^* = \Omega_{N2}(1-\epsilon)L$$

where $\Omega_{N1}$ and $\Omega_{N2}$ are constant functions of various parameters in the system given in the Mathematical Appendix. The internal symmetry of each industry leads all firms in an industry to hire the same amount of labor:

$$l_1 = \epsilon L/N_1$$
$$l_2 = (1-\epsilon)L/N_2 \quad (19)$$

The rates of return in the two industries are

$$r_{1j} \equiv r_1 = \Omega_{r1}l_1 \quad (21)$$
$$r_{2j} \equiv r_2 = \Omega_{r2}l_2 \quad (22)$$

where $\Omega_{r1}$ and $\Omega_{r2}$ are constant functions of various parameters in the system given in the Mathematical Appendix. These expressions for the rates of return depend on $l$, which is individual firm size, not on $L$ alone, which is related to population size. An increase in $L$ raises demand by the processed goods sector for intermediate goods and thereby raises profit of the existing intermediate goods firms. Thus there is a scale effect at the firm level, but it is only temporary. The increase in profit induces entry of new firms and raises $N_i$ to return $L_i/N_i$ and the rate of return $r$ to their original levels. The long-run constancy of the rate of return implies that the long-run growth rate is constant, too (see the Mathematical Appendix), so the scale effect of first-generation models is absent here.\(^{27}\)

\(^{27}\)With no entry cost, the number of firms jumps instantaneously to the equilibrium value, and the transition is immediate. With a positive entry cost, the transition would take time, during which profit and the return to R&D are abnormally high, raising the growth rate temporarily. That means that measured "productivity" will be higher after the increase in the population even though the steady state growth rate is not. Our type of model with a positive entry cost can explain the finding by Frankel and Romer (1999), Alesina, Spolaore, and Wacziarg (2000), and Alcala and Ciccone (2004) that trade can affect the levels (not growth rates) of productivity and income.
It is important to recognize that the scale effect is a by-product of a more fundamental problem with first-generation growth models, which is that they treat firm size as a constant. Making firm size endogenous eliminates the scale effect, but, more important, it eliminates the underlying reason for the scale effect and thereby changes other results emerging from the model, as we see later.

4.1 Complete Specialization

When condition (16) holds with strict inequality, both countries completely specialize in the class of goods in which they have a comparative advantage. The final goods industry is competitive and has a Cobb-Douglas production function, so the final good producer in the home country pays compensation \((1 - \epsilon)Y_H\) to the producers of intermediate industry 2, which are foreign firms. Similarly, the final good industry in the foreign country pays compensation \(\epsilon Y_F\), measured with the final good price from the home country, to the intermediate producers of industry 1, which are firms in the home country. Trade balance requires \(1 \cdot Y_H (1 - \epsilon) \lambda = P_{Y_F} \cdot Y_F \epsilon \lambda\), which after some substitution and rearrangement can be written as \(P_{Y_F} = [(1 - \epsilon) L_H / \epsilon L_F]^{1 - \lambda}\). The numerator is the total amount of the home country’s labor force that is using intermediate goods produced in the foreign country, and the denominator has the converse meaning. Substituting this value for \(P_{Y_F}\) yields the condition for complete specialization:

\[
\frac{A_{H2}}{A_{F2}} \left( \frac{Z_{F2}}{Z_{H2}} \right)^{(\delta + \gamma)/(1 - \lambda)} > \left[ \frac{1 - \epsilon}{\epsilon L_F} \right]^{1 - \lambda} \frac{A_{H1}}{A_{F1}} \left( \frac{Z_{F1}}{Z_{H1}} \right)^{(\delta + \gamma)/(1 - \lambda)}
\]

This condition means that each country is "technologically big enough" to satisfy its trading partner’s demands, as will become clearer when we discuss incomplete specialization.

Complete specialization is equivalent to an integrated economy with \(G_1\) produced by the technologies from the home country and \(G_2\) produced by the technologies from the foreign country. Under complete specialization, the home and foreign countries abandon intermediate goods industries 2 and 1, respectively. Once production of an intermediate good has stopped, R&D to improve its quality also stops because R&D is done in-house by the producing firms, which now have shut down. As a result, \(Z_{H2}\) and \(Z_{F1}\) stop growing, but \(Z_{H1}\) and \(Z_{F2}\) continue to grow. That widens the price interval within which complete specialization occurs, so if the world economy starts in a state of complete specialization, it stays there forever. Complete specialization is dynamically stable.

This model offers an explanation for intra-industry trade in terms of specialization arising from comparative advantage as in Davis (1995) rather than the increasing returns emphasized by Krugman (1979). Think of the intermediate goods sector as an industry and its two subdivisions as sub-industries. Under autarky, each country is active in both sub-industries. Once trade opens, each country specializes in one sub-industry and imports the products of the other. At the industry level, each country engages in intra-industry trade.

4.1.1 Level effect

Opening the world to trade has an immediate impact on the trading countries' income levels. The home country’s final output under autarky and trade are

\[
Y_H^{\text{Autarky}} = \kappa_H' \left[ \left( \frac{Z_{H1}^{\delta + \gamma}}{P_{G_{H1}}} \right) Z_{H2}^{1 - (\delta + \gamma)} \epsilon L_H \right]^\epsilon \left[ \left( \frac{Z_{H2}^{\delta + \gamma}}{P_{G_{H2}}} \right) Z_{H1}^{1 - (\delta + \gamma)} (1 - \epsilon) L_H \right]^{1 - \epsilon}
\]

\[
Y_H^{\text{Trade}} = \kappa_H' \left[ \left( \frac{Z_{H1}^{\delta + \gamma}}{P_{G_{H1}}} \right) Z_{F2}^{1 - (\delta + \gamma)} \epsilon L_H \right]^\epsilon \left[ \left( \frac{Z_{F2}^{\delta + \gamma}}{P_{G_{F2}}} \right) Z_{H1}^{1 - (\delta + \gamma)} (1 - \epsilon) L_H \right]^{1 - \epsilon}
\]

\(^{28}\)For more discussion of this point, see Peretto (1998b).
where \(\kappa_H'\) is a constant. The first term in brackets is the contribution from \(X_{H1}\) to \(Y_H\), and the second term in brackets is the contribution from \(X_{H2}\) to \(Y_H\). Trade affects the initial level of final output through two channels. The first channel is the direct impact of importing an intermediate good and is captured by the inverse of the quality-adjusted price \(Z_{F2}^{\lambda/(1-\lambda)(\delta+\gamma)}\) in (25). Imports have a lower quality-adjusted price, so substituting the imported \(G_{F2}\) for the domestically produced \(G_{H2}\) yields higher quality for the same expenditure and thus greater output of \(Y\), which is just the standard gain from trade. The second channel is the spillover to industry 1 through \(Z_{F2}^{1-\lambda(\delta+\gamma)}\) in (25). The spillover is an externality that is positive or negative depending on whether \(Z_{F2} > Z_{H2}\) or \(Z_{F2} < Z_{H2}\). The quality level of the foreign good can be below that of the domestic good even though the quality-adjusted price of the foreign good is below that of the domestic good if the foreign good’s unadjusted price \(P_{G_{F2}}\) is sufficiently below the domestic good’s unadjusted price \(P_{G_{H2}}\). Trade does not change the intermediate goods prices, which still are given by the markup over marginal cost.\(^{29}\) Thus the externality can be either positive or negative. Note that the externality operates across different industries in the two countries and so can be positive in one country and negative in the other. For example, it is possible that \(Z_{F2} < Z_{H2}\) but \(Z_{H1} > Z_{F1}\), leading to a negative spillover in the home country and a positive spillover in the foreign country, a situation that may well have characterized many countries during the period of globalization around 1980, as we discuss below. The spillover also can be negative for both countries.

The trade pattern in (15) implies \((P_{G_{H2}}/P_{G_{F2}})^{\lambda/(1-\lambda)(\delta+\gamma)} > Z_{H2}/Z_{F2}\). Using that and the markup rules, we can summarize the conditions for trade’s impact on initial output to be positive or negative:

1. The necessary and sufficient condition for trade to decrease initial output is\(^{30}\)

\[
\left(\frac{A_{H2}}{A_{F2}}\right)^{\lambda/(1-\lambda)(\delta+\gamma)} > \frac{Z_{H2}}{Z_{F2}} > \left(\frac{A_{H2}}{A_{F2}}\right)^{\lambda/(1-\lambda)(\delta+\gamma)+2\gamma(\delta+\gamma)/\gamma} > 1
\]

The first inequality means that the productivity parameters and quality levels are consistent with the assumed trade pattern, and the second inequality means that imports have such a low quality that the externality is high enough to reduce the final output.

2. A sufficient condition for trade to increase initial output level is \(Z_{H2} < Z_{F2}\), which means the quality of imports is higher than the quality of the domestic goods.

This impact effect of trade on the initial level of income is the only result in the paper that depends on there being cross-country knowledge spillovers. None of the dynamic effects to be discussed next, which are the most important contributions of the paper, depend on the presence of spillovers.

### 4.1.2 Growth Rate Effect

Under autarky, each country’s balanced growth rate is

\[
g_k^* = \frac{\delta}{1-\delta} - \frac{\alpha_k\theta_k\alpha_k\theta_k}{1-\delta} - \frac{1}{1-\delta}\beta \tag{26}
\]

where \(k \in \{H, F\}\). On the balanced growth path, the growth rates of \(Z_1\) and \(Z_2\) are equal, and the ratio \(Z_1/Z_2\) is constant, so the following growth rates are equal:

\[
g^* = \frac{Z_1}{Z_1} = \frac{Z_2}{Z_2} = \frac{\dot{Y}}{\overline{C}} = \frac{\dot{X}_1}{X_1} = \frac{\dot{X}_2}{X_2} = \frac{\dot{G}_1}{G_1} = \frac{\dot{G}_2}{G_2} = \frac{\dot{w}}{w} \tag{27}
\]

---

\(^{29}\) The combination of low quality and even lower prices has been a vehicle for achieving economic development through trade in many countries, such as Germany in the 19th century and Japan in the 20th (Grubel and Lloyd, 1975, p. 100).

\(^{30}\) If \(Z_{H1} > Z_{F2}\) \(\Rightarrow Z_{H2}/Z_{F2} > 1\), so according to the trade pattern condition, \((P_{G_{H2}}/P_{G_{F2}})^{\lambda/(1-\lambda)(\delta+\gamma)} > Z_{H2}/Z_{F2} > 1\), and \(\lambda/(1-\lambda)(\delta+\gamma) > 0\). Thus \(P_{G_{H2}}/P_{G_{F2}} > 1\). The Mathematical Appendix shows \((P_{G_{H2}}/P_{G_{F2}})^{\lambda/(1-\lambda)(\delta+\gamma)} > (P_{G_{H2}}/P_{G_{F2}})^{\lambda/(1-\lambda)(\delta+\gamma)-2\gamma(\delta+\gamma)/\gamma}\) given \(\epsilon [1 - (\delta + \gamma)] > 0\) by assumption.
When the two countries trade and completely specialize, their balanced growth rates are equal with the same structure as under autarky but with some of the parameters replaced by the trading partner’s parameters:

\[ g_H = g_F = \frac{\delta}{1 - \delta} \sqrt[1-\epsilon]{\alpha_{H1}\theta_{H1}\alpha_{F2}\theta_{F2}} - \frac{1}{1 - \delta} \rho \]

Equality of the two countries’ individual growth rate means we also have a world balanced growth rate.

These expressions for the countries’ growth rates show some of the effects of the industrial structure’s endogeneity. The first effect concerns the role of fixed operating cost in determining growth rates. It is unsurprising that the growth rates depend positively on the R&D productivities \( \alpha_1 \) and \( \alpha_2 \): the higher the productivity of R&D, the higher the return to R&D, which implies a higher growth rate. However, the growth rates also depend positively on the fixed operating cost parameters \( \theta_1 \) and \( \theta_2 \). Why should higher fixed operating costs increase the growth rates? The reason lies in the subtle interactions taking place at the firm level. The higher the fixed operating cost, the lower the profit for incumbents. The lower profit leads to a smaller equilibrium number of firms and thus a larger market size \( l_1 = L_i / N_i \), for those firms. From eqs. (21) and (22) we see that the larger the firm’s market size, the higher the return to R&D and thus the higher the growth rate.\(^{31}\) This sort of effect is absent from first-generation and semi-endogenous growth models because they have an exogenous market structure (either the number of firms or their market size is exogenous) that by construction rules out endogenous responses of relevant aspects of the industrial structure to economic incentives.

A second effect of the model’s endogenous industrial organization is the absence of a scale effect at the aggregate level, as already mentioned. After trade, the total market size increases for each intermediate goods industry and unit costs change. However, neither of those changes affects the world balanced growth rate. For example, firm size in industry 1 in the home country under autarky and trade are:

\[ \left( \frac{l_{H1}}{N_{H1}} \right)_{\text{Autarky}} = \frac{\epsilon L_H}{N_{H1}} \]  
\[ \left( \frac{l_{H1}}{N_{H1}} \right)_{\text{Trade}} = \frac{L_H}{N_{H1}} \]

When the home country opens to trade, intermediate good producers in industry 1 face not only the domestic market but also the foreign market. Their total market size increases from \( \epsilon L_H \) to \( L_H \). When trade opens, the market size of incumbent firms increases and creates an incipient profit. That instantly causes \( N_{H1} \) to increase to prevent realization of the incipient profit. The larger total market size therefore does not affect the return to R&D and hence does not affect the growth rate. That is how endogenous entry eliminates the scale effect. The absence of a scale effect is consistent with the data (e.g., Backus, Kehoe, and Kehoe, 1992). The endogeneity of market structure in the present model differs from the analytical framework of virtually all the previous literature on trade and growth, such as Grossman and Helpman (1990, 1991), Rivera-Batiz and Romer (1991), Young (1991), Taylor (1993, 1994), Feenstra (1996), Redding (1999), and Alvarez, Buera, and Lucas (2013). Because there is no scale effect here, trade’s effect on growth has no positive bias, in contrast to Taylor (1994) and Redding (1999). Also, an equiproportional increase in \( L_H \) and \( L_F \) has no effect on the growth rate, in contrast to first generation results such as Grossman and Helpman’s (1990) Proposition 2, because new firms enter in response to the economic incentives created by the labor force increase so that firm size and also rates of return end up unchanged. Finally, entry and the concomitant absence of an aggregate scale effect guarantee that the larger country does not take over world R&D, in contrast to Grossman and Helpman (1991, Chapter 9).

\(^{31}\)See Peretto (1999b) for more discussion of this point.
The same underlying force that eliminates the scale effect provides a third example of the effect of endogenous industrial organization on the economy’s aggregate behavior. In contrast to the first-generation literature, the growth rate here does not depend on the unit costs of production, $A_1$ and $A_2$. Unit production costs have no direct effect on R&D activity, which is governed by an unrelated production process. Unit costs of production do have indirect effects on R&D, but they cancel. One indirect effect works through the rate of return: eqs. (21) and (22) show that a decrease in unit costs causes an increase in the rate of return to R&D and hence also in the growth rate. The other indirect effect works through firm size: a decrease in unit cost causes a higher incipient profit and induces entry, which reduces firm size $l_i = L_i/N_i$ and hence reduces the return to R&D. The solution of the model shows that these two effects exactly cancel, leaving both firm size and growth rates unchanged.

A fourth effect is that growth rates are equal in the two countries no matter which country is endowed with more resources (labor) at any moment. This result is in sharp contrast to Grossman and Helpman’s (1991, Chapter 7) model in which the growth rate is higher in the country with more of the productive resource. The reason for the difference is that here firms’ market size is determined endogenously whereas in Grossman and Helpman it is exogenous.\textsuperscript{32}

Trade may raise or lower the home country’s growth rate, irrespective of what happens to the initial level of income. Equation (28) shows that the balanced growth rate depends only on R&D abilities and not on unit production costs or initial quality levels. When trade opens, the home country imports the good with a lower quality-adjusted price and possibly also with a lower associated R&D ability. In that case, trade decreases the growth rate because it shifts R&D to firms less efficient at doing it. This result arises from the nature of comparative advantage and the industrial organization of the economy. Comparative advantage is determined by the quality-adjusted price ratio and does not guarantee that the home country imports the good with a higher associated R&D ability. Indeed, condition (23) shows R&D ability is irrelevant to the determination of the trade pattern, which depends only on unit costs and the initial value of qualities at the moment that trade opens. Conversely, the factors that determine the trade pattern have nothing to do with determining the growth rate. Comparative advantage is based on what is cheap now. In contrast, growth is based on the R&D done to make things cheap in the future. The crucial difference between the present theory and that of the preceding literature on trade and growth is that here R&D is done by the firms that produce the existing goods, not by a separate R&D sector as in, for example, Grossman and Helpman (1990), and not simply the outcome of exogenous growth, as in Alvarez, Buera, and Lucas (2013). When a country shuts down production, it also shuts down related R&D, something that does not happen in models that posit an independent R&D sector or exogenous growth. Note also that in Alvarez, Buera, and Lucas’s (2013) model with scale effects and exogenous growth, increased openness always increases the growth rates of the trading partners, whereas in our framework that is not necessarily so. Two examples of comparative advantage leading to lower world R&D activity are the furniture and shipbuilding industries in the United States, both of which have been taken over by low-cost firms in foreign countries that generally also have lower R&D ability than the United States. Of course, trade can and often does reduce production costs and also increase growth rates by reallocating R&D resources to the most efficient users. The United States produces software cheaply and also does the best R&D on it. The United States trades software to Japan for automobiles, which the Japanese produce more cheaply and which the Japanese seem to be especially good at improving.

It is worth comparing these results to those emerging from the early (first-generation) endogenous growth models. Those models generally delivered growth results that were the analog of what one found for levels in exogenous "growth" models such as Solow and Cass. If something raised the level of output in an exogenous growth model, it generally would raise the growth rate in a first-generation endogenous growth model. So, in a Cass model, an increase in productive efficiency (total factor productivity) would

\textsuperscript{32}In contrast, our result can be regarded as an extension of Grossman and Helpman (1991, Chapter 8), in which the country that starts ahead in R&D output (variety expansion in their model) stays ahead. However, in Grossman and Helpman’s model, countries all have the same R&D efficiency. If R&D efficiencies differed across countries, as in the present model, Grossman and Helpman’s model would deliver the result that the R&D-inefficient country will become the world R&D producer if it happens to be ahead when trade opens.
raise the steady state level of output, whereas in an early endogenous growth model, the same increase would raise the balanced growth rate. Our results are quite different. In standard static models of trade, opening a country to trade raises the level of output. One might then expect that opening the economy to trade in our growth model would raise the growth rate. In fact, trade need not raise the growth rate, as we have just seen. The early growth models are too simple in their industrial structure, having a counterfactual aggregate scale effect that transforms level effects in exogenous growth models into growth rate effects. Our model is a member of the class of second-generation endogenous growth models that has no aggregate scale effect. That permits level and growth effects of a given change to be unrelated to each other. In our framework growth is driven by R&D efficiency, not productive efficiency. Increasing the efficiency of producing output has no bearing on the efficiency of doing R&D. Consequently, as we have shown, it is possible for trade to reduce growth, even though it may raise output initially.

We emphasize that the plethora of possibilities emerging from our model does not mean the model has nothing to say. Quite the contrary. The model gives specific conditions under which each possibility arises, and in principle those conditions are testable. The model thus is very rich in terms of both theory and empirical implications.

The foregoing results raise an issue in estimating the effects of trade on growth. According to our theory, trade’s effects on growth can be quite different from one country to another. Consequently, it may be very misleading to estimate "the" effect of trade on growth by looking at trade’s effect on world growth. Indeed, the data in Table 1 suggest that is the case. The table is taken from Bhalla (2002) and reports growth rates for the world and for various regions in the world for the years before and during the "era of globalization, which Bhalla defines as 1960-1980 and 1980-2000, respectively. We can do an event study of trade’s effect on growth by comparing growth rates before and after globalization. The world’s growth rate was a fifth of a percentage point higher after globalization than before, a modest but still economically significant positive effect of trade on growth. However, the data for several regions of the world tell a very different story. The growth rate for industrialized nations fell by 1.7 percentage points, whereas the growth rate for non-industrialized nations rose by 0.5 percentage points. Regional data show similarly striking differences. Asia’s growth rate, and especially China and India’s growth rate, rose dramatically in the globalization era. In every other region reported in Bhalla’s data, growth rates fell by over 1.5 percentage points. In every region that Bhalla reports, the absolute value of the change in the growth rate was far higher than it was for the world as a whole. Among all developing countries, the growth rate rose 1 percentage point after globalization, but for developing countries excluding China and India, the growth rate fell 1.7 percentage points. These data suggest enormous differences in the response of growth rates to trade with some of the responses being hugely negative. Furthermore, the negative effects are not confined to the less developed world. Indeed, the industrialized countries as a whole seem to have suffered the negative effects, whereas the non-industrialized countries reaped the benefits. More recently, Wacziarg and Welch (2008) report estimation results further supporting large cross-country differences in the effect of trade on growth, with about one-third of the countries in their sample showing negative effects of increasing openness that are significant both statistically and economically. Table 2 reproduces their results. Looking at the world as a whole averages out a great deal of cross-country variation and gives a misleading picture of both the magnitude and sign of regions’ or countries’ response to a change in openness. That is part of the reason for the prolonged debate in the literature over whether trade raises growth rates (apparently it does, on average) or lowers them (which it often seems to do). See, for example, Dollar (1993) for a review of the older literature and Sachs and Warner (1995), Rodriguez and Rodrik (2000), Warner (2003), and Rodriguez (2007) for some of the debate on trade’s growth effects. Our theory offers a possible explanation for a positive average cross-country effect of trade on growth and at the same time for both the wide range of magnitudes and the large number of negative signs for the individual country effects that have been reported in the literature. The theory thus offers a reconciliation of the two sides in the trade-growth debate. It seems that both sides make valid points.

33See Peretto (1998c) and Howitt (1999) for the prototypes.
Two game-theoretic aspects of the model and its solution require brief mention. First, firms in the country with superior R&D ability may want to continue doing R&D even if they cannot sell output now because they are out-competed by their foreign competitors with a current lower quality-adjusted price. Eventually they would increase their quality enough to reduce their quality-adjusted price below that of their competitors and recapture the market. To do that, they would have to borrow the funds to pay for the R&D because they currently have no sales and no earnings to retain. However, monopolistic competition and the zero profit condition (9) make it impossible for current firms ever to have excess funds to repay their debts, rendering the strategy infeasible. Second, there is no strategic behavior in this model. Firms do not condition their R&D behavior on what other firms are doing. In an oligopoly setting, strategic behavior would be possible. Unfortunately, it is not known how to solve such an oligopoly model of endogenous growth when strategic behavior is present, even for a closed economy. See Peretto (1996).

4.1.3 Transition dynamics

When the two countries satisfy the condition for complete specialization but are not on the balanced growth path, the growth rates of their incomes are equal:

\[ g = \frac{\dot{Y}_i}{Y_i} = \Gamma \frac{Z_{H1}}{Z_{H1}} + (1 - \Gamma) \frac{Z_{F2}}{Z_{F2}}; \text{ where } i = H, F \]

(29)

where \( \Gamma \) is a constant. The intuition is straightforward. When the two countries are completely specialized, each does R&D to improve the qualities of one of the two sets of intermediate goods. Each country imports the good that it does not produce. Consequently, each country uses the same sets of intermediate goods, one made at home and one made abroad, and so enjoys the same quality improvements. As a result, their growth rates are the same weighted average of the growth rates of the two qualities \( Z_{H1} \) and \( Z_{F2} \). We will see later that this property does not hold in the case of incomplete specialization.

We show in the Mathematical Appendix that the transition dynamics of the qualities under complete specialization are governed by the following differential equation:

\[
\frac{d(Z_{H1}/Z_{F2})}{dt} = -\alpha_{H1} \theta_{H1} \left( \frac{Z_{H1}}{Z_{F2}} \right)^2 + \alpha_{F2} \theta_{F2}
\]

(30)

The positive root \( \sqrt{\alpha_{F2} \theta_{F2}/\alpha_{H1} \theta_{H1}} \) is stable, and the economy converges monotonically to its balanced growth path. Growth rates change along the transition path as they approach the balanced growth rate, but the growth rates of income in the two countries always equal each other when the countries are completely specialized.

The equality of growth rates in the entire region of complete specialization strengthens Acemoglu and Ventura’s (2002) conclusion that trade equalizes growth rates in that region and Alvarez, Buera, and Lucas’s (2013) similar conclusion that trade equalizes growth rate asymptotically. Acemoglu and Ventura use a first-generation variety expansion model with scale effects, AK production, and no R&D driving quality improvement, and Alvarez, Buera, and Lucas use a model of exogenous growth. Our framework eliminates those limitations, but it still produces the result that growth rates are equal when countries specialize. The result thus is generalized substantially. We return to a comparison of our results with those of the previous literature when we examine the region of incomplete specialization.

4.1.4 Effective Technology Transfer

An interesting implication of (28) and (29) is that the growth rate with trade looks the same as if technology transfer had taken place even though in this model it never does. If each country adopts the manufacturing and R&D technology of the other country for the good at which the other country has a manufacturing comparative advantage, we get the same growth rate as in (28) and (29). Thus
trade by itself can produce an equalization of technology the same as if technologies were exchanged, a result reminiscent of factor-price equalization. Such "effective technology transfer" has an implication for the measurement of technology transfer and its growth effect. A large literature, started by Coe and Helpman (1995) and Coe, Helpman, and Hoffmaister (2008), finds that a country that trades with technologically advanced partners has a higher growth rate than a country that trades with less advanced partners. The usual interpretation is that trade facilitates technology transfer. Our analysis suggests such an interpretation may be unwarranted because trade itself can lead to the same growth rate results as technology transfer even when technology transfer does not occur.\footnote{Coe and Helpman (1995) are aware of effective technology transfer, remarking (p. 860): “The benefits from foreign R&D can be both direct and indirect. Direct benefits consist of learning about new technologies and materials, production processes, or organizational methods. Indirect benefits emanate from imports of goods and services that have been developed by trade partners. In either case foreign R&D affects a country’s productivity.”}

Effective technology transfer, together with our earlier result on negative spillovers, may explain some rather puzzling findings in the empirical literature on technology transfer. Several studies report that "forward spillovers" (in which the buyer learns from the supplier, as in our model) are negative.\footnote{See Koymen and Sayek (2010) and the large number of papers cited there.} That result is difficult to interpret in terms of technology transfer. Why would a country import technology that reduced productive efficiency? In our model, trade can lead to a drop in the current level of output and also to a drop in the growth rate of output. The results look as if a negative technology transfer has occurred even though no such thing has happened. If one restricts attention to technology transfer, as in the literature in question, one would interpret these outcomes as negative effects of technology transfer.

Effective technology transfer also is part of the reason our model gives somewhat different conclusions than Acemoglu and Ventura's (2002) on trade’s growth effect. We find that growth rates are equal everywhere in the region of complete specialization, whereas Acemoglu and Ventura find that growth rates differ except on the balanced growth path. In our model, complete specialization combined with effective technology transfer equalizes growth rates. In contrast, Acemoglu and Ventura do not have technical progress embodied in any traded goods, they prohibit trade in one factor of production (physical capital), and their model is a three-sector variant of the two-sector model. Those characteristics imply unequal growth rates except on the balanced growth path. The models thus are substantially different in structure with neither one obviously superior to the other. Empirical tests are needed to see which is more consistent with the data.

### 4.2 Incomplete specialization

Balance of trade requires that the relative price $P_{Y_F}$ (actually, $P_{Y_F}/1$) be inside the closed interval given in condition (16). Condition (23) shows that when the quantity $\frac{(1 - \epsilon) L_H / \epsilon L_F}{1 - \lambda}$ is inside that interval, $P_{Y_F}$ will equal it. However, there is no reason that $\frac{(1 - \epsilon) L_H / \epsilon L_F}{1 - \lambda}$ need be inside the interval. If it is outside, then $P_{Y_F}$ cannot equal it and will be at whichever boundary is closest to it. In that case, we have a corner solution. One of the two countries will be completely specialized, producing only one class of intermediates and trading for the other, and the other country will not be completely specialized but instead will produce both types of intermediate goods.

The incomplete specialization case has been given virtually no attention in the literature. When it is mentioned at all, it typically receives a few quick words acknowledging its existence and then is ignored. In fact, the behavior of the economy under incomplete specialization is quite different than under complete specialization and can explain aspects of the data that are inconsistent with the solution under complete specialization. To our knowledge, the analysis that follows is the first to examine the dynamics of the world economy when some countries do not completely specialize. Some of the results we obtain have no parallel in the previous literature.
Which country specializes depends on which boundary \( P_Y \) hits. The results are completely symmetric in the two possible cases, so without loss of generality we assume the following condition, which guarantees that the foreign country specializes and the home country does not:

\[
\left( \frac{(1 - \epsilon) L_H}{\epsilon L_F} \right)^{1 - \lambda} > \frac{A_{H2}}{A_{F2}} \left( \frac{Z_{F2}}{Z_{H2}} \right)^{(\delta + \gamma)(1 - \lambda)/\lambda} > \frac{A_{H1}}{A_{F1}} \left( \frac{Z_{F1}}{Z_{H1}} \right)^{(\delta + \gamma)(1 - \lambda)/\lambda}
\]  

(31)

In that case, \( P_{Y_F} \) "tries" to equal \([(1 - \epsilon) L_H/\epsilon L_F]^{1 - \lambda}\) and so hits the upper bound of the interval. We can see what that implies by rearranging terms:

\[
\frac{A_{H2}}{A_{H1}} \left( \frac{Z_{H1}}{Z_{H2}} \right)^{(\delta + \gamma)(1 - \lambda)/\lambda} = \frac{A_{F2}}{A_{F1}} \left( \frac{Z_{F1}}{Z_{F2}} \right)^{(\delta + \gamma)(1 - \lambda)/\lambda} = P_{Y_F}
\]

The expressions at the left and right extremes are the relative prices of the two classes of intermediate goods that would prevail under autarky in the home and foreign countries, respectively. When \( P_{Y_F} \) equals the left term, the world price equals the autarkic home price, indicating that the home country derives no price advantage from importing either good from abroad. In contrast, \( P_{Y_F} \) is larger than the autarkic foreign price, so the foreign country still finds it advantageous to specialize in intermediate good 2 and import good 1. In effect, the foreign country is not “technologically big enough” to satisfy the home country’s requirement for class-2 intermediates, i.e., \((1 - \epsilon) L_H/\epsilon L_F\) is too high relative to \( (A_{H2}/A_{F2}) \left( \frac{Z_{F2}}{Z_{H2}} \right)^{(\delta + \gamma)(1 - \lambda)/\lambda} \) at the moment that trade opens.

Recall that \((1 - \epsilon) L_H/\epsilon L_F\) is the ratio of the fraction of the home country’s labor force that uses the foreign-produced intermediate to the fraction of the foreign country’s labor force that uses the home-produced intermediate. It can be large for two reasons. First, the home country’s population can be large relative to that of the foreign country. That is a straightforward relative size effect. As mentioned earlier, it means that the foreign country is simply too small to meet the demands of the home country, so that the home country finds it worthwhile to continue producing intermediate good 2. This is not a scale effect. Increasing the size of the two countries’ populations equi-proportionally leaves everything unchanged, whereas shifting population from one country to another can move the world from the interior of the critical interval to the boundary (or vice versa) even if world population as a whole is unchanged. What matters is the relative size of the home country, not the absolute size. Second, the elasticities \( \epsilon \) and \( 1 - \epsilon \) also play a role in determining whether the world is inside the critical interval or at the boundary. Intuitively, if intermediate good 2 gets a heavy weight (i.e., \( 1 - \epsilon \) is high) in final good production, the more of it the home country wants. The final determination of whether the world is in the interior of the critical region or on its boundary depends on the interaction of the relative population sizes and the final good production elasticities.

4.2.1 Level Effect

The level effect on output of opening to trade is similar to but not exactly the same as that under complete specialization. The home country imports class-2 intermediates and hence gets \( Z_{F2} \) from the foreign country, which causes an externality to the home country’s processed goods industry \( X_{H1} \). Because the home country uses two types of intermediate good 2, the spillover to industry 1 is a combination of the two quality levels \( Z_{H2} \) and \( Z_{F2} \) given by \( Z_{H2} = Z_{H2}^{\eta} Z_{F2}^{1 - \eta} \). The home country’s final output under autarky and trade are

\[
Y_{H}^{\text{Autarky}} = \kappa \left[ \left( \frac{Z_{H1}^{(\delta + \gamma)} P_{G_{H1}}^{(\gamma)}}{P_{H1}^{(\gamma)}} \right) \left( Z_{H2}^{1 - (\delta + \gamma)} (\epsilon L_H) \right) \right]^{\epsilon} \left[ \left( \frac{Z_{H1}^{(\delta + \gamma)} P_{G_{H1}}^{(\gamma)}}{P_{H1}^{(\gamma)}} \right) Z_{H1}^{1 - (\delta + \gamma)} (1 - \epsilon) L_H \right]^{1 - \epsilon}
\]

\[
Y_{H}^{\text{Trade}} = \kappa \left[ \left( \frac{Z_{H1}^{(\delta + \gamma)} P_{G_{H1}}^{(\gamma)}}{P_{H1}^{(\gamma)}} \right) \left( Z_{H2}^{1 - (\delta + \gamma)} (\epsilon L_H) \right) \right]^{\epsilon} \left[ \left( \frac{Z_{F2}^{(\delta + \gamma)} P_{G_{F2}}^{(\gamma)}}{P_{F2}^{(\gamma)}} \right) Z_{H1}^{1 - (\delta + \gamma)} (1 - \epsilon) L_H \right]^{1 - \epsilon}
\]

36This relative size effect is similar to Acemoglu’s (2002) market size effect in his discussion of directed technical change.
where $\kappa'$ is a constant. Equilibrium requires that the domestic and foreign quality-adjusted prices $Z_H^{(\delta + \gamma)}/P_G^{\lambda/1-\lambda}$ and $Z_F^{(\delta + \gamma)}/P_G^{\lambda/1-\lambda}$ be the same. At the moment trade begins, the second term in the expressions for final output with and without trade are the same. The difference is the first term. In autarky, the spillover from industry 2 to industry 1 is $Z_H^{1-(\delta + \gamma)}$, whereas with trade, it is $(Z_H^{1-(\delta + \gamma)})^{1-(\delta + \gamma)}$. The necessary and sufficient condition for trade to increase initial output level for home is $Z_{F2} > Z_H$ and to decrease initial output is the reverse, $Z_{F2} < Z_H$. The level effect for the home country depends only on the externality because the home imports goods with the same quality-adjusted price as the domestic goods. The foreign country specializes in class-2 intermediates and imports good 1 from the home country. Thus the level effect is exactly like the effect under complete specialization. On the one hand, comparative advantage ensures a lower quality-adjusted price for the imports, and on the other hand, the quality of the imports can have a positive or negative externality on the other industry.

### 4.2.2 Balanced Growth Rate Effect

The balanced growth rate under incomplete specialization is

$$g^{\text{Trade}} = \frac{\delta}{1-\delta} \sqrt{\alpha_{H1}\theta_{H1} (\alpha_{H2}\theta_{H2})^\eta (\alpha_{F2}\theta_{F2})^{1-\eta}} - \frac{\rho}{1-\delta}$$  \hspace{1cm} (32)

Along the balanced growth path, the two quality ratios $Z_{H1}/Z_{H2}$ and $Z_{H2}/Z_{F2}$ are

$$\left(\frac{Z_{H1}}{Z_{H2}}\right)^* = \left(\frac{\alpha_{F2}\theta_{F2}}{\alpha_{H2}\theta_{H2}}\right)^{-1} \sqrt{\frac{\alpha_{F2}\theta_{F2}}{\alpha_{H1}\theta_{H1}}} \left(\frac{\alpha_{F2}\theta_{F2}}{\alpha_{H2}\theta_{H2}}\right)$$  \hspace{1cm} (33)

$$\left(\frac{Z_{H2}}{Z_{F2}}\right)^* = \frac{\alpha_{F2}\theta_{F2}}{\alpha_{H2}\theta_{H2}}$$  \hspace{1cm} (34)

which we will use in discussing the transition dynamics.

Comparing equation (32) with the autarky growth rate given in equation (26), we see the following effects of trade under incomplete specialization:

1. The home country’s growth rate increases if the foreign country has a higher R&D ability in the good that the home country imports, i.e., if $\alpha_{F2}\theta_{F2} > \alpha_{H2}\theta_{H2}$. The effect of $\alpha_{F2}\theta_{F2}$ is small if $(1-\eta)$ is small.

2. The foreign country’s growth rate increases if the home country has a higher R&D ability in the good that the foreign country imports, i.e., if $\alpha_{H1}\theta_{H1} > \alpha_{F1}\theta_{F1}$ and/or $\alpha_{H2}\theta_{H2} > \alpha_{F2}\theta_{F2}$. The home country’s R&D ability in class-2 intermediates enters the growth rate of the foreign country, even though the foreign country does not import that good. The reason is that $Z_{H2}$ affects the accumulation of $Z_{H1}$ which is embodied in the $G_{H1}$ good that the foreign country does import.

### 4.2.3 Transition Dynamics

The balanced growth path under incomplete specialization is only saddle-path stable, so the transition dynamics are especially interesting. To the best of our knowledge, some of the results we obtain are completely new and also of considerable practical significance.

As in the case of complete specialization, the growth rates of home and foreign income along the transition path are weighted averages of the growth rates of the quality levels:

$$\frac{Y_T^{\text{Trade}}}{Y_H^{\text{Trade}}} = \Gamma \left\{ \frac{Z_{H1}}{Z_{H1}} + \{\eta [1-(\delta + \gamma)] \epsilon + (\delta + \gamma) (1-\epsilon) \} \frac{Z_{H2}}{Z_{H2}} + \{(1-\eta)[1-(\delta + \gamma)] \epsilon \} \frac{Z_{F2}}{Z_{F2}} \right\}$$  \hspace{1cm} (35)
\[
\frac{Y^\text{Trade}_H}{Y^\text{Trade}_F} = \Gamma \frac{Z_{H1} - \delta \epsilon Z_{H2} + \eta \epsilon + \delta}{Z_{H1}} \frac{Z_{H2}}{Z_{F2}} + \left(1 - \epsilon + \delta\right) Z_{F2}
\]

where \(\Gamma\) is the same constant as before. Comparing these growth rates with the corresponding growth rates under complete specialization given in equation (29) reveals two notable differences. First, both countries’ income growth rates now are weighted averages of the three quality growth rates \(Z_{H1}/Z_{H1}\), \(Z_{H2}/Z_{H2}\), and \(Z_{F2}/Z_{F2}\), rather than just the two quality growth rates \(Z_{H1}/Z_{H1}\) and \(Z_{F2}/Z_{F2}\) that appear in (29). Second, the two countries’ income growth rates now differ from each other. Using (35) and (36), we get the difference between the two income growth rates:

\[
\frac{Y^\text{Trade}_H}{Y^\text{Trade}_F} = \frac{\eta \epsilon - \eta (\delta + \gamma) \epsilon + (\delta + \gamma)}{(\eta \epsilon - \eta (\delta + \gamma) \epsilon + (\delta + \gamma))} \left(\frac{Z_{H2}}{Z_{F2}} - \frac{Z_{H2}}{Z_{F2}}\right)
\]

where \(\eta \epsilon - \eta (\delta + \gamma) \epsilon + (\delta + \gamma) > 0\). The growth rates of income differ whenever the growth rates of \(Z_{H2}\) and \(Z_{F2}\) differ. This result contrasts with the vast majority of the literature, which restricts attention to the region of complete specialization and so obtains equal growth rates for countries that trade with each other (e.g., Taylor 1993, 1994, and Acemoglu & Ventura, 2002). Actual growth rates for trading partners often differ substantially and so are inconsistent with the usual result but are consistent with our extended theory that allows countries to be inside the region of incomplete specialization. We shall see momentarily that incomplete specialization can explain other important characteristics of the data as well.

Once again, getting the industrial organization right affects the growth rate implications of various changes in the economy. For example, in Grossman and Helpman’s (1990) Proposition 3, an increase in the population of the country that is relatively efficient in R&D raises the country’s growth rate. This same is true of Alvarez, Buera, and Lucas (2013, section 4.2). No such result holds in the present model. Under complete specialization, an increase in population has no effect on relative growth rates, which remain equal. The difference from Grossman and Helpman’s result arises because in their model R&D depends on the aggregate resource, whereas here it depends on the resource per firm, and endogenous entry keeps the resource per firm constant when population changes. In addition, an increase in population push the country into a corner solution and thereby may reduce the growth rate. When the country moves into the corner, it re-opens the industry that previously was shut down. If that industry is relatively inefficient at R&D, the country’s growth rate goes down because the growth rate is an average of all the growth rates for the three industries (home 1, home 2, and foreign 2). The crucial link is that in this model R&D activity is done by incumbents and so is tied to production activity. Firms that are active in production also are active in R&D and vice versa. Many first-generation growth models are of the creative destruction type, in which R&D is done by outsiders, a structure that is inconsistent with the facts. So on at least two levels - resource per firm rather than in the aggregate and in-house R&D rather than independent - the present model coincides with what we know about the industrial organization of R&D, in contrast to the structure of the first-generation models, and as a result obtains different implications for aggregate behavior.

To study the transition behavior of the incompletely specialized world economy, we need the equations for the relative growth rates of \(Z_{H1}, Z_{H2}\), and \(Z_{F2}\). The easiest way to proceed is to analyze the ratios \(u = Z_{H1}/Z_{H2}\), \(v = Z_{H1}/Z_{F2}\), and \(w = Z_{H1}/Z_{F2}\). Because \(v = u \cdot w\), the evolution of the world economy is described by the evolution of just \(u\) and \(w\). The Mathematical Appendix shows that the differential equations for \(u\) and \(w\) are nonlinear, so we linearize by Taylor expansion around the steady state values \(u^*\) and \(w^*\) to obtain

\[
\dot{u} = - \left[2\alpha_{H1}\theta_{H1}u^* (w^*)^{-\eta} \right] (u - u^*) - \left[\alpha_{H1}\theta_{H1} (1 - \eta) (u^*)^2 (w^*)^{-\eta} \right] (w - w^*)
\]

\[
\dot{w} = - \left[\frac{\epsilon}{(1 - \delta - \gamma) + \gamma} \left(\alpha_{H2}\theta_{H2}w^* - \alpha_{F2}\theta_{F2}\right)(w^*)^{-2} \right] (u - u^*) + \left[\frac{\delta}{(1 - \delta - \gamma) + \gamma} \alpha_{H2}\theta_{H2} (w^*)^{-1} \right] (w - w^*)
\]
where \( u^* \) and \( w^* \) are given by equations (33) and (34). The equilibrium loci \( \dot{u} = 0 \) and \( \dot{w} = 0 \) are

\[
\begin{align*}
\dot{u} &= -\left( \frac{1 - \eta}{2} \frac{u^*}{w^*} \right) w + (3 - \eta) u^* \\
w &= w^*
\end{align*}
\]

(40)  \hspace{1cm} (41)

The crucial variable turns out to be \( w \). The trade pattern condition (31) under incomplete specialization and the definition of \( w \) imply that the initial value of \( w \) must satisfy

\[
w > \left\{ \left( \frac{1 - \epsilon}{\epsilon L_Y} \right)^{1-\lambda} \frac{A_{H2}}{A_{F2}} \right\}^{\frac{\lambda}{\lambda + \eta (1 - \delta - \gamma)}}
\]

(42)

The evolution of the world economy depends on the relation between the initial value of \( w \) and the steady state value \( w^* \). In general, \( w^* \) can be above or below the right side of (42). Let us consider each possibility.

Suppose first that \( w^* \) is larger than the right side of (42). There are three possible cases depending on where the initial value of \( w \) is with respect to \( w^* \):

1. If \( w < w^* \), then \( \dot{w} < 0 \). At some finite time, \( w \) falls below the right side of (42). At that point, the economy switches to complete specialization. Its dynamics cease to be governed by (35)-(39) but instead become governed by (29)-(30) discussed earlier. We already have seen that the regime of complete specialization has a locally asymptotically stable balanced growth path, so once the economy crosses from incomplete to complete specialization, it remains completely specialized. Also, \( \dot{w} < 0 \) requires that \( Z_{F2} \) grows faster than \( Z_{H2} \), which in turn requires from (37) that \( Y_F \) grows faster than \( Y_H \).

2. If \( w = w^* \), then \( w \) is on its equilibrium locus and does not change, and \( u \) converges to \( u^* \). The world economy converges to a saddle-path stable balanced growth rate with perpetual incomplete specialization.

3. If \( w > w^* \), then \( \dot{w} > 0 \), and the world economy remains incompletely specialized forever. The difference of the growth rates of two countries converges to the constant

\[
\left( \frac{Y_H}{Y_H} - Y_F \right) \rightarrow \left( \delta + \frac{\delta^2}{\eta \epsilon (1 - \delta - \gamma) + \gamma} \right) \alpha_{H2} \theta_{H2} \frac{1}{u^*}
\]

The home country’s growth rate is perpetually above that of the foreign country, and the difference is bounded away from zero. The world goes asymptotically to a state in which the foreign country constitutes a vanishing fraction of world output. It is important to notice, though, that this result on the ratio of home to foreign output does mean that the foreign country is worse off over time or is worse off under free trade than under autarky. Trade may well raise the growth rate of the foreign country. It just does not raise it all the way to the growth rate of the home country. The foreign country therefore may grow faster under trade than under autarky yet still continue to fall behind the home country, just not as fast as it might have been falling behind under autarky.

Finally, suppose \( w^* \) is below the right side of (42). Then, if the world finds itself in a state of incomplete specialization when trade opens, it necessarily will be in case (3) above because incomplete specialization requires that (42) be satisfied. Figure 1 shows the phase diagram for the world economy under incomplete specialization when \( w^* \) is larger than the right side of (42).

These results stand in sharp contrast to those of Grossman and Helpman (1991, Chapter 9). In their model, the country that starts with a technological lead (i.e., higher \( Z \)) takes over all R&D asymptotically. Thus the balanced growth path is unstable in their model. In the present model,
comparative advantage guarantees that each country will produce something, and profit maximization then guarantees that the firms doing the production also will do R&D to improve quality, so it never happens that one country takes over world R&D. Furthermore, the balanced growth path in the interior is stable.

Our analysis shows an effect of growth on trade in the corner solution. Growth may push the world economy out of the region of incomplete specialization, or it may push it farther into that region, an outcome not possible in most of the previous literature. Both cases are interesting and correspond to observed phenomena. Which one prevails depends on the parameter values and the starting value of $w$. To the best of our knowledge, these results on the world economy’s dynamic behavior under incomplete specialization are completely new. As we now see, the case of incomplete specialization can explain the observed behavior of some countries’ growth rates.

Case 1 above offers a possible explanation for the growth behavior of Asia vis-a-vis the West over the last hundred years or so. Table 3 reports the growth rates for several world regions. Here, we take "the West" to mean the Table’s categories called "Total Western Europe" and "United States," and we take "Asia" to mean "Total Asia (excluding Japan)" and "Japan." The West initially was the "technologically large" country corresponding to the home country in our analysis and so was not specialized. Asia initially was the "technologically small" foreign country and was completely specialized (at least in tradable goods). Once significant trade between the two regions began, Asia’s growth rate jumped above the West’s growth rate but then began to approach it. Our theory suggests that the West and Asia still are in the region of incomplete specialization but are approaching its boundary with the region of complete specialization, with Japan being closer than the rest of Asia.

Case 3 above corresponds to Africa on the one hand and the rest of the world on the other. Table 3 shows that Africa’s growth rate has lagged behind that of Western Europe for the last 1,000 years, which seems about as good an approximation to the infinite horizon as one can expect to find in Earthly economic data. Africa’s behavior relative to the rest of the world is consistent with Africa being in that part of the incomplete specialization region in which its growth rate forever lags behind that of the more developed world.

Putting these two possibilities together offers a straightforward explanation for the emergence of "twin peaks" in the world income distribution, made famous by Quah (1997). One thousand years ago, countries of the world had much the same growth rates, barely greater than zero, so the world income distribution was approximately stable in the sense that countries’ relative positions were changing little. Since then, growth rates in most of the world have improved for a number of reasons (Clark, 2007; Maddison, 2001), but some regions have experienced faster growth than others. Also, some regions whose growth rates initially lagged behind the leaders’ rates later caught up, whereas other regions continued to lag. Our theory offers a possible explanation for the behavior of the catch-ups, who would be the countries (e.g., those in East Asia) that upon opening to trade found themselves in the region of incomplete specialization with growth rates unequal to those of the leaders but in that part of the region that leads to complete specialization, where growth rates are equal. Other countries (those in Africa) found themselves in that part of the region of incomplete specialization that leads deeper into that region, with growth rates permanently below those of the leaders. The original stable world income

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37For example, in Acemoglu and Ventura (2002) and Alvarez, Buera, and Lucas (2013) trade equalizes growth rates, at least asymptotically.
38Our analysis is consistent with and extends that of Ventura (1997). Ventura shows in a partial equilibrium analysis driven by a scale effect that trade can move a small open economy from low growth to high growth. Our model extends his analysis to a complete framework of dynamic general equilibrium without a scale effect, showing that Ventura’s main result survives and adding a complete description of the world’s transition dynamics.
39Our theory is not a complete explanation for Africa’s lag because the gap between the growth rates of the West and Africa has increased over time rather than converged to a constant. However, that shortcoming may result from a problem with virtually all the endogenous growth literature. World economic history does not show an asymptotic approach to a balanced growth path but rather if anything a divergence, at least over the last 1,000 years. Most existing growth theories generally cannot explain such behavior, with the possible exception of Peretto (2012). The increasing gap between African and other growth rates may be a manifestation of the growth in growth rates with Africa lagging behind.
distribution thus bifurcated into two groups of rich and poor countries. Recall that in our model quality improvement and cost reduction (productivity improvement) are isomorphic, so our explanation of the twin peaks in the world income distribution is consistent with Feyer’s (2008) finding that the twin peaks arise from twin peaks in productivity, not in availability of physical or human capital.

Figure 2 shows another way to picture the dynamic behavior of the economy. The horizontal axis is divided into three sections. The middle section is the region of complete specialization, denoted CS in the Figure, in which the home country does R&D on quality $Z_{H1}$ and the foreign country does R&D on quality $Z_{F2}$. Outside the CS region are the two regions of incomplete specialization, denoted IS in the Figure. In the IS region to the left of CS the home country completely specializes and does R&D on quality $Z_{H1}$, whereas the foreign country remains unspecialized and does R&D on both the qualities $Z_{F1}$ and $Z_{F2}$. In the IS region to the right of CS, the home country is unspecialized and does R&D on $Z_{H1}$ and $Z_{H2}$, whereas the foreign country is specialized and does R&D on $Z_{F2}$. It is the latter IS region that we have analyzed above. The boundaries of the regions depend on the quality ratios $Z_{F1}/Z_{H1}$ and $Z_{F2}/Z_{H2}$. When the quantity $[(1 - \epsilon) L_H / \epsilon L_F]^{1 - \lambda}$ is inside the CS region, as at point 1, the two quality levels $Z_{H1}$ and $Z_{F2}$ grow through R&D, and the two quality levels $Z_{H2}$ and $Z_{F1}$ are constant (because no R&D is done on them). The quality ratio $Z_{F1}/Z_{H1}$, and the quality ratio $Z_{F2}/Z_{H2}$ rise, causing the CS boundaries to spread farther apart. The quantity $[(1 - \epsilon) L_H / \epsilon L_F]^{1 - \lambda}$ remains inside the CS region, and the two economies remain completely specialized forever. Behavior is different when $[(1 - \epsilon) L_H / \epsilon L_F]^{1 - \lambda}$ is in one of the IS regions. For example, point 2 corresponds to the case analyzed above, where the home country is not specialized and the foreign country specializes in producing good $G_{F2}$. In that case, R&D is active for the three quality levels $Z_{H1}$, $Z_{H2}$, and $Z_{F2}$, so they all grow, and no R&D is performed on $Z_{F1}$, which therefore is constant. The lower (left) boundary of the CS region moves ever lower as time passes, but the movement of the upper (right) boundary depends on whether $w < w^*$ or $w > w^*$ (assuming for expository ease that $w^*$ itself is in the right IS region). If $w < w^*$, $Z_{F2}$ grows faster than $Z_{H2}$, and the upper boundary of the CS region increases over time, eventually passing point 1 and bringing the world into complete specialization. If $w > w^*$, $Z_{F2}$ grows more slowly than $Z_{H2}$, and the upper boundary of the CS region decreases over time, moving away from point 2 and leaving the world farther and farther inside the IS region. The possibility of technical progress changing the trading regime shows that growth can have a non-trivial effect on trade.

Finally, the behavior of growth rates under incomplete specialization is strikingly different from that under complete specialization. Under complete specialization, growth rates always are the same, even on the transition path. Under incomplete specialization, they always are different except on the knife-edge saddle-path-stable balanced growth path. Behavior in the region of incomplete specialization thus differs materially from that obtained by Acemoglu and Ventura (2002), who found that trade always equalized growth rates. The reason for the difference is that Acemoglu and Ventura restricted attention to the region of complete specialization. They assume that countries are endowed with sets of goods that they and no one else can produce, which is equivalent to assuming that countries always are completely specialized. Our results on incomplete specialization pertain to territory that Acemoglu and Ventura did not explore and thus complement and extend their analysis.

5 Welfare Analysis

We have seen that comparative advantage may raise or lower the level of output at the moment of that trade opens, so as in the standard static analysis, it is possible that trade reduces welfare because of the knowledge externality. We also have seen that trade may raise or lower output’s growth rate, and that can lower welfare through a dynamic effect apparently new to our analysis.

The home country’s flow utility (12) is

$$\log u_H^{Autarky} (t) = \log c_H (t) = \log Y_H (t) + \log \frac{c_H (t)}{Y_H (t)}$$
A little algebra gives

\[
\log u_{H}^{Autarky} (t) = \log \kappa'_{H} + \log L_{H} + (1 - \lambda) + \log \left[ \frac{Z_{H1}^{\delta+\gamma} (0)}{P_{G_{H1}}} \right] + (1 - \epsilon) \log \left[ \frac{Z_{H2}^{\delta+\gamma} (0)}{P_{G_{H2}}} \right] + \epsilon \left( 1 - \delta - \gamma \right) \log Z_{H2} (0) + (1 - \epsilon) \left( 1 - \delta - \gamma \right) \log Z_{H1} (0) + \Gamma \int_{0}^{t} g_{H1}^{A} (s) \, ds + (1 - \Gamma) \int_{0}^{t} g_{H2}^{A} (s) \, ds
\]  

Flow utility under trade with complete specialization is

\[
\log u_{H}^{Trade} (t) = \log \kappa'_{H} + \log L_{H} + (1 - \lambda) + \log \left[ \frac{Z_{H1}^{\delta+\gamma} (0)}{P_{G_{H1}}} \right] + (1 - \epsilon) \log \left[ \frac{Z_{F2}^{\delta+\gamma} (0)}{P_{G_{F2}}} \right] + \epsilon \left( 1 - \delta - \gamma \right) \log Z_{F2} (0) + (1 - \epsilon) \left( 1 - \delta - \gamma \right) \log Z_{H1} (0) + \Gamma \int_{0}^{t} g_{H1}^{F} (s) \, ds + (1 - \Gamma) \int_{0}^{t} g_{F2}^{A} (s) \, ds
\]

Let trade open at time \( t = 0 \). Assume that before \( t = 0 \) the home country is on its autarkic balanced growth path. At \( t = 0 \) the change in welfare is

\[
\Delta U_{H0} = \int_{0}^{\infty} e^{-\rho t} \log \frac{u_{H}^{T} (t)}{u_{H}^{A} (t)} \, dt
\]

where flow utility after trade relative to before trade is

\[
\log \frac{u_{H}^{T} (t)}{u_{H}^{A} (t)} = \log \frac{Y_{H}^{T} (t)}{Y_{H}^{A} (t)} = \log Y_{H}^{T} (t) - \log Y_{H}^{A}
\]

\[
= (1 - \epsilon) \log \left[ \frac{Z_{F2}^{\delta+\gamma} (0)}{P_{G_{F2}}} \right] - \log \left[ \frac{Z_{H1}^{\delta+\gamma} (0)}{P_{G_{H1}}} \right] + \epsilon \left( 1 - \delta - \gamma \right) \log Z_{F2} (0) - Z_{H2} (0) + \Gamma \int_{0}^{\infty} \left[ g_{H1}^{F} (s) - g_{H1}^{A} \right] \, ds + (1 - \Gamma) \int_{0}^{\infty} \left[ g_{F2}^{A} (s) - g_{H2}^{A} \right] \, ds
\]

The first term captures the standard static welfare gain from trade and is always positive because comparative advantage guarantees that foreign class-2 intermediates are cheaper in terms of quality-adjusted price at the moment that trade happens. The second term also is a static term. It captures the externality arising from the quality spillover across industries within the home country. Its sign is ambiguous, depending on whether the imported good’s quality is higher or lower than the domestic good it replaces. If it is negative, trade may reduce welfare. This second term arises here as a natural consequence of the nature of technical progress in our model, namely, labor-augmenting improvements embodied in physical capital, but it is nonetheless a static effect that owes nothing to the dynamic elements of our model. In sharp contrast, the last term is very different from the first two terms and is a new result emerging from our theory. It is a purely dynamic effect, capturing the change in the economy’s growth rate caused by trade. The third term has an ambiguous sign because the growth rates \( g_{1}^{T} (s) \) and \( g_{2}^{T} (s) \) can be either higher or lower than the balanced growth rate under autarky \( g_{H}^{A} \).

Whether trade raises or lowers the home country’s growth rate depends on whether the home country or the foreign country is more efficient at the R&D for good 2, i.e., whether \( \alpha_{F2} \theta_{F2} > \alpha_{H2} \theta_{H2} \). Even if the first two terms in equation (46) are positive, a sufficiently negative third term will mean that trade reduces welfare. The intuition is straightforward. Productive efficiency drives trade, but there is no
necessary relation between productive efficiency and R&D efficiency, as explained earlier. Productive efficiency gains may induce a country to stop producing goods for which that country has an advantage in doing R&D. In that case, trade reduces the growth rate of output even though it increases productive efficiency, and output in the open economy eventually will be lower than it would have been under autarky. If the crossing point comes early enough, the present value of lost future output will exceed the present value of increased output in the near term, and welfare will be reduced by trade.

The possibility that trade reduces welfare through a reduction in the economy’s growth rate is a type of dynamic inefficiency. It is different in nature from the kind of dynamic inefficiency originally discussed by Diamond (1965) in the context of an overlapping generations model. Diamond’s inefficiency is an intergenerational externality, arising from the current generation ignoring the costs it imposes on future generations by its investment decisions today. It is not present if people are altruistic toward their children (Barro, 1974), so it clearly is an intergenerational phenomenon. The dynamic inefficiency in the present model is completely different. It does not result from intergenerational considerations because there is only one infinitely-lived generation in the model. Rather, it is a cross-functional externality. Traders ignore the effects of their purchasing decisions on the R&D activity of the firms making the products that the traders decide to buy. Choosing a good also chooses the R&D efficiency associated with that good, but traders have no interest in the R&D efficiency and ignore it in making their purchasing decision. The result is that today’s purchasing decision affects tomorrow’s quality, but traders today do not see the connection because they have no market incentive to see it. The effect therefore is an externality. Being dynamic in nature, it obviously cannot arise in the standard static models of trade, so it shows clearly the contribution that a dynamic analysis brings to the evaluation of trade’s effects on welfare.

The case where the home country incompletely specializes after trade opens is straightforward. Under incomplete specialization, as discussed in section (3.3), the home country produces both goods and imports class-2 intermediates. The first term in (46) is zero because the quality-adjusted prices of class-2 intermediates are equal for the foreign and domestic goods. The sign of the second term is ambiguous as in the case of complete specialization. Qualitatively, the results are the same as under complete specialization.

In summary, then, trade may increase or decrease welfare on impact, may increase or decrease it over time, and may result in any possible pattern of these possibilities across the two trading partners. Which effect emerges depends on the relative R&D efficiencies of the two countries and on the structure of knowledge spillovers. Dynamic considerations therefore introduce the possibility of immiserising trade, a term we use somewhat hesitantly. It is deliberately reminiscent of Bhagwati’s (1958) immiserising growth, which by choice of words contrasts a possible bad outcome from something (i.e., growth) usually considered to be unquestionably good. Our result on trade and welfare has that same character. However, our mechanism is completely different from Bhagwati’s, having nothing to do with elasticities of demand and everything to do with knowledge externalities and R&D efficiency.

6 Conclusion

We have studied the interaction of trade and growth in the context of an endogenous growth model built to be consistent with several important facts about the nature of technical progress, the industrial organizational structure of the economy, and the nature of international trade. The analysis shows that trade affects growth and growth affects trade. The interaction of an endogenous industrial structure with aggregate general equilibrium dynamics is crucial for understanding the two types of effects.

Trade’s effects in a dynamic setting has interesting and sometimes surprising effects on the economy. Trade may raise or lower the initial income of one or both trading partners through knowledge externalities common in endogenous growth models. Trade also may raise or lower growth rates and thus may

40 Related inefficiencies are discussed by Krugman (1987) in a static, partial-equilibrium model and Redding (1999) in a first-generation growth model based on learning by doing that is restricted to the case of complete specialization.
effects on future income levels that reinforce or contradict the effect on initial income levels. These ef-
fects arise from the endogeneity of market structure. Under complete specialization and in some cases of
incomplete specialization, trade equalizes growth rates at least eventually and thus stabilizes the world
income distribution. In the remaining cases of incomplete specialization, the growth rate of the country
that is not completely specialized is always higher than the growth rate of its completely specialized
trading partner, and the difference between them asymptotically converges to a constant so that one
country forever grows faster than another. Trade can yield growth outcomes that mimic those arising
from technology transfers. Several phenomena in the data concerning the evolution of twin peaks in the
world income distribution of both levels and growth rates of income and concerning observed negative
effects of increased openness on growth rates are consistent with these theoretical outcomes.

Not only does trade affect growth, but growth affects trade. Technical progress can move the world
economy from one trading regime to another, shifting the pattern of specialization in production. This
result on regime switching apparently is new.

The welfare effects of trade in a dynamic setting may be strikingly different from what the standard
static analysis delivers. The dynamic effects of trade may reinforce the standard static effect of gains
from trade, may reduce it, or even may reverse it so that trade may actually reduce welfare. The
welfare effects on two trading partners may be of the same or opposite signs. The possible negative
effects arise from the static externality concerning knowledge spillovers and more importantly from the
dynamic inefficiency concerning the possible reallocation of world R&D resources from a country that
is relatively efficient at R&D to one that is relatively inefficient.

Finally, the model offers a single, unified explanation for a wide array of phenomena, including
frequently observed negative growth effects of trade, the catch-up of some countries to the industrial
leaders, and the emergence of "twin peaks" in the world distribution of national incomes.
References


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Figure 1: Phase diagram, incomplete specialization

Figure 2: Regions of specialization

\[ IS|_{Z_H, Z_F, Z_{F2}} \]

\[ CS|_{Z_{H1}, Z_{F2}} \]

\[ IS|_{Z_{H1}, Z_{H2}, Z_{F2}} \]

\[ \frac{A_{H1}}{A_{F1}} \left( \frac{Z_{F1}}{Z_{H1}} \right)^{\frac{(\delta+\lambda)(1-\lambda)}{\lambda}} \]

\[ \frac{A_{H2}}{A_{F2}} \left( \frac{Z_{F2}}{Z_{H2}} \right)^{\frac{(\delta+\lambda)(1-\lambda)}{\lambda}} \]
Table 1
Globalization & Growth
(annualized per capita growth rates, percentage points)

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Source: Bhalla (2002), Table 2.1

Table 2
Growth Rate Changes

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<th>Country</th>
<th>Growth Difference</th>
<th>Year of Liberalization</th>
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<td>1988</td>
<td>Israel</td>
<td>-0.96</td>
<td>1985</td>
</tr>
<tr>
<td>Ghana</td>
<td>1.99</td>
<td>1985</td>
<td>Botswana</td>
<td>-1.99</td>
<td>1979</td>
</tr>
<tr>
<td>Guinea</td>
<td>1.85</td>
<td>1986</td>
<td>Mexico</td>
<td>-2.16</td>
<td>1986</td>
</tr>
<tr>
<td>Guyana</td>
<td>1.80</td>
<td>1988</td>
<td>Hungary</td>
<td>-2.41</td>
<td>1990</td>
</tr>
<tr>
<td>Benin</td>
<td>1.74</td>
<td>1990</td>
<td>Guinea-Bissau</td>
<td>-2.95</td>
<td>1987</td>
</tr>
<tr>
<td>Mali</td>
<td>1.19</td>
<td>1988</td>
<td>Jordan</td>
<td>-4.28</td>
<td>1965</td>
</tr>
</tbody>
</table>

Source: Wacziarg & Welch (2008), Table 7

Table 3
Rates of Growth of GDP per Capita
(annual average compound growth rates, percentage points)

<table>
<thead>
<tr>
<th>Region</th>
<th>Years</th>
<th>1000-1500</th>
<th>1500-1820</th>
<th>1820-70</th>
<th>1870–1913</th>
<th>1913-50</th>
<th>1950-73</th>
<th>1973-98</th>
</tr>
</thead>
<tbody>
<tr>
<td>Western Europe</td>
<td></td>
<td>0.13</td>
<td>0.15</td>
<td>0.95</td>
<td>1.32</td>
<td>0.76</td>
<td>4.08</td>
<td>1.78</td>
</tr>
<tr>
<td>United States</td>
<td></td>
<td>0.36</td>
<td>0.34</td>
<td>1.82</td>
<td>1.61</td>
<td>2.45</td>
<td>1.99</td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td></td>
<td>0.03</td>
<td>0.09</td>
<td>0.19</td>
<td>1.48</td>
<td>0.89</td>
<td>8.05</td>
<td>2.34</td>
</tr>
<tr>
<td>Asia excluding Japan</td>
<td></td>
<td>0.05</td>
<td>0.00</td>
<td>-0.11</td>
<td>0.38</td>
<td>-0.02</td>
<td>2.92</td>
<td>3.54</td>
</tr>
<tr>
<td>Africa</td>
<td></td>
<td>-0.01</td>
<td>0.01</td>
<td>0.12</td>
<td>0.64</td>
<td>1.02</td>
<td>2.07</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Source: Maddison (2001), Table B-22.